Time-frequency based detection of fast maneuvering targets

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Abstract— **The time-frequency representation is a powerful tool for analysis of non-stationary signals. In the past decades, time-frequency representations have been primarily devoted to the analysis tasks in the sense that they were introduced so as to depict time-frequency structure of time-varying signals and non-stationary processes in the time-frequency plane. Also, there has been a permanent interest in tackling decision problems by means of time-frequency representations. In this paper, we present a time-frequency based detection scheme in the high-frequency surface-wave radar (HFSWR) for detection of maneuvering air targets in the presence of noise. Performance of the proposed method is evaluated by using both synthetic and experimental data. In addition, the proposed time-frequency detection scheme is examined in detail with various signal-to-noise ratio. This time-frequency based detection method is then compared to the Fourier-based detector. Results clearly demonstrate that the time-frequency based detector can significantly improve detection performance in the HFSWR, as well as add new physical insight, compared to the conventional Fourier-based detector, currently used by HFSWRs.**

I. Introduction

Traditionally, radar signals have been analyzed in either time or frequency domain. The Fourier Transform (FT) is at the heart of a wide range of techniques that are generally used in radar data analysis and processing. However, the change of frequency content with time is one of the main features we generally observe in radar data. Most of radar signals belong to the class of non-stationary signals. The analysis of non-stationary signals requires technique that extends the notion of a global frequency spectrum to a local frequency description. Joint time-frequency analysis, using time-frequency representations (TFR) or wavelet transforms, has improved the analysis of non-stationary signals by revealing time-varying information embedded in signals [1], [2].

During the past ten years, time-frequency analysis has been a major area of research in radar signal processing. One of the main challenges in radar detection is in the unknown nature of the target's motion. Commonly used technique for radar signal detection is a Fourier-based one, which assumes time invariance of the Doppler frequency. However, in real-world radar detection scenarios, when a target exhibits complex motion, such as acceleration or maneuvering, standard Fourier-based methods fail to reveal a complete picture of the temporal localization of the signal's spectral components. The TFR extend the fundamental concept of spectrum to non-stationary signals and facilitate time-varying spectral analysis by representing signal characteristics jointly in terms of time and frequency [1]-[7].

Detection of an unknown deterministic signal in a high noise environment is of crucial interest in many real-world applications. In the case of a stationary signal, for example, sinusoidal signal with constant frequency, the FT concentrates all the signal's energy at one frequency point, while the noise is distributed over many frequencies. Thus, it is easy to conclude that the FT-based detection method provides the optimal detection in the case of stationary signal and white noise, when noise components are distributed uniformly over whole frequency range. However, for non-stationary signals, i.e., when the frequency content of a signal changes over time, the spectral content of such signals becomes time-varying. Then, the FT-based detector will not provide the optimal result. In this paper, we present a time-frequency based detection scheme in the high-frequency surface-wave radar (HFSWR) for the detection of ma-

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neuvering air targets in the presence of high noise. The basic idea is to use a method that produces highly-concentrated energy of the desired signal around the instantaneous frequency (IF) and then to apply integration along the IF line. In the case of high noise, an algorithm for finding possible IF paths is proposed. In this way, the detection performance will be as high as in the case of constant frequency estimation using the FT method. The time-frequency based detection method is compared with the FT-based detector. The proposed method is then applied to the real radar signals with artificially added and increased noise.

The paper is organized in the following manner. Signal detection via the FT is reviewed in Section 2. Section 3 deals with the time-frequency tools in detection, including the definition of an algorithm for path finding. Efficiency of the proposed method is demonstrated in Section 4, along with its comparison to the FT-based detector and application to the real radar data. In Section 5, the conclusions are presented.

II. Signal Detection by using the Fourier Transform

Let us consider a single component deterministic signal with unknown discrete frequency ω_0

$$x(n) = Ae^{j\omega_0 n}, \tag{1}$$

and observation

$$s(n) = \xi \cdot x(n) + \varepsilon(n),$$

where $\varepsilon(n)$ is a complex zero mean Gaussian white noise with independent real and imaginary parts, with variance σ_ε^2. Variable ξ can take values $\xi = 0$ (absence of the signal $x(n)$) and $\xi = 1$ (presence of the signal $x(n)$). Suppose that there are N samples of the discrete signal $s(n)$. The finite FT of the signal $s(n)$ is

$$S(k) = \xi NA\delta(k - k_0) + \varepsilon_F(k), \tag{2}$$

where $\varepsilon_F(k)$ is also complex zero mean Gaussian white noise, with variance $\sigma_F^2 = N\sigma_\varepsilon^2$ and $k_0 = \omega_0 N/(2\pi)$. The expected value of the signal spectrum is

$$E[|S(k)|^2] = N^2|A|^2\delta(k - k_0) + N\sigma_\varepsilon^2 \text{ for } \xi = 1$$

$$E[|S(k)|^2] = N\sigma_\varepsilon^2 \text{ for } \xi = 0 \tag{3}$$

Now we can make decision about presence of the deterministic signal $x(n)$ in noisy samples $s(n)$ as

$$\hat{\xi} = \begin{cases} 1 \text{ for } \max[|S(k)|^2] > R_{S^2}, \\ 0 \text{ for } \max[|S(k)|^2] \le R_{S^2}, \end{cases} \tag{4}$$

where R_{S^2} is the threshold level. False alarm probability P_{FA} can be determined by analyzing the statistical properties of the noise, while magnitude of the signal $x(n)$ must be known in order to determine the probability of signal detection P_D. A common method for determining the threshold level R_{S^2} is a constant false alarm rate method (CFAR), where the probability P_{FA} is kept constant. In the analyzed case we have

$$\begin{aligned} P_{FA} &= P[|\varepsilon_F(k)|^2 > R_{S^2} \text{ for at least one } k] \\ &= 1 - P[|\varepsilon_F(k)|^2 \le R_{S^2} \text{ for every } k] \\ &= 1 - \prod_{k=0}^{N-1} P[|\varepsilon_F(k)|^2 \le R_{S^2}]. \end{aligned} \tag{5}$$

where $P[\cdot]$ denotes probability of event $[\cdot]$.

