Interference Analysis of Multicarrier Systems based on Affine Fourier Transform

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Abstract— **Multicarrier techniques based on affine Fourier transform (AFT) have been recently proposed for transmission in the wireless channels. The AFT represents a generalization of the Fourier and fractional Fourier transform. We derive the exact and approximated interference power, upper bound and measure of applicability for the AFT based multicarrier (AFT-MC) system. It is demonstrated that the AFT-MC system effectively minimizes interference in time-varying multipath channels with line-of-sight component and narrow beamwidth of scattered components that often occurs in aeronautical and satellite communications.**

I. Introduction

The Fourier transform (FT) plays a significant role in modern wireless communications. The orthogonal frequency division multiplexing (OFDM), based on the FT, is one of the most important classes of multicarrier modulations with equally spaced subcarriers and overlapping spectra [1]. It has been implemented in physical layer of many standardized wireless systems.

Multicarrier techniques based on orthonormal basis formed by chirps, complex exponentials with linearly varying instantaneous frequencies, have been recently proposed for transmission in the wireless channels [2], [3]. These multicarrier modulations can be described by a family of transforms known as the affine Fourier (AFT) or linear canonical transforms. The AFT was introduced in the optics as a generalization of the fractional Fourier transform (FrFT) [4]. However, mathematical basis can be found earlier in [5] and [6]. The FrFT has been also used in numerous applications in quantum mechanics [7], [8], and recently, in time-frequency signal analysis [9], [10].

In the multicarrier communications, the chirp based modulation has been introduced in [11] as the chirped-OFDM. A multicarrier system, with the orthogonal signal basis of the chirp type that corresponds to the FrFT, has been proposed in [2]. It has been suggested that the FrFT type can be implemented with a complexity that is equivalent to the traditional fast Fourier transform. The AFT based multicarrier (AFT-MC) system has been introduced in [3]. It has been shown that the AFT-MC system can efficiently combat intercarrier interference when the propagation channels have few multipath components affected by independent frequency offsets. It can be efficiently implemented by adding a phase-correction block to the standard OFDM modulators and demodulators.

In this letter, we derive the exact and approximated interference power, upper bound and measure of the AFT-MC applicability. Since the AFT-MC can be considered as a generalization of the OFDM, derived expressions represent generalization of results from [12], [13] and [14]. In a practical example we clearly demonstrate that the AFT-MC system effectively minimizes interference in aeronautical channels that characterize communications when the aircraft is airborne.

The letter is organized as follows. The performance analysis is presented in Section II, following by AFT-MC applicability analysis in Section III. Examples of practical implementation are given in Section IV. Finally, conclusions are presented in Section V.

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II. Performance Analysis

Assume that the data symbols $\{c_{n,k}\}$, where n and k correspond to the symbol interval and the subcarrier number, respectively, in the AFT-MC system are statistically independent, identically distributed, and with zero-mean and unit-variance. The translations and modulations by a chirp basis of a single normalized pulse shape $g(t)$ can be written as

$$g_{n,k}(t) = = g(t-nT)e^{j2\pi(c_1(t-nT)^2+c_2k^2+\frac{k}{T}(t-nT))}, \quad (1)$$

where T is the symbol period, and c_1 and c_2 are the AFT parameters. The baseband equivalent of the AFT-MC signal is defined as

$$s(t) = \sum_{n=-\infty}^{\infty}\sum_{k=0}^{M-1} c_{n,k}g_{n,k}(t), \quad (2)$$

where M is total number of subcarriers. The baseband doubly dispersive channel can be modeled by the multipath fading linear operator $\mathbf{H}$. The signal at the receiver is given as [15]

$$r(t) = (\mathbf{Hs})(\mathbf{t}) + \mathbf{n}(\mathbf{t}), \quad (3)$$

where $n(t)$ represents the additive white Gaussian noise (AWGN), with the one-sided power spectral density N_0. Reconstruction of the symbols at the receiver is performed by projecting the received signal on the signal set $\{g_{n,k}(t)\}$. The received symbols $\{\widehat{c}_{n,k}\}$ can be represented as [13]

$$\widehat{c}_{n,k} = \langle \mathbf{Hs}, \mathbf{g_{n,k}}\rangle + \langle n, g_{n,k}\rangle$$
$$= \sum_{n'=-\infty}^{\infty}\sum_{k'=0}^{M-1} c_{n',k'}\langle \mathbf{Hg_{n',k'}}, \mathbf{g_{n,k}}\rangle + \langle n, g_{n,k}\rangle . \quad (4)$$

