Chaos detection in chaotic systems with large number of components in spectral domain

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Abstract— **A novel technique for chaos detection is proposed by using a time-frequency (TF) representation. The proposed method classifies signals from chaotic oscillators by using entire spectral domain, and it is able to produce accurate results for signals with large number of spectral components in periodic regime. The detector accuracy has been proven on the Colpitts oscillator and logistic map system. The algorithm is tested in the case of moderately noisy signal.**

I. Introduction

A technique for classification of signals from chaotic non-linear oscillatory circuits has been recently proposed [1]. It has been applied to circuits producing signals having a maximum in spectral domain that corresponds to the first harmonic component. This class of circuits includes some important chaotic systems: Chua's, Duffing's, Rossler, etc. However, it cannot be used for oscillatory circuits producing signals with large number of spectral components in periodic regime. These signals are observed in some important chaotic systems such as the Colpitts oscillator [2] and logistic map system [3]. In this letter we propose a new technique for estimating chaotic state, based on counting samples in the entire TF plane. It can properly distinguish between the chaotic and periodic regimes for signals with large number of spectral components within periodic regime.

This letter is organized as follows. Review of the STFT based detector is given in Section II. The proposed detector is described in Section III. Simulation results are given in Section IV.

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II. Detector based on the STFT - an overview

Here we briefly review the chaos detector based on the STFT proposed in [1].

The STFT is used as a main tool in this detector, and it is defined as [4]:

$$STFT(t,f) = \int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j2\pi f\tau}d\tau, \quad (1)$$

where $x(t)$ is signal of interest and $w(\tau)$ is the window function, where $w(\tau) = 0$ for $|\tau| > T/2$ and T is window width. It can be assumed that in periodic regime signal is represented with a sum of finite number of sinusoids (or signals with slight variations in frequency). However, the signal in the chaotic regime is broadband and noise like. It means that in the spectral domain the signal content for periodic regime would be spread over the entire frequency domain. This consideration motivates the specific measure of chaotic state for signals from oscillatory circuits based on counting samples with high energy between DC (direct current - frequency $f = 0$) and dominant frequency component (or just the first harmonic component). This technique can be applied for chaotic systems that have one spectral component dominant over all others. Numerous systems, such as the Chua circuit, have this property.

The STFT concentration measure is defined as [1]:

$$m(t) = \int_{0}^{f_m(t)} u_{\Omega(t)}(t;f)df, \quad (2)$$

where $f_m(t)$ is the frequency of the main spectral component, determined as the position of the $TF(t,f)$ maxima:

$$f_m(t) = \arg\max_f TF(t,f). \qquad (3)$$

Function $u_{\Omega(t)}(t;f)$ is given as:

$$u_{\Omega(t)}(t;f) = \begin{cases} 1 & TF(t,f) \geq \Omega(t) \\ 0 & \text{elsewhere,} \end{cases} \qquad (4)$$

where $\Omega(t)$ is the threshold. Decision of the system state is made by comparing detector response function $m(t)$ with a detector threshold $C(t)$:

$$d(t) = \begin{cases} 1 & \text{for } m(t) \geq C(t) \quad \text{chaotic regime} \\ 0 & \text{for } m(t) < C(t) \quad \text{periodic regime.} \end{cases} \qquad (5)$$

Selection of $\Omega(t)$ and detection response threshold $C(t)$ is described in details in our papers [1] and [5].

This extremely efficient technique has been applied to chaotic circuits and systems with single dominant frequency component $f_m(t)$, but it can not be applied in this manner to the chaotic systems having numerous components with similar amplitudes.

In next section we develop chaotic detector that can be applied for these systems.

III. Proposed technique

The basic idea of the detector proposed in [1] is to count the STFT samples that are above specific threshold for a given instant. For a regime that has several sinusoids this measure will be small, while for a chaotic region this measure will be large. However, for the periodic regime in the Colpitts circuit, due to the large number of spectral components, this measure will also be large, thus causing difficulties in distinguishing between chaotic and such periodic regime.