The square absolute value of complex random variable with Gaussian probability distribution is a random variable with Chi-square probability distribution with two degrees of freedom so we have

$$P_{FA} = 1 - \left(1 - e^{(-\frac{R_{S^2}}{N\sigma_\varepsilon^2})}\right)^N, \tag{6}$$

Now we can determine the threshold level R_{S^2}, which depends on the probability P_{FA},

$$R_{S^2} = -\ln(1 - \sqrt[N]{1 - P_{FA}})N\sigma_\varepsilon^2. \tag{7}$$

To determine the threshold level, the noise variance must be known. This variance can be estimated by using data samples of the signal $s(n)$ according to [3] as

$$\begin{aligned} \sigma_\varepsilon^2 \cong 1.1[&\underset{1\le i<N}{median}(|\operatorname{Re}[s(i) - s(i-1)]|)^2 + \\ &+\underset{1\le i<N}{median}(|\operatorname{Im}[s(i) - s(i-1)]|)^2]. \end{aligned} \tag{8}$$

In many cases, the discrete frequency of the deterministic signal does not satisfy the relation $\omega_0 = 2\pi k_0/N$, where k_0 is an integer. In these cases, when $\omega_0 \neq 2\pi k_0/N$, the detection result can be improved (probability P_D increased) by zero padding before the FT calculation.

If the deterministic signal $x(n)$ is non-stationary it can be written as

$$x(n) = A(n)e^{j\varphi(n)}, \qquad (9)$$

where $\varphi(n)$ is a nonlinear function. In this scenario, the Fourier-based detector is not the optimal one. When signals are non-stationary, the detection capability of the Fourier-based detector is limited. In these cases, the detection problem can be solved in a better way by using time-frequency analysis of the signal $s(n)$. Before we start time-frequency formulation, in order to better illustrate its efficiency, we will introduce an intermediate step, the parametric processing of non-stationary signals.

A. Parametric Processing Extension of the Fourier Transform

A non-stationary signal of the form $x(n) = A(n)e^{j\varphi(n)}$ can be processed by using a parametric form of the FT

$$X(k) = \sum_{n=0}^{N-1} A(n)e^{j\varphi(n)}e^{-j\psi(n;a_0,a_1,\dots,a_P)}e^{-j2\pi kn/n}, \qquad (10)$$

where N is the signal's duration and $\psi(n;a_0,a_1,...,a_P)$ is a function with P parameters. If we are able to match the form of $\varphi(n)$ with $\psi(n;a_0,a_1,...,a_P)$ and find the parameters $a_0,a_1,...,a_P$ such that $\varphi(n) = \psi(n;a_0,a_1,...,a_P)$ up to a linear phase term (constant frequency), then $e^{j\varphi(n)}e^{-j\psi(n;a_0,a_1,\dots,a_P)}$ would be a pure sinusoid. Then, all the conclusions valid for the FT based detector could be applied [15]. In order to match the form of $\varphi(n)$ we can apply instantaneous frequency estimation techniques, the fractional Fourier transform [11], [12] or discrete polynomial Fourier transform [13], if the signal phase is polynomial.

In order to illustrate these methods, which will lead to a non-parametric time-frequency based signal detection, consider the discrete short time Fourier transform

$$STFT(n,k) = \sum_{\tau=-N/2}^{N\ /2-1} w(\tau)x(n+\tau)e^{-j2\pi k\tau/N} \qquad (11)$$

with Hanning window $w(\tau)$ of discrete length $N = 32$ and $N = 256$ samples along the frequency axis. The spectrogram, a squared module of the STFT, will be used here as the TFR. We will consider two signals: a signal with linear frequency modulation, and a signal with sinusoidal frequency modulation. Signal-to-noise ratio in both cases is 0 dB.

Figures 1 and 2 illustrate the basic principle of the time-frequency based detection. Subplots in the first row in Figure 1 present TFR of the analyzed linearly frequency modulated signal, illustrating the paths in the time-frequency plane and the summation of TFR values along the paths. In this case, the paths are parallel to the time axis and the summation along these paths is proportional to the FT of the analyzed signal. The second row in Figure 1 presents the summation along the paths adjusted to the signal's instantaneous frequency, by determining the parameters of the linear FM (LFM) signal. The maximum value of the summation along the paths is higher in the second case. This distinctly demonstrates that the detection of these types of signals is simple and straight-forward using this approach.

Figure 2 illustrates the time-frequency based detection of a sinusoidally frequency modulated deterministic signal. The first row presents the summation along the paths equivalent to the FT, the second row presents the summation along the paths adjusted to the LFM signal from the previous figure, and the third row presents the summation along the paths adjusted to the instantaneous frequency and parameters of the analyzed signal. The maximum value is obtained when the path coincides with signal's instantaneous frequency. The proposed detection method is based on searching the best path (with the maximum summation of the TFR values along the path).

