Local Polynomial Fourier Transform Receiver for Nonstationary Interference Excision in DSSS Communications

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Abstract— **The problem treated in this paper is monocomponent nonstationary interference excision in direct sequence spread spectrum (DSSS) communication systems by means of the local polynomial Fourier transform (LPFT). First, the interference is optimally concentrated in the time-frequency (t-f) plane and then its t-f signature is removed via a binary mask. The LPFT receiver is derived in matrix form and its optimization is performed, having in mind an influence of the binary mask on the received signal. The conventional (suboptimal) and the optimal LPFT receiver performances are compared by means of simulations carried out on the received signal corrupted by different FM types of interferences. The short-time Fourier transform (STFT) receiver is considered as a special case of the LPFT receiver and its performance is assessed simultaneously with the LPFT receiver, both in conventional and optimal case.**

I. Introduction

Direct sequence spread spectrum (DSSS) is a type of the spread spectrum (SS) communication systems where a pseudo-noise (PN) code sequence modulates an information signal before its transmission. Since the PN sequence varies faster than the information signal, the modulation expands its spectrum at the rate of *processing gain* $G = T_b/T_c$, where T_b is the information symbol (bit) duration and T_c is the PN sequence symbol (chip) duration. DSSS systems exhibit narrowband and broadband interference resistance, since the information signal is restored to its original frequency band (i.e., despread) by multiplying the received signal with a synchronized replica of the PN sequence at the receiver side, while everything else is additionally spread. In this way, despreading provides resistance to the multipath fading, since it also spreads all versions of the SS signal delayed by more than one chip duration. Moreover, since the PN sequence is known only to the transmitter and receiver, the privacy of the transmission is provided.

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The interference (jammer) may be intentionally (military communications) or unintentionally (commercial communications) superimposed on the transmitted signal. Low power interferences are substantially neutralized by despreading, but signals contaminated by high power interferences have to be preprocessed before the correlation at the receiver side is done. Numerous interference suppression techniques have been proposed in the literature in order to enhance the performance of the DSSS receiver in such severe interfering environment [1]-[8]. Most of these techniques employ an interference suppression filter preceding the correlator in the receiver scheme. This is usually an adaptive filter which exploits the pseudo-white properties of the SS signals. Recently, time-frequency (t-f) based methods have appeared, which have been shown to be very effective in improving the receiver performance when the desired signal is corrupted by broadband interferences characterized by narrowband instantaneous bandwidths.

Among significant contributions, several papers will be mentioned. Amin and coauthors [1]-[3] proposed several methods that used adaptive filtering for the nonstationary interference suppression. The instantaneous frequency (IF) of the interference can be successfully estimated by means of t-f distributions and used to construct a finite impulse response filter that reduces an interference power with a minimum possible distortion of the desired signal [1]. The optimum receiver implementing the short-time Fourier transform (STFT) interference excision system is developed in [3]. Another t-f method, namely generalized Wigner-Hough transform, has been proposed for the multiple interferences rejection [4]. The fractional Fourier transform can be a useful tool for the case of linear FM interferences [5]. A comprehensive analysis of DFT-based frequency excision algorithms has been presented in [6]. A nonparametric approach for the multiple jammers excision by using the local polynomial Fourier transform (LPFT) has been presented in [7].

This paper presents an optimization of the method proposed in [7]. In particular, introduced time-varying filtering procedure employes a binary mask to remove the interference, which was previously optimally concentrated in the t-f plane by means of the LPFT [9]. The binary mask is a two-dimensional function of time and frequency with values 0 (interference exists in (t, f)) and 1 (interference does not exist in (t, f)). However, the binary mask introduces a distinction between the received and original PN sequence, and therefore their correlation is suboptimal. This paper develops an optimal LPFT based receiver which takes into account a distortion effect of the binary mask on the received PN sequence. This receiver is shown to depend on the analysis window, the binary excision mask, the white noise power and optimal LPFT coefficients.