The parameters $q_{n,k,n',k'} = \langle \mathbf{Hg_{n',k'}}, \mathbf{g_{n,k}}\rangle$ in (4) represent the coupling produced by the channel, between the transmitted and received pulses. The inner product can be written in the form

$$q_{n,k,n',k'} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau,\nu)g_{n',k'}(t-\tau) \times g^*_{n,k}(t)e^{j2\pi\nu t}dtd\tau d\nu, \quad (5)$$

where $h(\tau,\nu)$ denotes the spreading function of the channel as a function of delay time τ and Doppler shift ν. Usually, frequency offset correction block, that can be modeled as $e^{j2\pi c_0 t}$, is inserted in the receiver. Substituting $t' = t - nT$, after some calculations we have

$$q_{n,k,n',k'} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau,\nu)e^{-j2\pi\theta(\tau,\nu)} \times [(g(t')g^*(t'-((n'-n)T+\tau)) \times e^{-j2\pi t'(\frac{1}{T}(k'-k)+\nu-c_0-2c_1((n'-n)T+\tau))}\Big]^* \times dtd\tau d\nu, \quad (6)$$

where

$$\theta(\tau,\nu) = c_1((n'-n)T+\tau)^2 + c_2(k'^2-k^2) - \frac{k'}{T}((n'-n)T+\tau) - \nu nT.$$

Now, the expression for $q_{n,k,n',k'}$ can be written as

$$q_{n,k,n',k'} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau,\nu)e^{j2\pi\theta(\tau,\nu)} \times A^*(\tau_p,\nu_p)d\tau d\nu, \quad (7)$$

where

$$\tau_p = (n'-n)T+\tau,$$
$$v_p = \frac{1}{T}(k'-k)+\nu-c_0 - 2c_1((n'-n)T+\tau). \quad (8)$$

The linearly transformed ambiguity function $A(\tau_p,\nu_p)$ can be defined as

$$A(\tau_p,\nu_p) = \int_{-\infty}^{\infty} g(t)g^*(t-\tau_p)e^{-j2\pi\nu_p t}dt, \quad (9)$$

in analogy with the classical ambiguity function definition $A(\tau,\nu) = \int_{-\infty}^{\infty} g(t)g^*(t-\tau)e^{-j2\pi\nu t}dt$ [13]. Under the assumption of wide sense stationary uncorrelated scattering (WSSUS) channel [15], the mean-squared values of the coefficients satisfy

$$E\left[|q_{n,k,n',k'}|^2\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\tau,\nu) \times |A(\tau_p,\nu_p)|^2 d\tau d\nu, \quad (10)$$

where $S(\tau,\nu)$ denotes a scattering function that completely characterizes the WSSUS

$$E\left[h(\tau,\nu)h^*(\tau_1,\nu_1)\right] = S(\tau,\nu)\delta(\tau-\tau_1)\delta(\nu-\nu_1). \tag{11}$$

The useful power is obtained for $n = n'$ and $k = k'$

$$P_U = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\tau,\nu)\left|A(\tau_p,\nu_p)\right|^2_{\substack{n=n'\\k=k'}} d\tau d\nu. \tag{12}$$

The interference power in the n-th interval on the k-th subcarrier is obtained for $n \neq n'$ or $k \neq k'$

$$P_I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\tau,\nu) \sum_{(n,k)\neq(n',k')} \left|A(\tau_p,\nu_p)\right|^2 d\tau d\nu. \tag{13}$$

Consider the AFT-MC system with rectangular pulses $g(t) = 1/\sqrt{T},\ -T/2 \leq t \leq T/2$. Usually, a guard interval (GI) of length T_{GI} is inserted in the transmitting pulses, that efficiently eliminates all effects of multipath delays. The linearly transformed ambiguity function $A(\tau_p,\nu_p)$ for $n' = n$ and $k' = k$ can be expressed as

$$\left|A(\tau_p,\nu_p)\right|^2_{\substack{n=n'\\k=k'}} = \text{sinc}^2\pi T(\nu-c_0-2c_1\tau). \tag{14}$$

In practical wireless channels, both time and frequency spread have finite support and $S(\tau,\nu)$ has nonzero values only for $\tau \in [0,\tau_{\max}]$ and $\nu \in [-\nu_d,\nu_d]$. Since $\left|A(\tau_p,\nu_p)\right|^2_{n=n',\ k=k'}$ and $S(\tau,\nu)$ are given with unit power, total power $P_I + P_U$ is equal to 1. Now, the interference power can be expressed as

$$P_I = 1- \\ -\int_{-\nu_d}^{\nu_d}\int_{0}^{\tau_{\max}} S(\tau,\nu)\text{sinc}^2\pi T(\nu - c_0 - 2c_1\tau) d\tau d\nu. \tag{15}$$

The AFT-MC system reduces respectively to the fractional FT (FrFT) and OFDM based system for $c_1 = \cot\alpha/(4\pi)$ and $c_1 = 0$ in (8). Furthermore, all derived expressions can also be used for FrFT and OFDM, by inserting appropriate c_1. Note that interference power expression for AFT-MC can be consider as a generalization of derivation for OFDM in [12] and [13].

