Modification of the ICI Rule Based IF Estimator for High Noise Environments

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Abstract— **The non-parametric algorithm for instantaneous frequency (IF) estimation, based on the intersection of the confidence intervals (ICI) rule and the Wigner distribution (WD), is modified in order to produce an accurate IF estimate for a high noise environment. Original approach is developed under the assumption that a small amount of noise can move the WD maxima only within the auto-term. This is not true for high noise environments. Probability that the WD maxima are outside the auto-term is high for narrow windows used in the WD calculation. Estimates obtained with these windows are used as an initial guess in the adaptive algorithm. In this paper we set the initial estimate as the one produced by the narrowest window for which the probability of error due to high noise is smaller than a threshold. The error probability is estimated based on the estimates of signal amplitude and noise variance. The IF estimates for some windows are additionally improved by applying a median filter directly to the IF estimate.**

I. Introduction

Numerous instantaneous frequency (IF) estimators are used in practical applications such as communications, seismology, radars, sonars, biomedicine, speech processing, underwater acoustics, oceanography, (for details see [1], [2]). Many of them are based on time-frequency (TF) distributions [1]-[13]. Common non-parametric estimator is based on the positions of TF distributions maxima [1], [4], [8], [9], [13]. Here, we will consider the IF estimator based on the Wigner distribution (WD). Sources of error in the WD based IF estimator are analyzed in [8], [13], [14]:

- Bias caused by the IF higher order derivatives;
- Errors due to small noise that can move the WD maxima within the auto-term;
- Errors due to high noise that can move the WD maxima outside the auto-term.

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The bias and small noise influence have opposite behavior with respect to the window width [8]. Namely, the bias is increasing function of the window width, while the variance is a decreasing one. Therefore, the mean squared error (MSE) can be minimized by using an appropriate window width. A non-parametric algorithm for estimation of the optimal window width is proposed in [8]. It is shown that the IF estimate obtained by using the adaptive window width has smaller MSE than in the case of any constant size window from the considered set. Unfortunately, for high noise cases the values of WD maxima can be found outside the auto-term. Complexity of the non-parametric algorithm application, in this case, is additionally increased by the fact that probability of errors due to high noise is highest for the narrowest window from the set, which is used to produce the initial guess in the algorithm. If the initial guess is outside the auto-term, the non-parametric algorithm fails to produce acceptable accuracy. In order to treat that kind of signals, we will present a modification of the non-parametric algorithm.

Based on the estimate of the signal amplitude and noise variance, we estimated the probability of errors due to high noise. The initial guess in the algorithm is obtained by the window that produces probability of error smaller than a threshold. For such windows, when probability of errors due to high noise is not negligible, the IF estimation is improved by using the median filter. The WD could have very emphatic inner interferences for signals with highly nonlinear IF. Then, the WD maximum can be dislocated to the position of these interferences. In order to avoid the interference problem, the proposed algorithm can be applied to the reduced interference distributions. Note that the median filter is used for improvement of the IF estimation accuracy in

[15], [16]. The approach is extended in this paper to the adaptive window lengths in the IF estimation, by applying the ICI rule based algorithm. In this way the estimate robust to the high noise errors and the estimation bias is obtained.

The paper is organized as follows. Brief theoretical overview is given in Section II. Modified algorithm is presented in Section III, while numerical examples and statistical analysis are given in Section IV.

II. Theoretical Overview

A. Wigner Distribution and Signal Model

Consider a signal $f(t) = A\exp(j\phi(t))$ corrupted by a complex white Gaussian noise $\nu(t)$ with variance σ^2:

$$x(t) = f(t) + \nu(t). \tag{1}$$

Signal is sampled with the interval Δt, $x(n) = x(n\Delta t)$. Our goal is to estimate the IF, $\omega(n) = \phi'(t)|_{t=n\Delta t}$, by using the WD:

$$WD_N(n,\omega) = \sum_{k=-N/2}^{N/2-1} w_N(k)x(n+k)x^*(n-k)e^{-j2\omega k}, \tag{2}$$