The paper is organized as follows. Section II presents the conventional LPFT receiver for the nonstationary interference excision in matrix form, along with a simple signal synthesis procedure. The optimal LPFT receiver is analyzed in Section III. The original PN sequence at the receiver side is modified so that it optimally corresponds to a distorted received PN sequence. The modification is obtained by means of a multiplicative matrix $\mathbf{C}$. The LPFT receiver with maximum output signal-to-noise ratio (SNR_{out}) is referred to as optimal receiver and the matrix $\mathbf{C}$ that maximizes SNR_{out} is calculated. Section IV presents results, showing analytically and numerically (by simulations) obtained SNR_{out} values and numerically obtained bit error rate (BER) values for two types of simulated FM interferences with varying parameters, linear and sinusoidal FM interferences.

II. Conventional local polynomial Fourier transform receiver

The DSSS signal description can be found in [7], [11]. The baseband received signal $x(n)$ comprises three sequences as follows

$$x\left(n\right) = s\left(n\right) + j\left(n\right) + \xi\left(n\right) \qquad (1)$$

where $s(n)$ is an SS sequence of unit amplitude, $j(n)$ is a jammer sequence and $\xi(n)$ is an additive white Gaussian noise sequence with zero mean and variance σ_ξ^2. The mutual uncorrelatedness of all the three sequences is assumed. For the SS signal of unit amplitude, SNR is defined as $\mathrm{SNR} = -20log_{10}(\sigma_\xi)$. The jammer can be analytically expressed as

$$j\left(n\right) = a_j \cos\left(\varphi\left(n\right)\right) \qquad (2)$$

where $\varphi(n)$ is the phase and a_j is the amplitude of the jammer. Jammer-to-signal ratio (JSR) is defined as $\mathrm{JSR} = 10log_{10}(P_j/P_s)$, where P_j and P_s represent the power of the jammer and SS signal, respectively [11], [13]. Besides, the SS signal characterized by one sample per chip is assumed, when perfectly flat spectrum of the SS signal is obtained [6]. Therefore, $s\left(n\right)$ equals $p\left(n\right)$ when bit "1" is transmitted and $-p\left(n\right)$ when "-1" is transmitted, where $p\left(n\right)$ is a PN sequence characterized by the length L and

$$\begin{aligned} E\left[p\left(n\right)\right] &= 0 \\ E\left[p\left(n\right)p^*\left(m\right)\right] &= \delta\left(n-m\right) \end{aligned}$$

where $E\left[\cdot\right]$ is the expectation operator, $\delta\left(n\right)$ the Dirac delta and "$*$" the conjugation operator.

The LPFT has been introduced in the t-f analysis by Katkovnik [7], [9], and the Mth order discrete form of the LPFT of the sequence $x(n)$ is defined by

$$\mathrm{LPFT}(n,k) = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} x\,(n+m)w(m) \times e^{-j\sum_{i=1}^{M}\omega_i \frac{m^{i+1}}{(i+1)!}} e^{-j\frac{2\pi}{N}mk} = \mathrm{DFT}\left(x(n+m)w(m)\,e^{-j\sum_{i=1}^{M}\omega_i \frac{m^{i+1}}{(i+1)!}}\right) \quad (3)$$

where $w(m)$ represents an analysis window, N is the number of frequency bins and ω_i is the ith transform parameter. Hereinafter, all DFTs will be calculated at a number of frequency bins equal to the PN sequence length L [3]. As (3) indicates, the LPFT of the received signal can be calculated analogously to the STFT, i.e., by sliding the analysis window $w(m)$ over the modulated received signal

$$x(n+m)e^{-j\sum_{i=1}^{M}\omega_i \frac{m^{i+1}}{(i+1)!}}$$

and implementing the FFT routine on the product of the modulated signal and window at the current position.

In order to analyze the LPFT receiver, equation (3) will be expressed in matrix form. First, a zero-padding of the sequence x should be performed by concatenating $N/2$ zeros both to the beginning and the end of x, thus creating the sequence x_z. Then, the $N \times N$ matrix $\mathbf{X}$ is formed as follows:

$$\mathbf{X} = \begin{bmatrix} x_z(1)\Theta(1,1) & \cdots & x_z(N)\Theta(1,N) \\ x_z(2)\Theta(2,1) & \cdots & x_z(N+1)\Theta(2,N) \\ \vdots & \ddots & \vdots \\ x_z(N)\Theta(N,1) & \cdots & x_z(2N-1)\Theta(N,N) \end{bmatrix} \quad (4)$$