In order to develop chaos detector, consider the STFT of the sinusoidal signal calculated by using the Hanning window. If amplitude of the sinusoid is A, number of samples within the window is N and frequency of sinusoid is on the frequency grid, then the STFT would have amplitude on the frequency of sinusoid equal to $AN/2$, while the two adjacent STFT samples would have amplitudes $AN/4$. We want to clearly distinguish this event from the chaotic regime where in the spectral domain we have no clear spectral peaks. For this purpose we introduce the following function:

$$B(f) = \begin{cases} 1/2 & \text{for } f = 0 \\ 1/4 & \text{for } f = \pm 1/T \\ \text{-}1/4 & \text{for } f = \pm 2/T, \pm 3/T \\ 0 & \text{elsewhere.} \end{cases} \qquad (6)$$

Three central non-zero samples of $B(f)$ form the optimal detector for the STFT of sinusoid on the frequency grid, while 4 negative samples are introduced to produce zero output for approximately flat spectrum in chaotic domain.

The convolution of $B(f)$ with the magnitude of the STFT is calculated as:

$$G(t,f) = |STFT(t,f)| *_f B(f), \qquad (7)$$

where $*_f$ denotes convolution in the frequency domain. The expected value of the $G(t,f)$ for the position that corresponds to the frequency of sinusoid is $3AN/8$. For parts of the TF plane without sinusoidal components, where the magnitude of the STFT is approximately constant, $G(t,f)$ would approximately be 0. Then we use the mean of these two values as a threshold in the process of recognizing the points in the TF plane that belong to the sinusoidal component. This value is equal to $3AN/16$, i.e., $3|STFT(t,f)|/8$ for sinusoidal component frequency.

Based on this analysis, chaos detection is performed by using two measures:

$$m_i(t) = \int_f u_i(t;f)df, \qquad i = 1,2, \qquad (8)$$

where $u_1(t,f) = 1$ for $G(t,f) \geq \frac{3|STFT(t,f)|}{8}$ and $u_1(t,f) = 0$ elsewhere, $u_2(t,f) = 1$ for $|STFT(t,f)| \geq \Omega(t)$ and $u_2(t,f) = 0$ elsewhere. For non-noisy signals parameter $\Omega(t)$ can be set as $\Omega(t) = \varepsilon \max_f |STFT(t,f)|$. In our experiments we select $\varepsilon = 0.001$. Selection of $\Omega(t)$ for noisy signals is discussed within numerical examples, Section IV.C.

Now, the entire time-frequency plane is used for calculation of the functions $m_i(t)$ for $i = 1,2$. This is the main difference with respect to the technique described in Section II. The

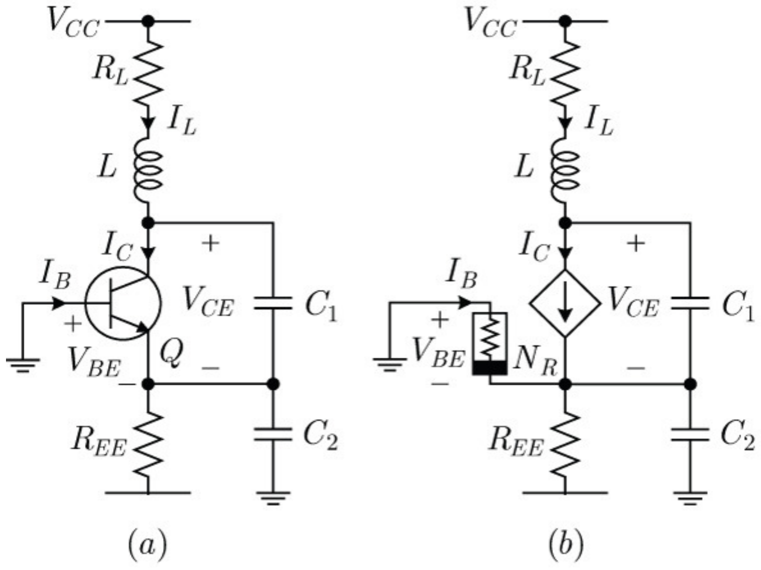

Fig. 1. (a) Colpitts oscillator with bipolar transistor. (b) Equivalent circuit.

first measure counts samples in the TF plane that belong to the sinusoidal component, while the second criterion counts STFT samples that have significant energy. This value is not critical and any relatively small value can be used. Both $m_i(t,f)$, $i=1,2$, are averaged within a short interval in order to reduce noise influence:

$$m_i'(t) = \frac{1}{p}\int_{t-p/2}^{t+p/2} m_i(\tau)d\tau, \quad i=1,2. \tag{9}$$

In our experiments we set p to be equal to the used window width. Finally, the detection of the chaotic state is performed in the following manner:

$$\begin{cases} m_2'(t)/m_1'(t) \geq C & \text{circuit is within chaotic regime} \\ m_2'(t)/m_1'(t) < C & \text{circuit is within periodic regime.} \end{cases} \tag{10}$$

Threshold value C can be selected within a wide range, and in our experiments it is set to $C=3$.