The above example shows that the detection of non-stationary signal can only be considered

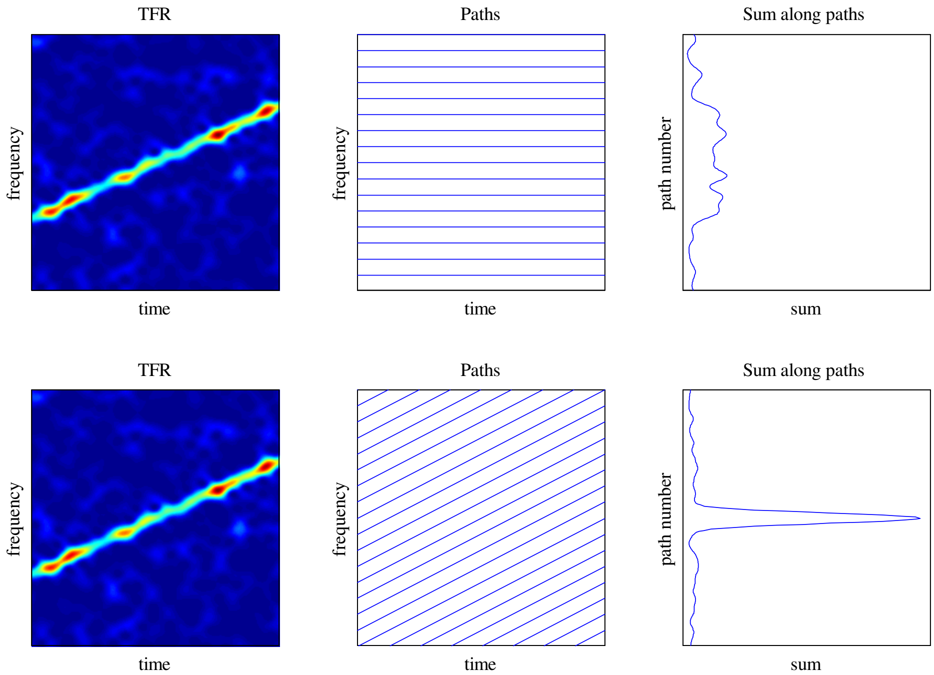

Fig. 1. Detection example - linear FM signal. First row - paths parallel to the time axis, equivalent to FT-based detector. Second row - paths parallel to the instantaneous frequency, equivalent to the time-frequency detector. First column - TFR. Second column - paths. Third column - sum of the TFR values along the paths.

by FT tools if we know the signal form and we are able to adjust the signal parameters to the instantaneous frequency. However, in practice the signal form is not *a priori* known and the parametric approach to these types of applications is quite limited. We will next show that similar principles can be used in non-parametric formulation of the detection, without using *a priori* knowledge about the signal form.

III. Signal Detection by using Time-Frequency Analysis

For signals whose spectral content varies over time, time-frequency distributions are introduced in order to obtain energy distribution over time and frequency. The basic quadratic time-frequency distribution is the Wigner distribution (WD) [1]. The discrete WD is defined as

$$WD(n,k) = \sum_{\tau=-N/2}^{N/2} w(\tau)w(-\tau)x(n+\tau)x^*(n-\tau)e^{-j4\pi k\tau/N}, \quad (12)$$

where $w(\tau)$ is time window. The WD has the best concentration among quadratic distributions [1], [14]. However it can not be used in practice due to the very exhibited cross-terms.

A method which is based on the idea of preserving auto-terms as in the WD, with elimination or significant reduction of the cross-terms, is introduced as the S-method (SM) [16]. It has been derived based on the relationship between the STFT and the pseudo WD

$$WD(n,k) = \sum_{i=-N/2}^{N/2} STFT(n,k+i)STFT^*(n,k-i) \quad (13)$$

This relation has led to the definition of a time-

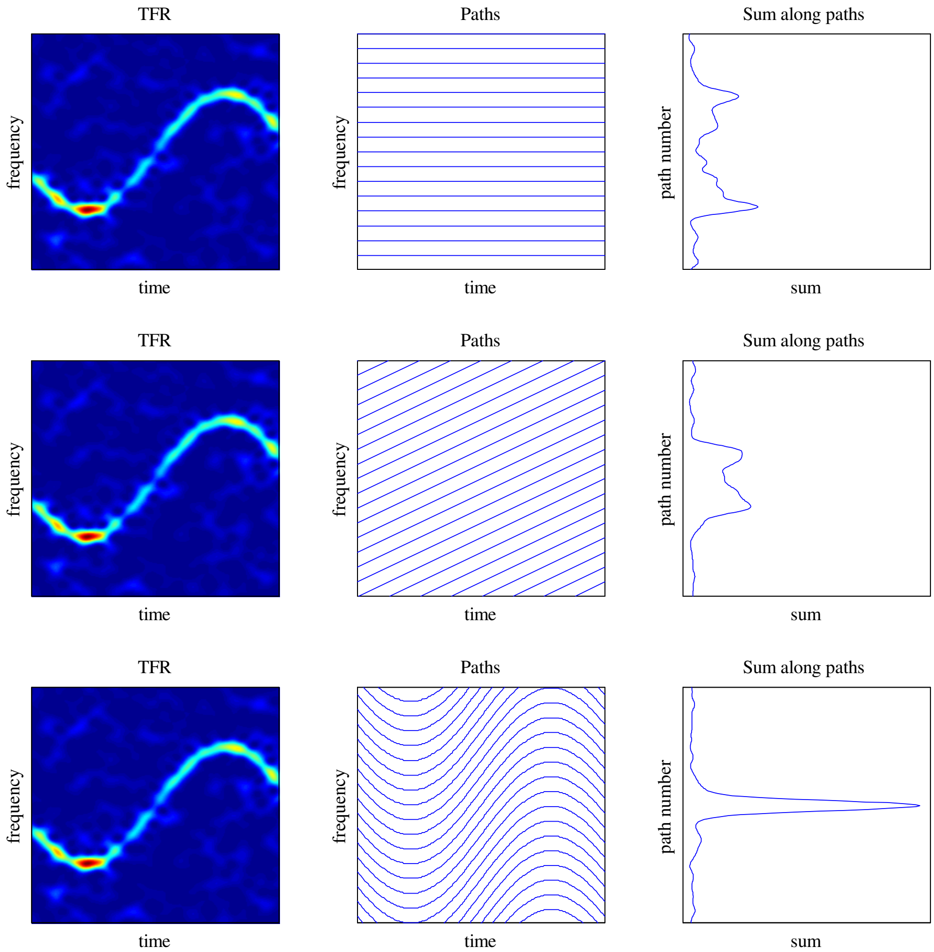

Fig. 2. Detection example - sinusoidal FM signal. First row - paths parallel to the time axis, equivalent to FT-based detector. Second row - paths used in the LFM example. Third row - paths parallel to the instantaneous frequency, equivalent to the time-frequency detector. First column - TFR. Second column - paths. Third column: sum of the TFR values along paths.