where $\boldsymbol{\Theta}$ is the $N \times N$ matrix with the (n,m) entry

$$\Theta\,(n,m) = e^{-j\sum_{i=1}^{M}\omega_i(n)\frac{\left(m-\frac{N}{2}-1\right)^{i+1}}{(i+1)!}} \quad (5)$$

where $\omega_i\,(n)$ is the ith LPFT parameter at the nth window position. Also, define the $N \times N$ matrix

$$\mathbf{W} = \begin{bmatrix} w(1)W_N^0 & w(1)W_N^0 & \cdots & w(1)W_N^0 \\ w(2)W_N^0 & w(2)W_N^1 & \cdots & w(2)W_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ w(N)W_N^0 & w(N)W_N^{N-1} & \cdots & w(N)W_N^{(N-1)^2} \end{bmatrix} \quad (6)$$

with $W_N = e^{-j\frac{2\pi}{N}}$. Now, the LPFT of the received signal $x\,(n)$ may be written in matrix form as

$$\mathbf{LPFT} = \mathbf{X}\ \mathbf{W}. \quad (7)$$

The LPFT is a linear transform and reconstruction (synthesis) of the signal from its LPFT at the time instant n is simply obtained by summing elements of the nth row of the matrix $\mathbf{LPFT}$ [7], i.e.,

$$x'\,(n) = \frac{1}{N}\sum_{k=1}^{N} LPFT\,(n,k)$$

where $x'\,(n)$ represents the reconstructed signal. In the previous equation, $w(0) = 1$ is assumed.

The LPFT parameters ω_i for $i = 1, 2, ..., M$ are calculated so as to optimally concentrate the interference in the t-f plane for a given analysis window. To that aim, an order adaptive algorithm is developed in [7] and it is shown to keep a calculation complexity at a relatively low level. Furthermore, it is shown that the second-order LPFT produces results almost independent of the parameters of FM interferences, thus preventing a need for a time-consuming calculation of a higher-order LPFT[1]. The jammer excision is then obtained in the optimal LPFT domain by removing its t-f signature via the binary mask $\mathbf{B}$, which is the matrix with the same dimensions as $\mathbf{LPFT}$ and values 0 in all points (n,k) of the

[1]For a stationary jammer, the STFT and the optimal LPFT are the same, and therefore their calculation complexities coincide. For a linear FM jammer, the optimal LPFT complexity exceeds the STFT complexity for a number of operations required by relations (16)-(19) given in [7]. Finally, for a highly nonstationary jammer, such as sinusoidal FM jammer, the optimal LPFT calculation requires an additional number of operations for the iterative optimization of the second order parameter.

LPFT corrupted by the jammer and 1 otherwise. The synthesis is then performed on the masked LPFT to recover the jammer-free desired signal as follows:

$$x'(n) = \frac{1}{N}\sum_{k=1}^{N} LPFT(n,k)\, B(n,k)$$
$$= \frac{1}{N}\sum_{k=1}^{N}\left[\sum_{l=1}^{N} X(n,l)\, W(l,k)\right] B(n,k)$$
$$= \frac{1}{N}\sum_{l=1}^{N} X(n,l)\sum_{k=1}^{N} W(l,k)\, B(n,k)\,.$$

Denoting the product of $\mathbf{W}$ and $\mathbf{B^T}$ as $\mathbf{W_B} = \mathbf{W}\,\mathbf{B^T}$, where "T" stands for the transpose operator, we obtain

$$x'(n) = \frac{1}{N}\sum_{l=1}^{N} X(n,l)\, W_B(l,n)$$
$$= \frac{1}{N}\sum_{l=1}^{N} \left(P(n,l) + \Xi(n,l)\right) W_B(l,n) \qquad (8)$$

which implies that the main diagonal of the matrix product $\mathbf{X}\,\mathbf{W_B}$ represents the scaled reconstructed signal. In addition, since it is assumed that the excision mask $\mathbf{B}$ completely removes the jammer[2] [3], the only two components of the matrix $\mathbf{X}$ that will be hereafter considered are the PN matrix $\mathbf{P}$ and the noise matrix $\mathbf{\Xi}$, which can be obtained from (4) by setting $x_z(n) = p_z(n)$ and $x_z(n) = \xi_z(n)$, respectively. The sequences $p_z(n)$ and $\xi_z(n)$ are the zero-padded PN sequence $p(n)$ and zero-padded noise sequence $\xi(n)$, respectively.