IV. Numerical examples

In order to show that this detector can be applied to different chaotic systems, we consider the well-know chaotic systems - Colpitts oscillator and logistic map system.

A. *Colpitts oscillator*

The Colpitts oscillator, given in Fig.1a, is a simple circuit used in communications [2]. The bipolar transistor, assuming that it operates in directly active or cutoff regimes, can be modeled as a two-segment piecewise linear voltage controlled resistor (Fig.1b). Then this circuit can be modeled with three state equations:

$$C_1\frac{dv_{CE}}{dt} = i_L - I_C$$

$$C_2\frac{dv_{BE}}{dt} = -\frac{V_{EE}+v_{BE}}{R_{EE}} - i_L - I_B$$

$$L\frac{di_L}{dt} = V_{CC} - v_{CE} + v_{BE} - i_L \tag{11}$$

with the characteristics of non-linear resistor given as:

$$I_B = \begin{cases} 0 & v_{BE} \leq V_{TH} \\ \frac{v_{BE}-V_{TH}}{R_{ON}} & v_{BE} > V_{TH} \end{cases} \quad I_C = \beta_F I_B, \tag{12}$$

where V_{TH} is threshold voltage, R_{ON} is on-resistance for small signals and β_F is direct current gain. This circuit, with proper selection of parameters, produces chaotic behavior. For example, this behavior can be obtained for: $C_1 = C_2 = 54$nF, $L = 98.5\mu$H, $R_{EE} = 400\Omega$, $V_{EE} = -5V$, $V_{CC} = 5V$, $\beta_F = 255$, $R_{ON} = 100\Omega$, $V_{TH} = 0.75V$. By linearly varying resistance R_L (bifurcation parameter) in the range between 67Ω and 5Ω the chaotic circuit switches states (chaotic and periodic) producing different behavior in the spectral domain. Fig.2a gives the TF representation of the voltage $v_{BE}(t)$. It can be seen that in periodic regime we have numerous spectral components with constant or slightly varying frequencies. However, in a chaotic regime, a signal is wideband and noise-like and sinusoidal components cannot be clearly recognized.

Results of experiments for the detector setup described in Section III are given in Fig.2b-d. The function $m_1'(t)$ is given in Fig.2b. It can be seen that this function is large for regions with sinusoidal components and small for chaotic region. However, region at the beginning of the signal that has small number of sinusoids cannot clearly be recognized, since the function $m_1'(t)$ for this region is between values achieved for other periodic and for chaotic regions. The second function, $m_2'(t)$ (Fig.2c), gives large value for both chaotic and for periodic regime with large number of components, but small value for periodic regime with small number of components. Obviously, this measure used in [1] for chaos detection in the Chua's circuit and similar chaotic systems can distinguish chaos from periodic regime only in the case of small number of components within periodic regime. Ratio $m_2'(t)/m_1'(t)$ is given in Fig.2d with the used threshold value $C = 3$ depicted with dashed lines. Region above the threshold corresponds to the recognized chaotic regime. Obviously, the proposed detector gives accurate recognition of the chaotic regime. In addition, it can be concluded that selection of the threshold C is not critical since it can be selected in a wide range.

B. Logistic map

One of the simplest known chaotic systems is logistic map described by the difference equation:

$$x_{n+1} = Ax_n\left(1 - x_n\right). \tag{13}$$

Even this very simple system can be used to model numerous chaotic phenomena [3]. In our experiment, parameter A is linearly varied in the range from 3.5 to 4. Initial condition is $x_0 = 0.1$. For $A = 3.57 (n = 2200)$ the accumulating point has been reached and after that instant we have the chaotic regime. However, in chaotic regime there are infinitely many periodic windows [3]. The proposed method accurately detected chaotic regime and three relatively long periodic windows Fig.3a. Measures $m_i'(t), i = 1, 2$ are given in Fig.3b, c while detector response is given in Fig.3d. From this figures one can notice that the chaotic state is detected accurately since the detector response function is above the threshold in the entire interval of chaotic state. In addition, all periodic windows are detected correctly.