frequency distribution [16]

$$SM(n,k) =$$

$$= \sum_{i=-L_d}^{L_d} P(i) STFT(n,k+i) STFT^*(n,k-i)$$

$$= |STFT(n,k)|^2 +$$

$$+2\,\mathrm{Re} \sum_{i=1}^{L_d} P(i) STFT(n,k+i) STFT^*(n,k-i) \tag{14}$$

where $P(i)$ is a finite frequency domain window (we also assume rectangular form), $P(i) = 0$ for $|i| > L_P$. Two special cases are: the spectrogram $P(i) = \delta(i)$ and the pseudo WD $P(i) = 1$. Distribution obtained in this way is referred to as the S-method [1], [16].

The S-method can produce TFR of a multicomponent signal such that the distribution of each component is its WD, avoiding cross-terms, if the STFTs of the components do not overlap in the time-frequency plane. For $x(n) = \sum_{m=1}^{M} x_m(n)$ the S-method has the form

$$SM(n,k) = WD_{at}(n,k) =$$

$$\sum_{m=1}^{M}\sum_{\tau=-N/2}^{N/2} w(\tau)w(-\tau)x_m(n+\tau)$$
$$\times x_m^*(n-\tau)e^{-j4\pi k\tau/N}$$

where $WD_{at}(\omega,t)$ denotes the sum of the pseudo WDs of the individual signal components (without cross-components terms).

A. *Time-Frequency based Signal Detection*

Let us consider a single component linearly frequency modulated signal of the form

$$x(n) = Ae^{jan^2}. \quad (15)$$

Let $S(n,k)$ denote TFR of the signal $s(n)$. Figure 3 shows the FT of the signal $s(n)$ for the cases $\xi = 1$ (first plot from the left) and $\xi = 0$ (last plot). TFR of the signal $s(n)$ for both cases is also shown in Figure 3. The S-method with rectangular window and $L = 12$ is used as the TFR of the signal [3]. The number of samples is $N = 256$, the noise variance is $\sigma_\varepsilon^2 = 4$, and the signal amplitude $A = 1$, so that the signal-to-noise ratio is approximately -6 dB. Reference level of the FT-based detector is calculated with the false alarm probability $P_{FA} = 0.0027$, and is shown in the first and last plots. The same realization of the noise is used for all four plots. Figure 3 clearly shows the limited ability of the FT-based detector. On the other hand, it is easy to see if a deterministic component $x(n)$ exists in the TFR of the signal $s(n)$.

This example shows that the time-frequency analysis can be used for non-stationary signal detection in the presence of a strong noise. The basic problem is to automate the signal decision procedure if the analyzed TFR contains a deterministic component. An algorithm for the signal detection in the arbitrary TFR is described below.

Let us consider TFR $S(n,k)$ of the signal $s(n)$, where $k = 0,1,\dots,M-1$ and $n = 0,1,\dots N-1$. Assume that the instantaneous frequency of a deterministic signal $x(n)$ is a continuous function. We define a path in the time-frequency plane as an array of N frequency indices $\pi(t)$, with $0 \le \pi(t) < N$ for every t. We then observe the ensemble of such paths having the property $|\pi(t) - \pi(t-1)| \le D$ for some specified value D and for $t = 1,2,\dots,N-1$. The value of D is the maximal allowed frequency index change for two consecutive time instants, or allowed frequency step. We then observe one path $\pi_m(t) \in \Pi_D$ and sum the TFR values along this path. That is,

$$J_m = \sum_{t=0}^{N-1} S(t,\pi_m(t)). \quad (16)$$

Denote the maximum of the observed sum over the ensemble Π_D as $J_{\max}$,

$$J_{\max} = \max_{\pi_m\in\Pi_D} J_m = \sum_{t=0}^{N-1} S(t,\pi_{\max}(t)), \quad (17)$$

where $\pi_{\max}$ is the best path. The quantity defined in this way represents a reliable criteria for determining the deterministic component existence in the TFR $S(n,k)$. Namely, if $J_{\max} > R_J$ holds, where R_J denotes the threshold level, it can be concluded that the deterministic component exists in the signal $s(n)$; in other words, $\hat{\xi} = 1$.

The basic problem in detector realization is in the threshold level R_J determination. In the case of the second order TFR, we can assume that the level R_J is proportional to σ_ε^2. For a specific TFR, known noise distribution and for a chosen probability P_{FA}, it is sufficient to determine the threshold level R_0 when the false alarm probability is equal to P_{FA} for the noise variance $\sigma_\varepsilon^2 = 1$. The threshold level for non-unity noise variance can be calculated as $R_J = R_0\sigma_\varepsilon^2$. The determination of R_0 in this way, demands the processing of many noise only data if P_{FA} is significantly small. However, this procedure should be used only once for a given TFR and for a given P_{FA}. The algorithm for the threshold level R_0 determination is described below.

1. Choose TFR, probability of false alarm P_{FA}, and the maximum allowed frequency step D.
2. For $i = 1,2,\dots,M_i$ where M_i is number of iterations
 (a) Take a realization of noise only signal $s(n)$ with unity noise variance $\sigma_\varepsilon^2 = 1$.
 (b) Calculate $S(n,k) = TFR[s(n)]$.