Naturally, if the LPFT is modified by the excision mask, then the synthesized PN sequence will no longer coincide with the original one. More precisely, the synthesized PN sequence becomes a time-varying convolution between the original PN sequence and binary mask.

[2]Strictly speaking, the jammer cannot be completely removed from the t-f plane because of a finite length of the analysis window. However, for a given time instant, one can neglect a portion of a jammer power concentrated in sidelobes of its spectrum, especially when windows characterized by highly suppressed sidelobes are used (e.g., Kaiser window [7]). Therefore, in this paper, to completely remove the jammer means to excise only its main lobe. In addition, the procedure for the matrix B obtaining is presented in [7, Section V].

In particular, by setting all elements of the matrix $\mathbf{\Theta}$ to 1, i.e., each ω_i to 0, the LPFT receiver becomes the STFT receiver, so the STFT receiver can be considered as the special case of the LPFT receiver. The comparison between the conventional STFT and LPFT receiver performances for received signals corrupted by nonstationary FM interferences with variable parameters is drawn in [7].

III. Optimal local polynomial Fourier transform receiver

For the detection of an information symbol, the decision variable d is formed as the correlation of the reconstructed signal $x'(n)$ and modified receiver PN sequence $\mathbf{q} = [\mathbf{q}(\mathbf{1}), \mathbf{q}(\mathbf{2}), \cdots, \mathbf{q}(\mathbf{N})]$, that is

$$d = \sum\nolimits_{n=1}^{N} x'(n)\, q(n)\,. \qquad (9)$$

The sequence $\mathbf{q}$ and the original one $\mathbf{p}$ will be related through the $N \times N$ matrix $\mathbf{C}$, i.e., $\mathbf{q} = \mathbf{C}\,\mathbf{p}$. The conventional receiver is obtained by setting $\mathbf{C} = \mathbf{I}$, that is $\mathbf{q} = \mathbf{p}$, where $\mathbf{I}$ is the $N \times N$ identity matrix. The correlation performed between $x'(n)$ and $\mathbf{q} = \mathbf{p}$ produces suboptimal results, since it does not take into account the modification of the received PN sequence induced by the binary mask $\mathbf{B}$.

The SNR at the output of the receiver correlator [12]

$$\mathrm{SNR}_{out} = \frac{E^2[d]}{Var[d]} \qquad (10)$$

depends on the matrix $\mathbf{C}$. The receiver with maximal SNR_{out} will be referred to as optimal receiver and the matrix $\mathbf{C}$ that maximizes SNR_{out} will be herein calculated.

To start with, let us calculate $E[d]$.

$$E[d] = \frac{1}{N}E\left[\sum_{n=1}^{N} q(n)\sum_{k=1}^{N} X(n,k)\, W_B(k,n)\right]$$
$$= \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{N} E\left[X(n,k)\, q(n)\right] W_B(k,n)$$
$$= \frac{1}{N}\sum_{n,k,i=1}^{N} C(n,i)\, E\left[X(n,k)\, p(i)\right] W_B(k,n)\,. \qquad (11)$$

For the sake of brevity, the abbreviated form of the triple summation is introduced in (11). Furthermore, since $E\left[\Xi\left(n,k\right)p\left(i\right)\right]=0$ holds, $E\left[d\right]$ reduces to

$$E\left[d\right]=\frac{1}{N}\sum_{n,k,i=1}^{N}C\left(n,i\right)\times E\left[P\left(n,k\right)p\left(i\right)\right]W_B\left(k,n\right). \quad (12)$$

Since $P\left(n,k\right)=p\left(n+k-\frac{N}{2}-1\right)\Theta\left(n,k\right)$, we get

$$E[P(n,k)p(i)]=\delta(i-n-k+\frac{N}{2}+1)\Theta(n,k)$$

and therefore

$$E[d]=\frac{1}{N}\sum_{n,k,i=1}^{N}\delta(i-n-k+\frac{N}{2}+1)\times W_B\left(k,n\right)C(n,i)\Theta\left(n,k\right). \quad (13)$$