C. Noise influence

In order to analyze robustness of the proposed detector to noise influence, it was assumed that the signal produced by the Colpitts oscillator is transmitted through a noise channel. Noise environment was Gaussian with $SNR = 10$dB. Here we used definition of the SNR adopted in [6] as:

$$SNR = 10\log_{10}\frac{\int_t |f(t) - \overline{f}(t)|^2 dt}{\int_t |\nu(t)|^2 dt} \tag{14}$$

where:

$$\overline{f}(t) = \frac{1}{T}\int\limits_{t-T/2}^{t+T/2} f(\tau)d\tau. \tag{15}$$

Parameter T is equal to the used window width in the STFT.

Noise increases the mean value of the spectrogram and all samples would be above the previously used threshold $\Omega(t) = \varepsilon \max_f |STFT(t,f)|$. Then, we have to increase the value of $\Omega(t)$ in some systematic way. The variance of the STFT is approximately equal to $\sigma^2_{STFT} = \sigma^2 \int_t w^2(t)dt$, [7], where σ^2 is the variance of the input Gaussian noise. Note that there are estimators of the

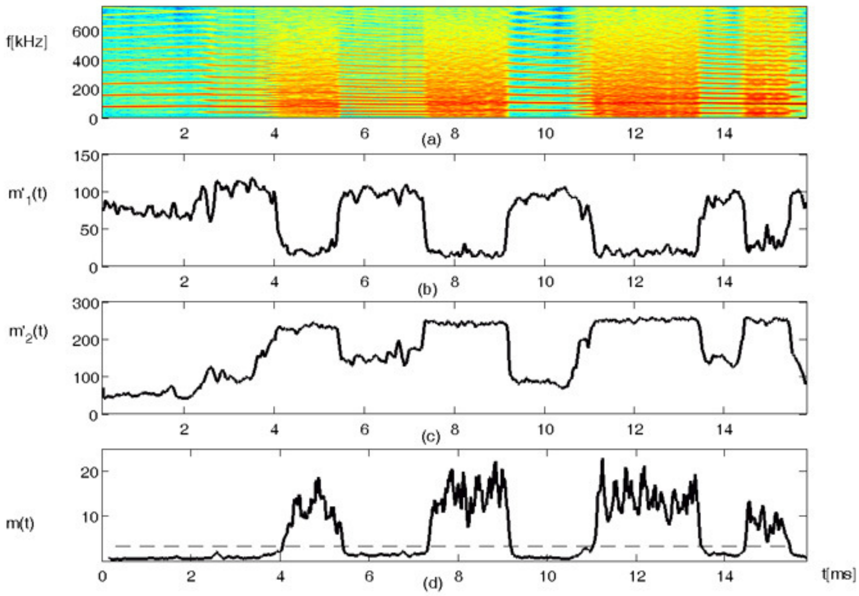

Fig. 2. Colpitts oscillator - Chaos detection: (a) logarithm of $|STFT(t,f)|$; (b) measure $m_1'(t)$; (c) measure $m_2'(t)$; (d) detector response function $m(t) = m_2'(t)/m_1'(t)$ - solid line; detector threshold C - dashed line.