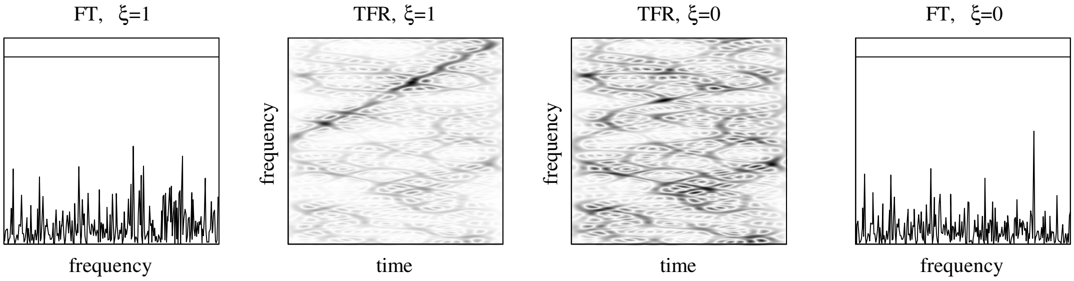

Fig. 3. FT (with threshold level) and TFR of the signal $s(n)$ in the cases of the deterministic signal $x(n)$ existence ($\xi = 1$) and non-existence ($\xi = 0$).

(c) Find the best possible path $\pi_{\max}(t)$ and calculate

$$J_{\max}(i) = \sum_{t=0}^{N-1} S(t, \pi_{\max}(t)). \quad (18)$$

3. Calculate the threshold level R_0 such that in $M_i P_{FA}$ iterations we obtain $J_{\max}(i) > R_0$ (and $J_{\max}(i) < R_0$ in the remaining $M_i(1 - P_{FA})$ iterations).

Second problem is in specifying the number of path ensembles Π_D and in the determination of the best path. In order to decrease the total number of paths (of the order N^M), we can apply the following approach:

1. For each time index n, find the maximum $S_{\max}(n)$ and the position of maximum $k_{\max}(n)$

$$S_{\max}(n) = \max_k S(n, k), \quad (19)$$
$$k_{\max}(n) = \arg\max_k S(n, k).$$

Assume that the best path $\pi_{\max}(n)$ passes through at least one of the selected maxima.

2. For each time index $n \in \{1, 2, \ldots, N\}$, form the path $\pi_n(t)$ starting at the point $(n, k_{\max}(n))$ in the time-frequency plane.

(a) Put point $(n, k_{\max}(n))$ into path: $\pi_n(n) = k_{\max}(n)$.

(b) For $t = n+1, n+2, \ldots, N$, the path point is

$$\pi_n(t) = \arg\max_{k \in K} S(t, k), \quad (20)$$

where set K includes frequency points $K = \{k | \pi_n(t-1) - D \le k \le \pi_n(t-1) + D\}$. Note that this procedure limits the frequency step between two consecutive time instants to D.

(c) For $t = n-1, n-2, \ldots 1$, the path point is

$$\pi_n(t) = \arg\max_{k \in K} S(t, k), \quad (21)$$

where $K = \{k | \pi_n(t+1) - D \le k \le \pi_n(t+1) + D\}$.

3. Calculate the sum $J_n(i) = \sum_{t=0}^{N-1} S(t, \pi_n(t))$.

4. The best path is the path with maximum $J_n(i)$.

Note that the number of analyzed paths in this procedure is equal to N. The number of paths can be further decreased if, in step 2 of the previous procedure, we perform the path forming only if the starting point $(n, k_{\max}(n))$ is not included in any of the previous analyzed paths $\pi_{n-1}(t), \pi_{n-2}(t), \ldots, \pi_1(t)$. This can de done because if the point $(n, k_{\max}(n))$ belongs to the path $\pi_p(t)$ for some p then the path $\pi_n(t)$ coincides with the path $\pi_p(t)$. Also, note that in step 2 of the previous procedure we can process time instants n in an arbitrary order. We suggest that the value of $S_{\max}(n)$ determines the order of processing the time instants. That is, we process the time instant with the highest $S_{\max}(n)$ first and the time instant with the smallest $S_{\max}(n)$ should be the last one processed. This re-ordering can not change the best path, but can change the total number of analyzed paths.

Figure 4 presents the threshold estimation for the spectrogram with 32-point Hanning window. It should be noted that $J_{\max}$ in Figure 4 is plotted in a descending order. Signal length is 256 samples and the STFT is calculated over 256 frequency bins. The probability of false alarm is $P_{FA} = 0.0027$. The threshold

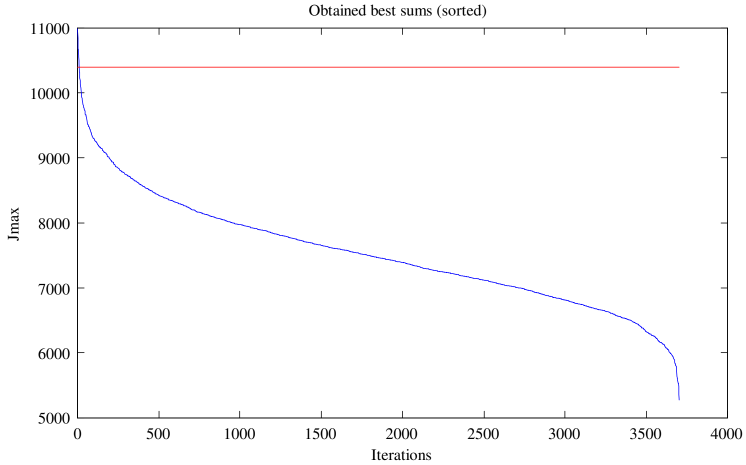

Fig. 4. Threshold estimation - The best acheived values of the path sum for 3704 realizations of random signal without deterministic component. The horizontal line represents the threshold $R_0 = 10397$ obtained with $P_{FA} = 0.0027$.

is estimated by analyzing the best paths in the noise only case with $\sigma_\varepsilon^2 = 1$. The number of realizations is $M_i = 3704$. The threshold is determined according to the number of expected false alarms $M_i \cdot P_{FA} = 10$. In this way, we can implement the desired probability of false alarms.

Figure 5 presents histogram of the number of analyzed paths in the procedure for searching the best path in the noise only case. The mean number of the analyzed paths per realization is 12.