where N is the window width, $w_N(k) = 0$ for $k \notin [-N/2, N/2)$. The IF estimate obtained by using the WD maxima is:

$$\hat{\omega}_N(n) = \arg\max_{\omega} WD_N(n,\omega). \tag{3}$$

The MSE of the IF estimation for moderate noise environment is approximately, [8]:

$$MSE(N;n) = \text{bias}^2\{\hat{\omega}_N(n)\} + \text{var}\{\hat{\omega}_N(n)\} \cong [bN^2\phi'''(n)]^2 + c\frac{\sigma^2}{A^2}(1+\frac{\sigma^2}{A^2})\frac{1}{N^3}, \tag{4}$$

where b and c include the parameters depending on the applied window function. Optimal value of the window width depends on the third phase derivative $\phi'''(n)$, and it cannot be determined from (4). An algorithm based on the ICI rule is proposed in [8] to determine a window width in the WD producing the MSE close to the minimal value. This algorithm produces accurate results for moderate noise environment. However, its original form cannot be used for high noise cases.

B. Errors due to the High Noise

The WD maxima can be found outside the auto-term when the noise amount is higher than a certain threshold [13]. Then, assumptions under which the original algorithm [8] is derived don't hold. For the linear FM signal, probability that the WD takes maximal value outside the auto-term position is [13]:

$$P_E(N) = 1 - \int_{-\infty}^{\infty} \left(1 - 0.5\text{erfc}\left(\frac{\xi}{\sqrt{2}\sigma_{WD}}\right)\right)^{N-1} \times e^{-(\xi - NA^2)^2/2\sigma_{WD}^2} d\xi, \tag{5}$$

where σ_{WD}^2 is variance of the WD, $\sigma_{WD}^2 = N\sigma^2(2A^2 + \sigma^2)$ [17], [18]. The probability of error (5) is high for small N, i.e., for narrow windows from the considered set. These windows are used for the initial guess in the algorithm.

Consider, for example, a signal whose amplitude is $A = 1$, variance of the noise is $\sigma^2 = 0.36$, and windows are of the width $N = 4, 8, 16, ..., 512$. The probability of error for the narrowest window is $P_E(4) = 60.48\%$, while for the next wider windows it is $P_E(8) = 23.48\%$ and $P_E(16) = 1.6\%$. For other windows from the set, the value of P_E is smaller than 0.01%. Thus, the first window produces error in more than half of the instants. It means that its application in the adaptive algorithm produces inaccurate initial estimate in more than a half of the instants. Therefore, the initial guess should be obtained by the next window, $N = 8$. However, this window also produces high probability of error, but accuracy can be improved by application of the median filter directly to the IF estimate [16].

This example illustrates the motivation for introducing a more accurate initial IF estimate in the algorithm. This estimate will be based on the value of $P_E(N)$. The modified algorithm is proposed in the next section, in order to deal with the high noise cases.

III. Adaptive Algorithm for the IF Estimation in High-Noise Environment

In order to avoid large bias in the initial estimate, the adaptive algorithm starts with the IF produced by the WD with a very narrow window. However, narrow windows are prone to errors due to high noise. Algorithm with inaccurate initial estimate cannot produce satisfactory final results. More sophisticated procedures are required for non-parametric IF estimation in the high noise environment.

An algorithm based on the graph theory for the IF estimation in extremely high noise environment is proposed in [19]. This algorithm uses two criteria: (a) The IF should pass through as high as possible values of the WD; (b) The IF estimate variations between two instants should be small. This algorithm is calculationally very demanding and, in addition, it cannot produce accurate results for signals with abrupt changes in the IF.

In this paper we will propose a modified algorithm based on the ICI rule. Two additional tools are used in the modified algorithm, in order to make it robust to the high noise influence: (a) The narrowest window, that can produce IF estimate satisfying minimal accuracy requirements, is determined based on the estimates of signal amplitude and noise variance; (b) Median filtering of the IF estimates is performed in order to reduce the impulse errors due to the high noise.

The algorithm for the IF estimation can be summarized as follows.

Step 1: Consider an instant $t = n\Delta t$ and a set of windows with dyadic widths N_i, $i = 0, ..., q$, where $N_i = 2^i N_0$ and $N_0 = 2^r$, $r \in \mathbf{N}^+$.