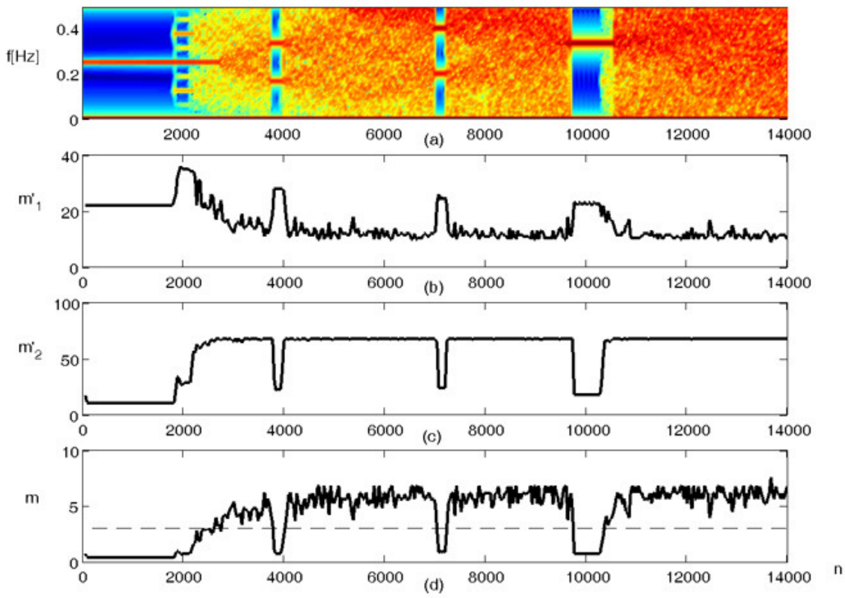

Fig. 3. Logistic map - Chaos detection: (a) logarithm of $|STFT(t,f)|$; (b) measure $m_1'(t)$; (c) measure $m_2'(t)$; (d) detector response function $m(t) = m_2'(t)/m_1'(t)$ - solid line; detector threshold C - dashed line.

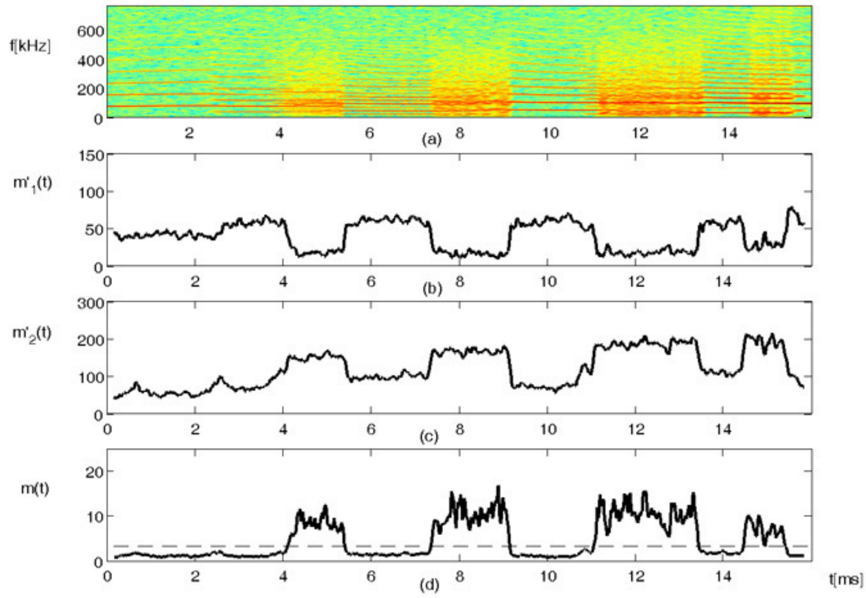

Fig. 4. Colpitts oscillator in noisy environment (SNR =10dB) - Chaos detection: (a) logarithm of $|STFT(t,f)|$; (b) measure $m_1'(t)$; (c) measure $m_2'(t)$; (d) detector response function $m(t) = m_2'(t)/m_1'(t)$ - solid line; detector threshold C - dashed line.

Gaussian noise variance with quite acceptable accuracy and in this research we used the technique from [8] (see Section III.B). Then, the threshold $\Omega(t)$ that can work for both noisy and non-noisy signals can be selected as $\Omega(t) = \max\{\varepsilon \max_f |STFT(t,f)|, \kappa\sigma_{STFT}\}$. The obtained results are not sensitive to κ, and we selected $\kappa = 3$.

Detector response is depicted in Fig 4. Proposed detector detects accurately all periodic regions and chaotic states for this moderate noise level.

V. Conclusion

Modification of the chaos detector in the non-linear circuits has been proposed. It enables application of the considered detector for systems producing large number of sinusoidal components within periodic state. The proposed detector has been tested on signals from the Colpitts oscillator and logistic map system where it produces accurate results. Also, it can be successfully used in a moderate noise environment.

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