Introducing the $N\times N$ matrix $\mathbf{S}$ with the (i,n) entry

$$S\left(i,n\right)=\sum_{k=1}^{N}\delta\left(i-n-k+\frac{N}{2}+1\right)\times\Theta\left(n,k\right)W_B\left(k,n\right) \quad (14)$$

we obtain

$$E\left[d\right]=\frac{1}{N}\sum_{n=1}^{N}\sum_{i=1}^{N}C\left(n,i\right)S\left(i,n\right). \quad (15)$$

Define

$$\mathbf{C_1^H}=\left[\mathbf{c_1}\ \mathbf{c_2}\ \cdots\mathbf{c_N}\right] \qquad \mathbf{S_1}=\begin{bmatrix}\mathbf{s_1}\\ \mathbf{s_2}\\ \vdots\\ \mathbf{s_N}\end{bmatrix} \quad (16)$$

where the superscript "H" stands for the Hermitian transpose, $\mathbf{c_i}$ is the $1\times N$ vector (ith row of the matrix $\mathbf{C}$), $\mathbf{s_i}$ is the $N\times 1$ vector (ith column of the matrix $\mathbf{S}$) and $i=1,2,...,N$. Now $E\left[d\right]$ can be expressed in the following manner:

$$E\left[d\right]=\frac{1}{N}\mathbf{C_1^H}\mathbf{S_1}. \quad (17)$$

We proceed with a $Var\left[d\right]$ calculation, which is, by definition, $Var\left[d\right]=E\left[d^2\right]-E^2\left[d\right]$. The second component of $Var\left[d\right]$ has already been calculated (17), and the first one is

$$E\left[d^2\right]=\frac{1}{N^2}\times\sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N}E\left[X\left(n_1,k_1\right)X^*\left(n_2,k_2\right)p\left(i_1\right)p^*\left(i_2\right)\right]\times W_B\left(k_1,n_1\right)C\left(n_1,i_1\right)W_B^*\left(k_2,n_2\right)C^*\left(n_2,i_2\right). \quad (18)$$

The expectation term within the summation in (18) contains two additive terms as follows:

$$\begin{aligned}&E[X(n_1,k_1)X^*(n_2,k_2)p(i_1)p^*(i_2)]=\\&E[P(n_1,k_1)P^*(n_2,k_2)p(i_1)p^*(i_2)]\\&+E[\Xi(n_1,k_1)\Xi^*(n_2,k_2)]E[p(i_1)p^*(i_2)]\end{aligned} \quad (19)$$

and components of $E\left[d^2\right]$ that correspond to these terms will be calculated separately. The first component, denoted as A, equals

$$\begin{aligned}A=&\frac{1}{N^2}\times\sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N}E\left[P\left(n_1,k_1\right)P^*\left(n_2,k_2\right)p\left(i_1\right)p^*\left(i_2\right)\right]\\&\times W_B\left(k_1,n_1\right)C\left(n_1,i_1\right)W_B^*\left(k_2,n_2\right)C^*\left(n_2,i_2\right)\\&=\frac{1}{N^2}\sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N}E[p\left(n_1+k_1-\frac{N}{2}-1\right)\\&\times p^*\left(n_2+k_2-\frac{N}{2}-1\right)p\left(i_1\right)p^*\left(i_2\right)]\\&\times\Theta\left(n_1,k_1\right)W_B(k_1,n_1)\,C(n_1,i_1)\\&\times\Theta^*\left(n_2,k_2\right)W_B^*(k_2,n_2)\,C^*(n_2,i_2).\end{aligned}$$

Since the PN sequence is assumed to be a white non-Gaussian random signal of unit variance, one gets (the conjugation operator "$*$" is discarded, since $p\left(n\right)$ is real)

$$\begin{aligned}E[p(i)p(j)p(k)p(l)]=&\ \delta(i-j)\delta(k-l)\\&+\delta(i-l)\delta(j-k)+\delta(i-k)\delta(j-l)\\&-2\delta(i-j)\delta(k-l)\delta(i-k)\end{aligned}$$