Interference power can be upper bounded by using Taylor series $\text{sinc}^2\pi T(\nu - c_0 - 2c_1\tau) \geq 1 - \frac{1}{3}\pi^2T^2(\nu - c_0 - 2c_1\tau)^2$ as

$$P_I \leq \frac{1}{3}m_{20}\left(c_0,c_1\right)\pi^2T^2 \tag{16}$$

where $m_{ij}\left(c_0,c_1\right)$ for $i,\ j \in N$ represent moments of the scattering function for AFT-MC

$$m_{ij}\left(c_0,c_1\right) = \int_{-\nu_d}^{\nu_d}\int_{0}^{\tau_{\max}} S(\tau,\nu) \\ \times(\nu - c_0 - 2c_1\tau)^i\tau^j d\tau d\nu. \tag{17}$$

Similarly, moments for the OFDM $m_{ij}\left(0,0\right)$ can be obtained for $c_0 = 0$ and $c_1 = 0$. Using binomial formula $(a+b)^i = \sum_{k=0}^{i}\binom{i}{k}a^{i-k}b^k$, moments $m_{ij}\left(c_0,c_1\right)$ for the AFT-MC can be calculated using $m_{ij}\left(0,0\right)$ for the OFDM as

$$m_{ij}\left(c_0,c_1\right) = \sum_{k=0}^{i}\sum_{l=0}^{i-k}(-1)^{l+k}\binom{i}{k}\binom{i-k}{l} \\ \times c_0^l\left(2c_1\right)^k m_{i-k-l,k+j}\left(0,0\right). \tag{18}$$

Note that (16) represents the AFT-MC equivalent of the upper bound for OFDM previously derived in [14].

The Taylor series of $\text{sinc}^2\theta$ around zero accurately represents function for $\theta \ll 1$. In the OFDM case, this restriction is simplified to $\nu_dT \ll 1$, which is a valid assumption for practical mobile radio fading channels, where the time-varying effects in the channel are sufficiently slow and symbol duration is always much smaller than the coherence time. However, in aeronautical and satellite communications, Doppler shifts larger than 10^3Hz may occur due to high velocity of the objects. For $\left(\nu_d + |c_0| + 2\left|c_1\right|\tau_{\max}\right)T > 1$ (e.g. symbol interval and velocity are large) the approximation diverges toward infinity, whereas the exact interference power converges towards power of diffused components σ^2_{diff}. Therefore, a modification of the interference power should be made. It can be easily shown that an approximate interference power for the wide range of channel parameters can be obtained as

$$P_I \cong \frac{\frac{1}{3}\sigma_{diff}^2 m_{20}\left(c_0, c_1\right)\pi^2 T^2}{\sigma_{diff}^2 + \frac{1}{3} m_{20}\left(c_0, c_1\right)\pi^2 T^2}. \tag{19}$$

III. AFT-MC Applicability Analysis

Let us now analyze multipath scenario with line-of-sight (LOS) component. The power of LOS σ_{LOS}^2 and the power of diffused components σ_{diff}^2, for unchanged mean throughput power, can be defined as a function of the Rician factor $K = \sigma_{LOS}^2/\sigma_{diff}^2$ as

$$\sigma_{LOS}^2 = \frac{K}{K+1}, \tag{20}$$

$$\sigma_{diff}^2 = \frac{1}{K+1}. \tag{21}$$

In this case, scattering function can be defined as

$$S(\tau,\nu) = \frac{K}{K+1}\delta\left(\tau\right)\delta\left(\nu - \nu_{LOS}\right) + \frac{1}{K+1} S_{diff}\left(\tau,\nu\right), \tag{22}$$

where ν_{LOS} is Doppler shift of the LOS component, and $S_{diff}(\tau,\nu)$ denotes scattering function of the diffused components. Non-line-of-sight (NLOS) channel follows from (22) for $K = 0$. In this case, $\sigma_{LOS}^2 = 0$, $\sigma_{diff}^2 = 1$, and all derived equations can be directly implemented for the NLOS communications.