B. Comparison with Fourier-Based Detector

Performance of time-frequency based signal detectors, with their comparison to the Fourier-based detector, are shown in Figure 6. The case of a linearly frequency modulated signal is considered

$$x(n) = e^{j\frac{\pi}{128}(n-128)^2}, \tag{22}$$

for $0 \leq n \leq 256$ in the presence of additive Gaussian white noise. For each SNR, 200 realizations are observed and the detection is performed by using the FT and the S-method with $L = 4$, $L = 16$ and $L = 32$. The dependency of the probability P_D on signal-to-noise ratio is shown in Figure 6. As we expected, the S-method with large enough L is a good signal detector, even if the signal-to-noise ratio is small. The false alarm probability for all analyzed detectors is $P_{FA} = 0.0027$. It should be noted that the threshold estimation for the Fourier-based detector is very simple compared to time consuming threshold estimation for proposed detector.

IV. Application to the Real Radar Data

Let us now consider detection of target signals in experimental high-frequency surface-wave radar system. Suppose that the target velocity is high enough so that the sea-clutter can be removed by high-pass filtering.

A. Stationary Case

For stationary targets, the FT is the optimal detector. Let us now consider a single range cell with the signal representing that range cell denoted by $x(n)$. The detection algorithm in the case of a constant false alarm rate is:

1. Choose the probability of false alarms P_{FA}.
2. Estimate the noise variance for the considered signal σ_ε^2. A good estimation can be obtained by using (8).
3. Calculate the reference level $R_{FT} = -\ln(P_{FA}/N)N\sigma_\varepsilon^2$, where N is the signal length.

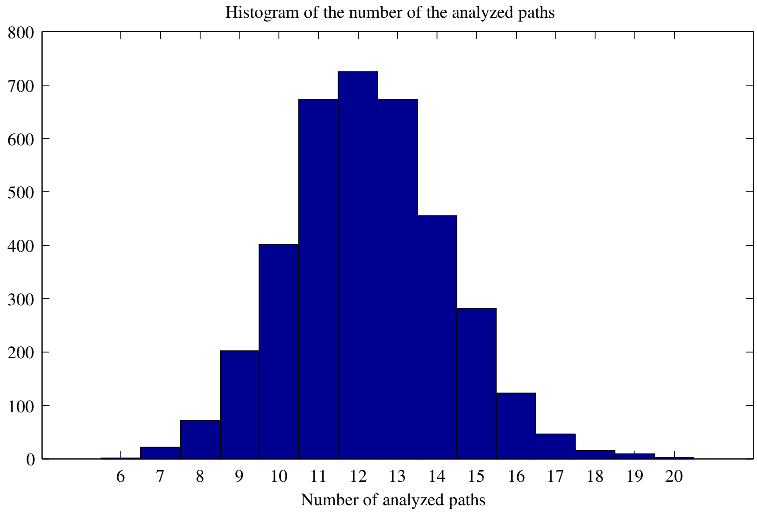

Fig. 5. Histogram of the number of analyzed paths for $M_i = 3704$ realizations. The average number of analyzed paths is 12.

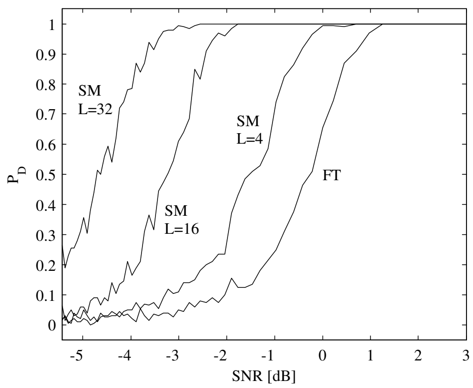

Fig. 6. Probability of the signal detection for the case of FT-based detector and the S-method based detector for different values of the parameter L.

4. Calculate the discrete FT of the signal $X(k) = DFT_N[x(n)]$.
5. If there exists k such that $|X(k)|^2 > R_{FT}$, then we make a decision that a target signal exists.

B. Non-Stationary Case

The algorithm mentioned above is optimal in the case of stationary signals. When the target velocity changes in the considered time interval, the target's signal becomes non-stationary, and the FT is no longer an optimal detector. In these cases, we can use the time-frequency based detector. Now, the detection algorithm is:

1. Choose the probability of the false alarm P_{FA}.
2. Choose the TFR.
3. Consider the noise-only case with unit variance. Estimate the reference level R_0 so that the criterion $J_{\max} > R_0$ gives the false alarm rate as chosen in the step 1.
4. Estimate the noise variance σ_ε^2.
5. Calculate the TFR of the analyzed signal.
6. Find $J_{\max}$ and compare it with the reference level $R_J = R_0\sigma_\varepsilon^2$. If $J_{\max} > R_J$, we can

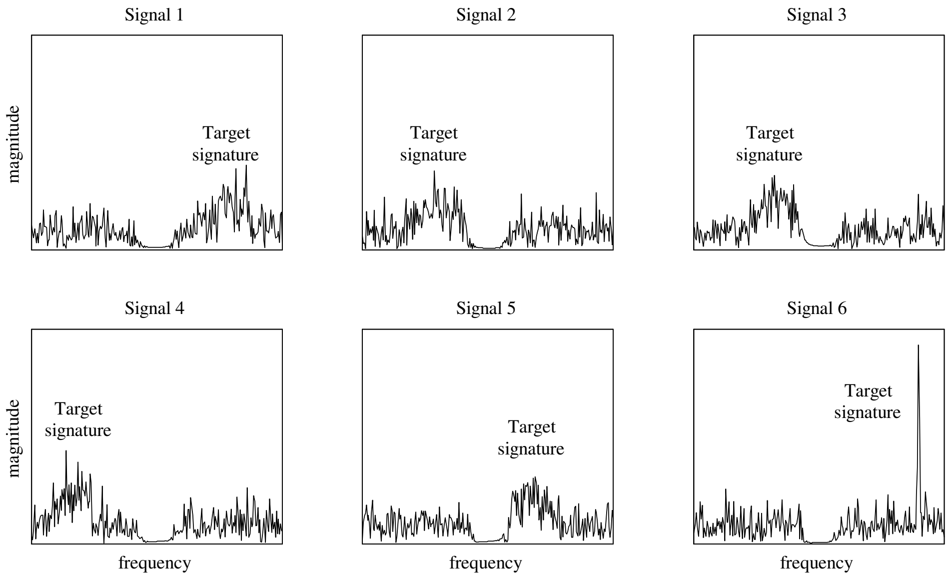

Fig. 7. FT of the analyzed signals with SNR=-2dB

make a decision that the target signal exists in the considered range cell.