which, in turn, yields

$$E[p\left(n_1+k_1-\frac{N}{2}-1\right) \times p\left(n_2+k_2-\frac{N}{2}-1\right)p\left(i_1\right)p\left(i_2\right)] =$$
$$\delta\left(n_1+k_1-n_2-k_2\right)\delta\left(i_1-i_2\right) + \delta\left(n_1+k_1-\frac{N}{2}-1-i_2\right) \times \delta\left(n_2+k_2-\frac{N}{2}-1-i_1\right) + \delta\left(n_1+k_1-\frac{N}{2}-1-i_1\right) \times \delta\left(n_2+k_2-\frac{N}{2}-1-i_2\right) - 2\delta\left(n_1+k_1-n_2-k_2\right)\delta\left(i_1-i_2\right) \times \delta\left(n_1+k_1-\frac{N}{2}-1-i_1\right). \quad (20)$$

Now, according to (20), the component A will be separated into four different components as $A = A_1 + A_2 + A_3 - 2A_4$. Let us start with A_1, i.e.,

$$A_1 = \frac{1}{N^2}\sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N}\delta\left(n_1+k_1-n_2-k_2\right)\delta\left(i_1-i_2\right) \times \Theta\left(n_1,k_1\right)W_B\left(k_1,n_1\right)C\left(n_1,i_1\right) \times \Theta^*\left(n_2,k_2\right)W_B^*\left(k_2,n_2\right)C^*\left(n_2,i_2\right)$$
$$=\frac{1}{N^2}\sum_{i_1,n_1,n_2=1}^{N}C(n_1,i_1)\,C^*(n_2,i_1) \times \sum_{k_1=1}^{N}\sum_{k_2=1}^{N}\delta(n_1+k_1-n_2-k_2)\Theta\left(n_1,k_1\right) \times W_B\left(k_1,n_1\right)\Theta^*\left(n_2,k_2\right)W_B^*\left(k_2,n_2\right).$$

Introducing the $N \times N$ matrix $\mathbf{T}$ with the (n_1, n_2) entry

$$T\left(n_1,n_2\right) = \sum_{k_1=1}^{N}\sum_{k_2=1}^{N}\delta\left(n_1+k_1-n_2-k_2\right)\Theta\left(n_1,k_1\right) \times W_B\left(k_1,n_1\right)\Theta^*\left(n_2,k_2\right)W_B^*\left(k_2,n_2\right) \quad (21)$$

A_1 may be written as

$$A_1 = \frac{1}{N^2}\sum_{i_1,n_1,n_2=1}^{N}C\left(n_1,i_1\right) \times C^*\left(n_2,i_1\right)T\left(n_1,n_2\right). \quad (22)$$

In order to express the component A_1 in matrix form, define the auxiliary matrix $\mathbf{t^1_{k_1k_2}}$ with the main diagonal entries $T(k_1, k_2)$ and 0 otherwise, that is

$$\mathbf{t^1_{k_1k_2}} = \begin{bmatrix} T\left(k_1,k_2\right) & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & T\left(k_1,k_2\right) \end{bmatrix}_{\mathbf{N\times N}}$$

and define the matrix $\mathbf{T_1}$ as

$$\mathbf{T_1} = \begin{bmatrix} \mathbf{t^1_{11}} & \mathbf{t^1_{12}} & \cdots & \mathbf{t^1_{1N}} \\ \mathbf{t^1_{21}} & \mathbf{t^1_{22}} & \cdots & \mathbf{t^1_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{t^1_{N1}} & \mathbf{t^1_{N2}} & \cdots & \mathbf{t^1_{NN}} \end{bmatrix}_{\mathbf{N^2\times N^2}}.$$

Thus we get

$$A_1 = \frac{1}{N^2}\mathbf{C_1^H T_1 C_1}. \quad (23)$$

Let us proceed with the calculation of the component A_2, i.e.,

$$A_2 = \frac{1}{N^2}\sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N}\delta\left(n_1+k_1-\frac{N}{2}-1-i_2\right) \times \delta\left(n_2+k_2-\frac{N}{2}-1-i_1\right) \times \Theta\left(n_1,k_1\right)W_B\left(k_1,n_1\right)C\left(n_1,i_1\right) \times \Theta^*\left(n_2,k_2\right)W_B^*\left(k_2,n_2\right)C^*\left(n_2,i_2\right)$$
$$= \frac{1}{N^2}\sum_{i_1,i_2,n_1,n_2=1}^{N}C\left(n_1,i_1\right)C^*\left(n_2,i_2\right) \times \sum_{k_1=1}^{N}\delta\left(n_1+k_1-\frac{N}{2}-1-i_2\right) \times \Theta\left(n_1,k_1\right)W_B\left(k_1,n_1\right) \times \sum_{k_2=1}^{N}\delta\left(n_2+k_2-\frac{N}{2}-1-i_1\right) \times \Theta^*\left(n_2,k_2\right)W_B^*\left(k_2,n_2\right)$$