Step 2: Estimate the noise variance $\hat{\sigma}^2(N)$ and the signal amplitude $\hat{A}$[1].

[1]Estimation of the amplitude and variance can be done in numerous ways. Here we use a simple method: $\hat{A}^2 + \hat{\sigma}^2 = \frac{1}{N}\sum_{n=-N/2}^{N/2-1} |x(n)|^2$, $\hat{\sigma}^2 \simeq \left[\frac{\text{median}\{|\,\text{real}(x(n)-x(n-1))|, n\in[1,N)\}}{0.6745\sqrt{2}}\right]^2 + \left[\frac{\text{median}\{|\text{imag}(x(n)-x(n-1))|, n\in[1,N)\}}{0.6745\sqrt{2}}\right]^2$, $\hat{A}^2 \simeq \frac{1}{N}\sum_{n=-N/2}^{N/2-1} |x(n)|^2 - \hat{\sigma}^2$, $\hat{\sigma}(N) = \widehat{\text{var}}\{\hat{\omega}_N(n)\} = c\frac{\hat{\sigma}^2}{\hat{A}^2}(1 + \frac{\hat{\sigma}^2}{\hat{A}^2})\frac{1}{N^3}$.

Step 3: Based on the estimate of the amplitude and variance, the probability of error $\hat{P}_i = \hat{P}_E(N_i)$ is estimated.

Step 4: Determine the WDs for windows from the considered set, $WD_i(n,\omega) = WD_{N_i}(n,\omega)$, for $i = l, ..., q$ where N_l is the narrowest window satisfying $\hat{P}_l \leq p_1$.

Step 5: Estimate the IF by using the WDs:

$$\hat{\omega}_i(n) = \arg\max_{\omega} WD_i(n,\omega),\ i = l, ..., q. \quad (6)$$

The initial estimate is the one produced by N_l. Set $j = l$.

Step 6: For all windows that satisfy $p_0 \leq \hat{P}_i \leq p_1$ the IF estimate is modified by:

$$\hat{\omega}_{m,i}(n) = \text{median}\{\hat{\omega}_i(n), n \in [n-K, n+K]\}, \quad (7)$$

where $2K+1$ is the width of used median filter. Estimates, obtained by using windows for which $\hat{P}_i < p_0$, are not modified. For these windows $\hat{\omega}_{m,i}(n) = \hat{\omega}_i(n)$.

Step 7: If the statistical test inequality of the ICI rule based algorithm [21]:

$$|\hat{\omega}_{m,j+1}(n) - \hat{\omega}_{m,j}(n)| \leq (\kappa + \Delta\kappa)[\hat{\sigma}(N_{j+1}) + \hat{\sigma}(N_j)], \quad (8)$$

is satisfied, then set $j = j+1$ and repeat Step 7. If (8) is not satisfied, go to Step 8. Inequality (8) represents test if the confidence intervals constructed around the estimates $\hat{\omega}_{m,j+1}(n)$ and $\hat{\omega}_{m,j}(n)$ intersect. Then, with probability depending on $(\kappa + \Delta\kappa)$ the true IF will be within this intersection. An appropriate range of $(\kappa + \Delta\kappa)$ values is analyzed in [21], while detailed analysis of the algorithm behavior in various noise environments and fine adjustment of the algorithm parameters is given in [22].

Step 8: Adaptive IF estimate is $\hat{\omega}(n) = \hat{\omega}_{m,j}(n)$, while adaptive window width is $\hat{N}(n) = N_j$.

Step 9: Consider new instant and go to Step 4.

For additional improvement of the accuracy it is possible to apply the median filter to the resulting estimate $\hat{\omega}(n)$.

Note that the basic algorithm can be obtained if Steps 3, 4 and 6, are skipped and if $\hat{\omega}_j(n)$ are used instead of $\hat{\omega}_{m,j}(n)$, in Step 7.