Note that steps 1-3 should be performed only once, so the time-consuming step 3 does not slow down the detection process.

C. Detection Examples

Signals in the following comparisons are experimental aircraft data collected by a high-frequency surface-wave radar (HFSWR), which used a 10-element linear receiving antenna array. The data was collected with a target present. Since the experimental data are collected with high SNR an artificial noise is added in order to simulate various SNR scenarios. The radar carrier frequency is 5.672 MHz and the pulse repetition frequency is 9.17762 Hz. There are 6 trials, each trial corresponds to a block of 256 pulses. A detailed description of the radar is given by [4]. We consider five non-stationary cases (signals 1-5) and one stationary case (signal 6). The signals are high-pass filtered in order to remove strong sea clutter. Figure 7 presents the FT of the filtered analyzed signals with SNR= -2dB. TFR of the analyzed signals are presented in Figure 8. The S-method is chosen as the TFR with large L ($L = 64$). The reference level $R_0 = 1320$ is determined according to the previously described procedure. The false alarm probability is $P_{FA} = 0.0027$.

In order to estimate threshold values of the detector, we add noise to the analyzed signal so that a noisy signal is be obtained. Note that in this example deterministic signal $x(n)$ is experimental data and noise $\varepsilon(n)$ is artificial. Detection algorithms of stationary and non-stationary cases are then applied to the noisy signal. Table I shows the number of detected target signals for 100 noise realizations with varying signal-to-noise (SNR). It is obvious that the time-frequency based approach outperforms the Fourier-based detection approach when the target signal is non-stationary. In the case of a stationary signal (signal 6), the proposed method is slightly worse than the Fourier-based approach. The Fourier-based detection is optimal in these cases. The estimated probability of target detection with varying SNR is given in Figure 9 for non-stationary cases using both the Fourier and the time-frequency based detectors. Figures 10 and 11 represent two typical realizations for non-stationary and stationary signals and its detection procedure, respectively. Both Ta-

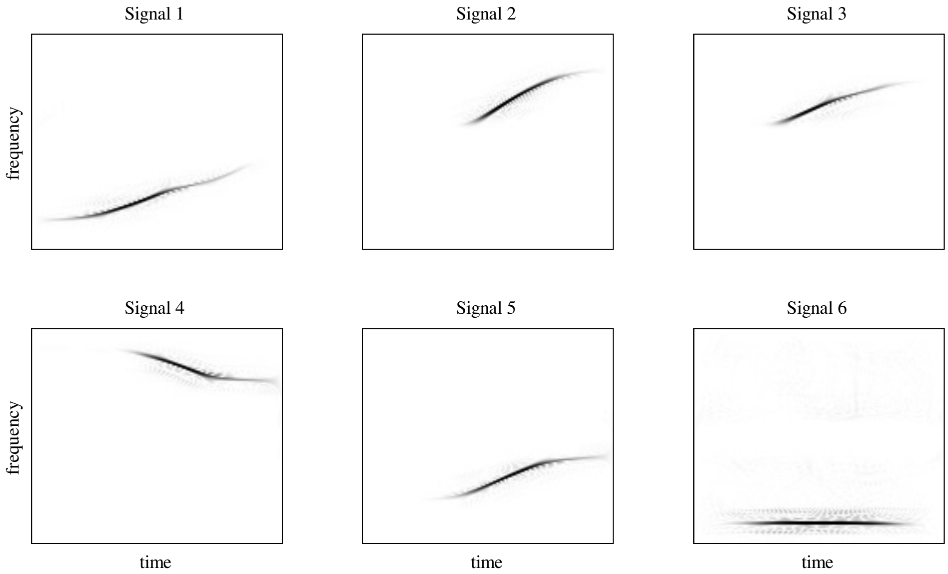

Fig. 8. TFR of the analyzed signals without added noise.

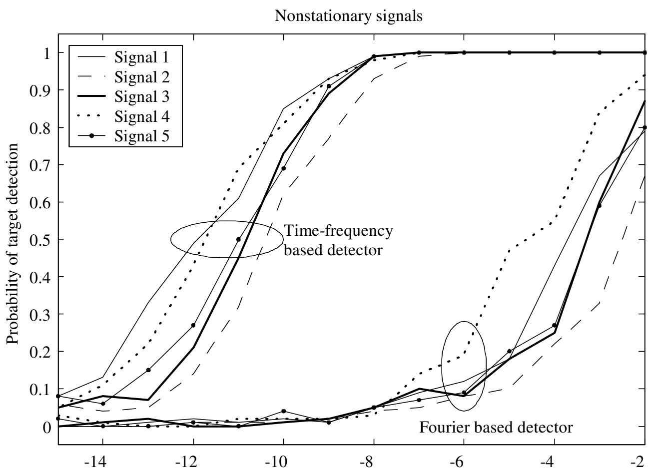

Fig. 9. Estimated probability of target detection versus signal-to-noise ratio for non-stationary cases using both FT and time-frequency based detectors.

TABLE I
NUMBER OF DETECTED TARGET SIGNALS IN 100 TRIALS FOR VARIOUS SNR. FT - FOURIER BASED DETECTOR, T-F - TIME-FREQUENCY BASED DETECTOR.