Note that if carrier frequency offset ν_{CFO} exists, it can lead to the interference in multicarrier systems. These effects can be modeled by $S(\tau,\nu) = \frac{1}{2}\delta\left(\tau\right)\left[\delta\left(\nu + \nu_{CFO}\right) + \delta\left(\nu - \nu_{CFO}\right)\right]$. In this case, it can be easily shown that $P_I = 1 - \text{sinc}^2 \pi T \nu_{CFO}$, and the moment $m_{20}\left(0, c_1\right) = \nu_{CFO}^2$. Now, the upper bound on interference and approximate interference power can be calculated by inserting $m_{20}\left(0, c_1\right)$ in (16) and (19), respectively.

The moments $m_{20}(0,0)$ and $m_{02}(0,0)$ have a special physical importance since they represent Doppler spread ν_m and delay spread τ_m of the channel in the OFDM system, respectively. Optimal coefficients c_{0opt} and c_{1opt}, can be calculated by minimizing $\nu_m(c_0,c_1)$ as [11]

$$c_{0opt} = = \frac{m_{02}\left(0,0\right) m_{10}\left(0,0\right) - m_{01}\left(0,0\right) m_{11}\left(0,0\right)}{m_{02}\left(0,0\right) - m_{01}^2\left(0,0\right)},$$

$$c_{1opt} = = \frac{m_{11}\left(0,0\right) - m_{01}\left(0,0\right) m_{10}\left(0,0\right)}{2\left(m_{02}\left(0,0\right) - m_{01}^2\left(0,0\right)\right)}. \tag{23}$$

A difference between Doppler spreads for the OFDM with the offset correction $\nu_m\left(c_0, 0\right)$ and the AFT-MC system $\nu_m\left(c_0, c_1\right)$ can be used to indicate advantages of the AFT-MC system with the respect to OFDM

$$\Delta\nu_m = \nu_m\left(c_0, 0\right) - \nu_m\left(c_0, c_1\right), \tag{24}$$

where $\nu_m\left(c_0, 0\right) = m_{20}\left(c_0, 0\right)$ and $\nu_m\left(c_0, c_1\right) = m_{20}\left(c_0, c_1\right)$ Now, $\Delta\nu_m$ can be obtained as

$$\Delta\nu_m = 4\int_{-\nu_d}^{\nu_d}\int_{0}^{\tau_{\max}} S\left(\tau,\nu\right) \times \left(c_1\nu\tau - c_0 c_1\tau - c_1^2\tau^2\right) d\tau d\nu. \tag{25}$$

Using definition of $m_{ij}(0,0)$ parameters, after some calculations it follows

$$\Delta\nu_m = 4 m_{02}\left(0,0\right) c_{1opt}^2. \tag{26}$$

It is obvious that $\Delta\nu_m$, with the optimal choice of c_0 and c_1, will always be positive, except in the case when m_{02} or c_{1opt} are equal to 0. Since the parameter $m_{02}(0,0)$ defines delay spread, it will be zero only when there are no multipath components of the signal. Parameter c_{1opt} is equal to 0 only if there is no LOS component or Doppler shift of the LOS component is equal to 0. In these cases, the AFT-MC system reduces to the ordinary OFDM, whereas in other cases the AFT-MC system would have smaller ν_m and better characteristics. The coherence time for the AFT-MC system, $T_c\left(c_0, c_1\right) = 1/\nu_m\left(c_0, c_1\right)$ can be expressed in terms of the coherence time for the OFDM system with the offset correction $T_c(c_0, 0) = 1/\nu_m\left(c_0, 0\right)$ as

$$T_c\left(c_0, c_1\right) = T_c(c_0, 0)\left(1 + \frac{\Delta\nu_m}{\nu_m(c_0, c_1)}\right). \tag{27}$$

It can be seen that $T_c(c_0, c_1)$ is greater than $T_c(c_0, 0)$ for $\Delta\nu_m > 0$. If $T_c(c_0, c_1) > T_c(c_0, 0)$, the AFT-MC system will be more suitable than the OFDM for the given channel setup. Note that in the NLOS environments with symmetrical Doppler power profile, that is typical for the land mobile communications, $c_{1opt}^2 = 0$, $\Delta\nu_m = 0$, $T_c(c_0, c_1) = T_c(c_0, 0)$ and AFT-MC has the same properties as OFDM.