Comments on the Algorithm: In practice the median filters are used for removing up to 50% of impulses in a sequence. However, the median filter does not produce satisfactory accuracy when it is applied to the IF estimation. Namely, the WD in the n-th and $(n+1)$-th instant uses $N-2$ samples which are the same. Thus, the IF estimate values are highly correlated, i.e., if the error in IF estimation exists in a considered instant, then it is highly probable that it will also exist in the neighboring instants. This is the reason why the probability threshold p_1 should be smaller than 50%. In our numerical examples we took $p_1 = 1/3$. Application of the median filter can introduce small error in the result when there are no errors due to high noise. Thus, in our simulations we set $p_0 = 0.1\%$.

For signals with non-linear IF function, the WD can have inner interferences. They could be very emphatic for wide windows. In order to improve accuracy for this application, we can use the proposed algorithm with the S-method, instead of the WD. The S-method is a reduced interference distribution proposed in [20]. It can produce signal auto-terms close to those in the WD, with significantly reduced interferences. In this application we used the following form of the S-method:

$$SM_N(n,\omega) = |STFT_N(n,\omega)|^2 + 2\,\mathrm{Re}\left\{\sum_{l=1}^{L} STFT_N(n,\omega + l\Delta\omega) \times STFT_N(n,\omega - l\Delta\omega)\right\}, \quad (9)$$

where $\Delta\omega$ is frequency step, L is width of the used frequency window, while $STFT_N(n,\omega)$ is the short-time Fourier transform:

$$STFT_N(n,\omega) = \sum_{k=-N/2}^{N/2-1} w_N(k) x(n+k) e^{-j\omega k}. \quad (10)$$

IV. Numerical Examples and Statistical Study

We consider signals of amplitude $A = 1$, embedded in the complex white Gaussian noise with values of the standard deviation within the interval $\sigma \in [0.05, 2.5]$. Windows of the width 4, 8, 16, 32, 64, 128, 256 and 512 are used. For $\sigma \in$ [0.05,0.20] probability of error obtained by using the narrowest window is less than 0.1%. The initial window is the narrowest one, without application of the median filter. This form then reduces to the case of nonparametric algorithm presented in [8]. For $\sigma \in [0.25, 0.35]$, the initial guess is produced by the narrowest window with median filter application. Window of the width $N = 8$ is used to produce the initial estimate for $\sigma \in [0.40, 0.50]$, $N = 16$ for $\sigma \in [0.55, 0.70]$, $N = 32$ for $\sigma \in [0.75, 0.90]$, $N = 64$ for $\sigma \in [0.95, 1.20]$, $N = 128$ for $\sigma \in [1.25, 1.50]$, $N = 256$ for $\sigma \in [1.55, 1.90]$, while for $\sigma \geq 1.95$ only window $N = 512$ produces satisfactory robustness to the high noise influence.

Three signals are used for numerical testing:

$$f_1(t) = \exp(jat|t|), \quad (11)$$

$$f_2(t) = \exp(jbt^3), \quad (12)$$

$$f_3(t) = \exp(jc|t|), \quad (13)$$

where $a = 128\pi$, $b = 256\pi$, $c = 128\pi$. The IFs are: $\omega_1(t) = 2a|t|$, $\omega_2(t) = 3bt^2$ and $\omega_3(t) = c\,\mathrm{sgn}(t)$. Signals are considered within $t \in [-0.5, 0.5]$ and sampled with the sampling rate $\Delta t = 1/512$. The IF estimate for signal $f_1(t)$ is shown in Figure 1. Columns represent results for noise of the variance $\sigma^2 = 0.5^2 = 0.25$, $\sigma^2 = 1$ and $\sigma^2 = 1.5^2 = 2.25$, respectively. Results obtained by using the original non-parametric algorithm are presented in the first row, while application of the proposed algorithm is given in the second row. Adaptive window width produced with the modified algorithm is given in the third row. The median filter, applied in the modified adaptive algorithm, is $2K+1 = 11$ samples wide. Graphical presentation of the same simulations for signals $f_2(t)$ and $f_3(t)$ are shown in Figures 2 and 3, respectively. Detailed statistical analysis is presented in Table I obtained with 100 realization of the noisy process. Columns of Table I represent the MSE produced by the constant-sized window from the considered set that gives minimal MSE, original non-parametric algorithm, original non-parametric algorithm with the median filter applied to the output and the proposed modified algorithm. It can be seen that proposed modification performs better than all the others, especially as the amount of noise increases. An average adaptive window width for cases from Figures 1-3 for 100 trials is shown in Figure 4.