SNR	Signal 1		Signal 2		Signal 3		Signal 4		Signal 5		Signal 6	
[dB]	FT	T-F	FT	T-F	FT	T-F	FT	T-F	FT	T-F	FT	T-F
−2	79	100	67	100	87	100	94	100	80	100	100	100
−3	67	100	33	100	60	100	84	100	59	100	100	100
−4	43	100	22	100	25	100	55	100	27	100	100	100
−5	18	100	10	100	18	100	47	100	20	100	100	100
−6	12	100	8	100	8	100	19	100	9	100	100	100
−7	9	100	5	99	10	100	14	100	7	100	100	99
−8	5	99	4	93	5	99	3	98	5	99	98	94
−9	1	93	2	77	2	89	2	93	1	91	93	87
−10	2	85	1	62	1	73	2	81	4	69	80	77
−11	1	61	1	32	0	45	2	79	0	50	59	45
−12	2	49	1	14	0	21	0	43	1	27	35	31
−13	1	33	0	5	2	7	0	22	0	15	21	18
−14	0	13	0	4	1	8	1	11	0	6	14	13
−15	0	8	0	6	0	5	3	5	2	8	9	5

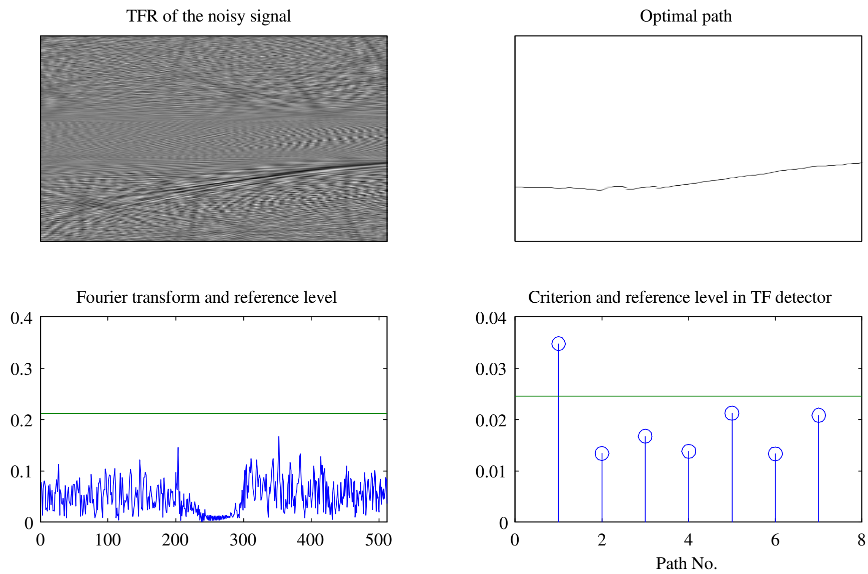

Fig. 10. Signal 5 (nonstationary target velocity) with $SNR = -8$ dB

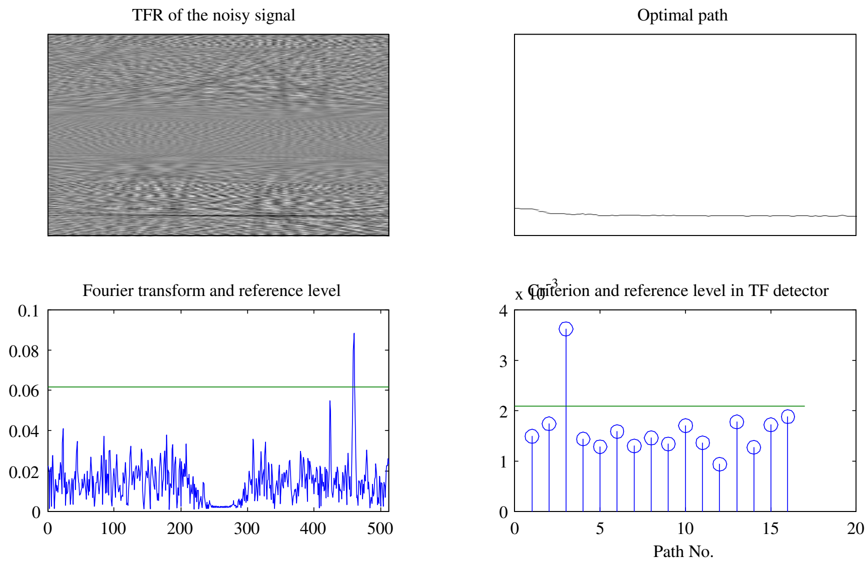

Fig. 11. Signal 6 (stationary target velocity) with $SNR = -8$ dB

ble I and Figure 9 also show that the time-frequency based detector is able to detect the non-stationary target signals correctly when the SNR is higher than −8 dB.

These results specifically suggest that the Fourier detector is optimal when the signals are stationary, whereas the time-frequency based detector is better for non-stationary signals.

V. Conclusion

In this paper, we present a time-frequency based detector, which yields substantial performance improvement over the traditional Fourier-based detector. In this method, we choose the S-method as a time-frequency tool due to its desirable properties. The proposed time-frequency detector requires training with noise and does not require full knowledge of the event to be detected. It is assumed that the deterministic signal is a single component signal with continuous frequency changes. Performance of the proposed method is evaluated by using both simulated and experimental data. Results demonstrate that the time-frequency based approach provides an effective technique for detecting and analyzing maneuvering air targets in heavy noisy environment. The proposed time-frequency based detector approach successfully detects the maneuvering target. When the target is stationary, the Fourier-based detector and the proposed method produces similar detection results. However, when the target is maneuvering or non-stationary, the time-frequency based detector approach produces reliable and robust results. Results also show that the proposed time-frequency detector outperforms the Fourier-based detector in terms of good detection with equal false alarm rates. The method presented here is not restricted to this particular application. It can be applied in various other settings of non-stationary signal analysis and filtering. More generally, it is believed that the time-frequency approach to the signal detection can provide new hints for handling open problems in a comprehensive way.

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