IV. Examples

The en-route scenario in aeronautical channels models ground-to-air or air-to-air communications when the aircraft is airborne [16]. It is a channel model with LOS path and cluster of scattered paths. In this case, $S(\tau, \nu)$ takes form

$$S(\tau,\nu) = \frac{K}{K+1}\delta(\tau)\,\delta(\nu - \nu_{LOS}) + \frac{1}{K+1}P_{diff}(\nu)\,\delta(\tau - \tau_{diff}), \tag{28}$$

where $P_{diff}(\nu)$ represents Doppler power profile and τ_{diff} denotes time delay of the scattered components. The beamwidth of the scattered components is narrow, leading to the asymmetrical $P_{diff}(\nu)$ that can be modeled by restricted Jakes model [17]

$$P_{diff}(\nu) = \psi \frac{1}{\nu_d\sqrt{1-\left(\frac{\nu}{\nu_d}\right)^2}}, \nu_1 \le \nu \le \nu_2, \tag{29}$$

where $\psi = 1/\left(\arcsin(\nu_2/\nu_d) - \arcsin(\nu_1/\nu_d)\right)$, denotes a factor introduced to normalize $P_{diff}(\nu)$. For this channel, optimal parameters are

$$\begin{aligned} c_{0opt} &= \nu_{LOS}, \\ c_{1opt} &= \frac{1}{2\tau_{diff}}\psi\left(\sqrt{\nu_d^2-\nu_1^2} - \sqrt{\nu_d^2-\nu_2^2}\right) \\ &\quad -\frac{1}{2\tau_{diff}}\nu_{LOS}. \end{aligned} \tag{30}$$

In the worst case the LOS component comes directly to the front of the aircraft and scattered components come from behind. Here, $\nu_1 = -\nu_d$ and $\nu_2 = -\nu_d(1 - \Delta\varphi_B/\pi)$, where $\Delta\varphi_B$ represents the beamwidth of the scattered components symmetrically distributed around the $\varphi = \pi$. Parameters for this scenario are: carrier frequency $f_c = 1.55$GHz (corresponding to the L band), $\Delta\varphi_B = 3.5°$, $\tau_{diff} = 66\mu$s, $T = 1056\mu$s and $K = 15$dB. Maximal Doppler shift depends on the velocity of the aircraft $\nu_d = v_{\max} f_c/c$, where c denotes speed of light and maximal velocity is assumed to be up to $v_{\max} = 250$ m/s. From Fig. 1, it can be observed that the AFT-MC system efficiently suppresses interference even for the large Doppler shifts. The approximated interference powers stay close to the exact ones (in AFT-MC case, they are practically indistinguishable). When GI is implemented, the interference power in the AFT-MC is negligible, comparing to its significant value in the OFDM. Thus, in channels with LOS path and cluster of scattered paths with narrow beamwidth, implementation of the AFT-MC leads to significant reduction of interference. Similar to the en-route scenario in aeronautical communications, this type of channel also occurs in various satellite communications. In each of these scenarios AFT-MC efficiently minimizes interference and simultaneously offers all advantages of multicarrier communications. Note that simulations in the typical NLOS environments would not show any differences between AFT-MC and OFDM, since $c_{1opt}^2 = 0$ and AFT-MC reduces to OFDM. A detailed analysis of bounds on interference in time-varying multipath channels and implementation in aeronautical and satellite channels will be reported elsewhere.

V. Conclusion

In this letter, the interference analysis of the AFT-MC in time-varying multipath channels with LOS component is presented. The AFT-MC system significantly improves the interference suppression in channels with LOS component and narrow beamwidth of scattered components, which is a typical scenario in aeronautical and satellite channels.

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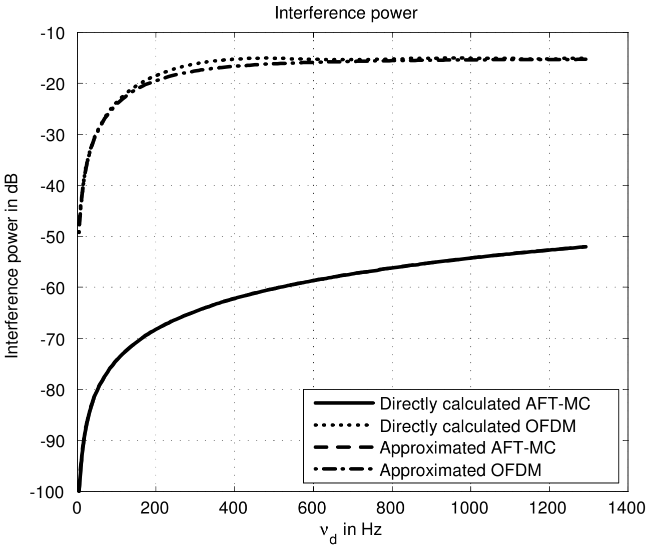

Fig. 1. Comparison of directly calculated and approximated interference power in the en-route scenario in aeronautical channels for AFT-MC and OFDM.

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