Comments: Accurate IF estimation is done for signal $f_1(t)$ in a wide range of the noise standard deviation. It can be explained by the fact that this is the linear FM signal far from the origin $t = 0$. In this region algorithm selects wide window that is also resistant to the high noise influence. Also, the bias around origin does not disturb the estimation accuracy in this case.

Relatively high accuracy is achieved in the case of signal $f_2(t)$, as well. However, this signal is quite different from the linear FM model. Note that relationship for the probability of errors due to the high noise (5) is derived under the assumption of linear FM noisy signal. If the signal shape is known in advance, the exact probability of error could be determined. In the case of signal $f_2(t)$ the WD values along the IF are smaller than for the linear FM signal for the same amplitude of the input signal, but at the same time, width of the auto-term is larger than in the case of the linear FM signal. These two effects partially compensate each other. However, the probability of error in this case is higher than the one given by (5). This effect produces slightly worse accuracy than in the previous case.

The main problem in the case of the third signal $f_3(t)$ is a significant bias around instant $t = 0$ for wide windows. This effect is especially emphatic for high noise case with $\sigma = 1.5$. It is caused by the cross-term appearing between signal components in $t < 0$ and $t > 0$. Here, we applied the proposed algorithm to the S-method (9) with $L = 16$. Note that determination of the optimal value of L is discussed in [20]. The IF estimation performed by using the proposed algorithm and the S-method is depicted in Figure 5 for $\sigma = 0.5$, $\sigma = 1$ and $\sigma = 1.5$. Average window widths obtained in 100 trials are presented in Figure 6. The MSE of the estimates obtained in 100 trials for algorithm with the S-method is given in Table I, last three rows.

V. Conclusion

The modified version of the non-parametric algorithm for the IF estimation is presented. This algorithm can produce good accuracy for different signals in a high noise environment. Algorithm is slightly more complex than the original algorithm since it needs estimation of the probability of error due to high noise and median filter application.

VI. Acknowledgment

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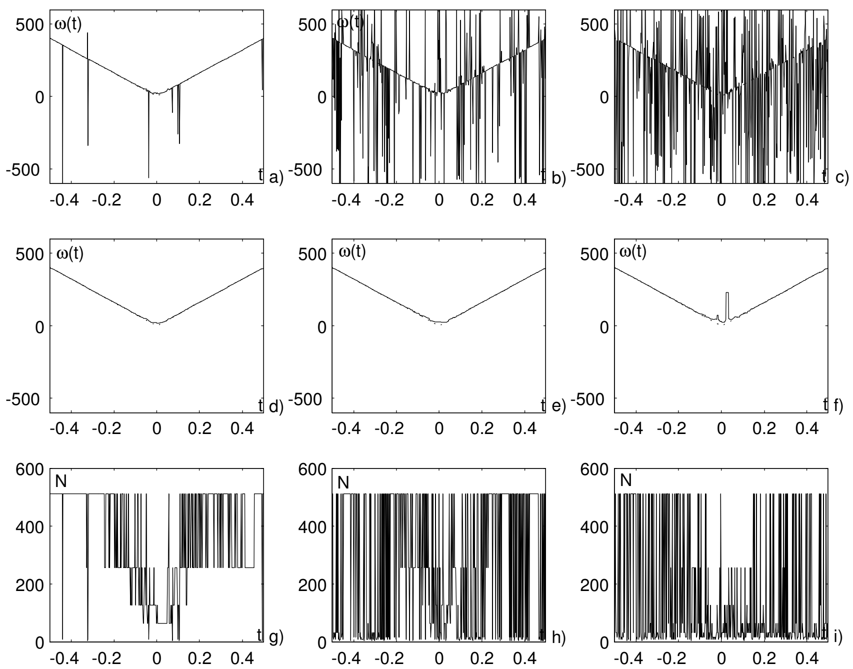

Fig. 1. IF estimation for signal $f_1(t)$: First column - $\sigma = 0.5$; Second column - $\sigma = 1$; Third column - $\sigma = 1.5$; First row - Original algorithm; Second row - Modified algorithm; Third row - Adaptive window width.

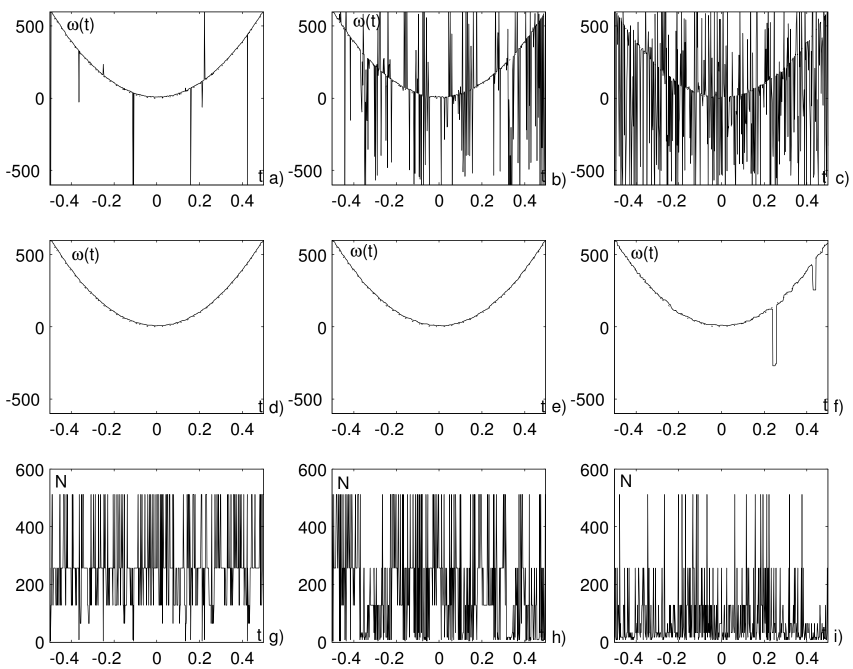

Fig. 2. IF estimation for signal $f_2(t)$: First column - $\sigma = 0.5$; Second column - $\sigma = 1$; Third column - $\sigma = 1.5$; First row - Original algorithm; Second row - Modified algorithm; Third row - Adaptive window width.

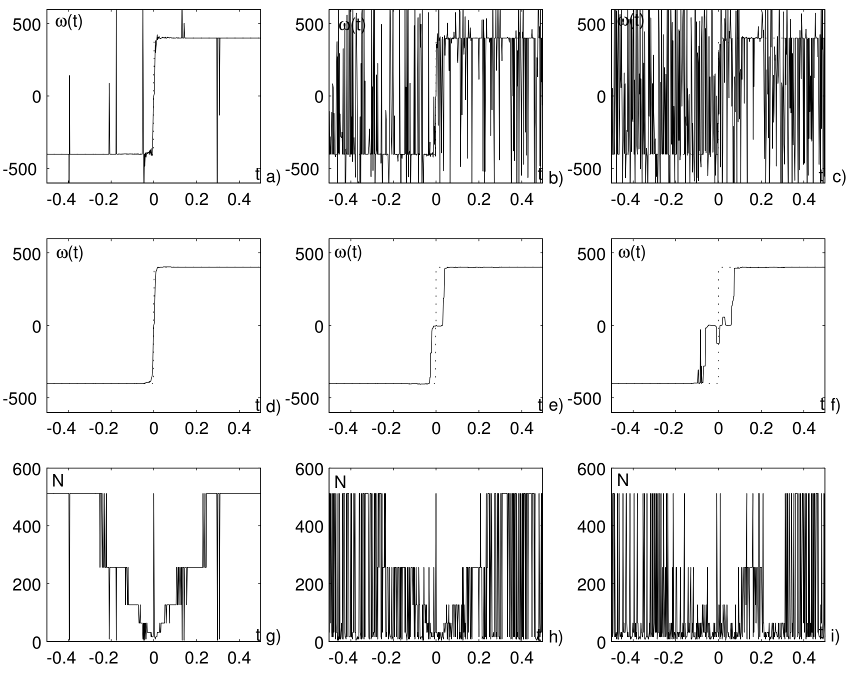

Fig. 3. IF estimation for signal $f_3(t)$: First column - $\sigma = 0.5$; Second column - $\sigma = 1$; Third column - $\sigma = 1.5$; First row - Original algorithm; Second row - Modified algorithm; Third row - Adaptive window width.

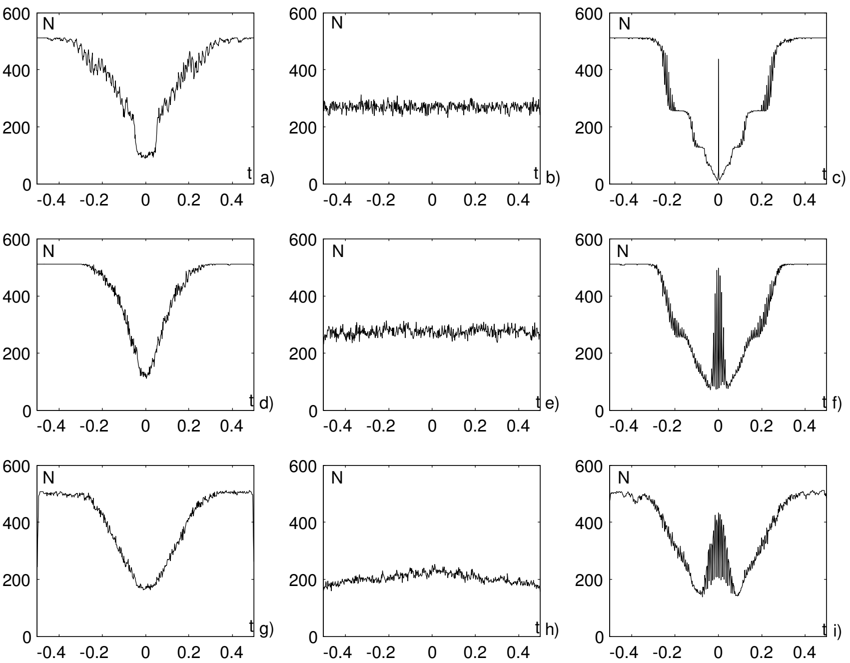

Fig. 4. Average adaptive window width obtained by 100 trials of modified algorithm: Columns - for signals $f_1(t)$, $f_2(t)$ and $f_3(t)$ respectively; Rows - standard deviations $\sigma = 0.5$, 1, and 1.5 respectively.

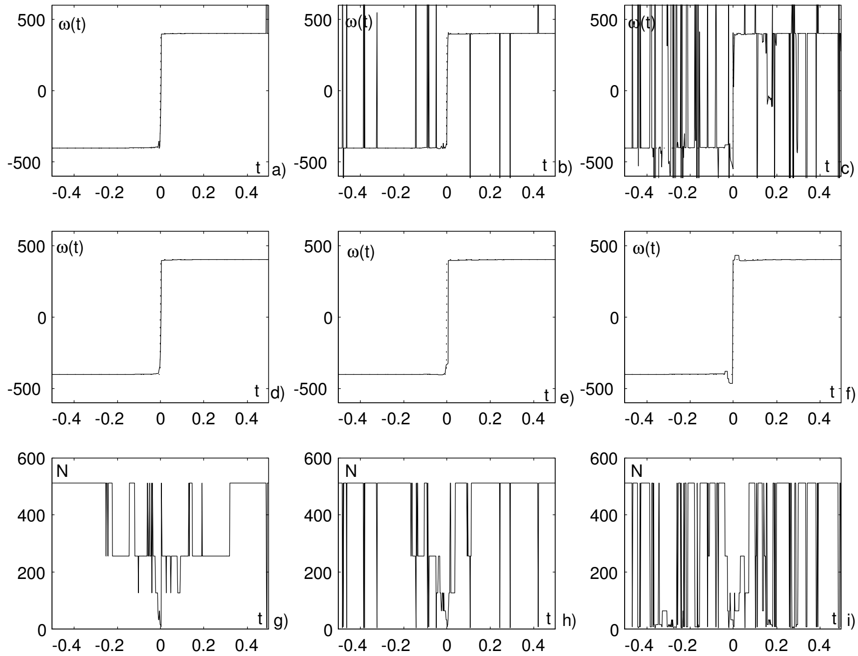

Fig. 5. IF estimation for signal $f_3(t)$ and S-method as the time-frequency representation: First column - $\sigma = 0.5$; Second column - $\sigma = 1$; Third column - $\sigma = 1.5$; First row - Original algorithm; Second row - Modified algorithm; Third row - Adaptive window width.

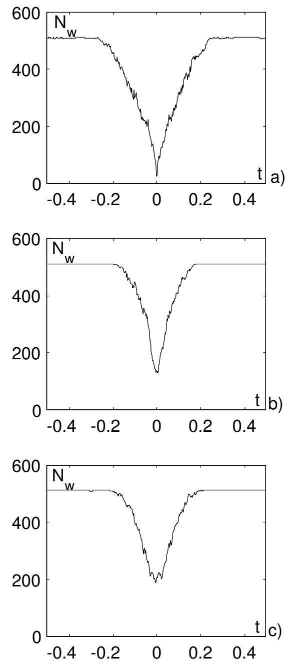

Fig. 6. Average adaptive window width obtained by 100 trials of modified algorithm applied to the S-method: Rows: standard deviations $\sigma = 0.5$, 1, and 1.5 respectively.

TABLE I

MSE OF THE IF ESTIMATE: σ - STANDARD DEVIATION; N_c - WINDOW THAT PRODUCES MINIMAL MSE; CONSTANT - MSE PRODUCED WITH N_c; ORIGINAL - MSE WITH ORIGINAL NON-PARAMETRIC ALGORITHM; ORIGINAL+MEDIAN - APPLICATION OF THE MEDIAN TO OUTPUT OF THE ORIGINAL NON-PARAMETRIC ALGORITHM, MODIFIED - MSE OF THE MODIFIED ALGORITHM. LAST THREE ROWS: IF ESTIMATION FOR SIGNAL $f_3(t)$ AND S-METHOD.

Signal	σ	N_c	Constant	Original	Original+Median	Modified
$f_1(t)$	0.5	64	8.50	359.4	11.11	7.96
	1.0	128	$1.52 \cdot 10^3$	$9.41 \cdot 10^3$	462.4	15.01
	1.5	256	$3.12 \cdot 10^4$	$1.78 \cdot 10^5$	$2.95 \cdot 10^4$	703.1
$f_2(t)$	0.5	64	63.32	$9.40 \cdot 10^3$	67.18	61.63
	1.0	128	398.2	$1.11 \cdot 10^5$	$6.97 \cdot 10^3$	62.64
	1.5	128	$7.45 \cdot 10^4$	$1.89 \cdot 10^5$	$3.98 \cdot 10^4$	$3.64 \cdot 10^3$
$f_3(t)$	0.5	16	$2.06 \cdot 10^3$	$7.86 \cdot 10^3$	628.47	627.09
	1.0	64	$1.86 \cdot 10^4$	$1.35 \cdot 10^5$	$1.36 \cdot 10^4$	$8.13 \cdot 10^3$
	1.5	256	$6.82 \cdot 10^4$	$2.64 \cdot 10^5$	$8.03 \cdot 10^4$	$2.11 \cdot 10^4$
$f_3(t)$	0.5	16	856.9	$2.20 \cdot 10^3$	400.90	440.7
	1.0	32	$3.22 \cdot 10^4$	$1.27 \cdot 10^5$	$6.67 \cdot 10^3$	$3.39 \cdot 10^3$
	1.5	64	$1.14 \cdot 10^5$	$2.40 \cdot 10^5$	$9.22 \cdot 10^4$	$1.04 \cdot 10^4$

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