which, by using (14), becomes

$$A_2 = \frac{1}{N^2} \sum_{i_1,i_2,n_1,n_2=1}^{N} C(n_1,i_1)\, C^*(n_2,i_2) \times S(i_2,n_1)\, S^*(i_1,n_2).$$

Finally, the matrix form of A_2 is

$$A_2 = \frac{1}{N^2} \mathbf{C_1^H S_2 S_3^T C_1} \tag{24}$$

where $\mathbf{S_2}$ and $\mathbf{S_3}$ are defined in the following manner:

$$\mathbf{S_2} = \begin{bmatrix} \mathbf{S}^* & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{S}^* & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{S}^* \end{bmatrix}_{\mathbf{N^2 \times N^2}}$$

and the $N^2 \times N^2$ matrix

$$\mathbf{S_3} = \begin{bmatrix} \mathbf{s_1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{s_2} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{s_N} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{s_1} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{s_2} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{s_N} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{s_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{s_2} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{s_N} \end{bmatrix}.$$

The boldface zero in $\mathbf{S_2}$ represents the $N \times N$ zero matrix, and in $\mathbf{S_3}$ represents the $N \times 1$ zero vector.

The component A_3 is given by

$$A_3 = \frac{1}{N^2} \sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N} \delta\left(n_1+k_1-\frac{N}{2}-1-i_1\right) \times \delta\left(n_2+k_2-\frac{N}{2}-1-i_2\right) \times \Theta(n_1,k_1)\, W_B(k_1,n_1)\, C(n_1,i_1) \times \Theta^*(n_2,k_2)\, W_B^*(k_2,n_2)\, C^*(n_2,i_2)$$

which, by using (17), leads to

$$A_3 = E^2[d] = \frac{1}{N^2} \mathbf{C_1^H S_1 S_1^H C_1}. \tag{25}$$

The component A_4 is defined by

$$\begin{aligned} A_4 &= \frac{1}{N^2} \sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N} \delta(n_1+k_1-n_2-k_2) \\ &\times \delta(i_1-i_2)\, \delta\left(n_1+k_1-\frac{N}{2}-1-i_1\right) \\ &\times \Theta(n_1,k_1)\, W_B(k_1,n_1)\, C(n_1,i_1) \\ &\times \Theta^*(n_2,k_2)\, W_B^*(k_2,n_2)\, C^*(n_2,i_2) \\ &= \frac{1}{N^2} \sum_{i_1,n_1,n_2=1}^{N} C(n_1,i_1)\, C^*(n_2,i_1) \\ &\times \sum_{k_1=1}^{N} \sum_{k_2=1}^{N} \delta(n_1+k_1-n_2-k_2) \\ &\times \delta\left(n_1+k_1-\frac{N}{2}-1-i_1\right) \Theta(n_1,k_1) \\ &\times W_B(k_1,n_1)\, \Theta^*(n_2,k_2)\, W_B^*(k_2,n_2). \end{aligned}$$

Introducing

$$\begin{aligned} R(i_1,n_1,n_2) &= \sum_{k_1=1}^{N} \sum_{k_2=1}^{N} \delta(n_1+k_1-n_2-k_2) \\ &\times \delta\left(n_1+k_1-\frac{N}{2}-1-i_1\right) \Theta(n_1,k_1) \\ &\times W_B(k_1,n_1)\, \Theta^*(n_2,k_2)\, W_B^*(k_2,n_2) \end{aligned} \tag{26}$$

A_4 can be rewritten in the form

$$A_4 = \frac{1}{N^2} \sum_{i_1,n_1,n_2=1}^{N} C(n_1,i_1)\, C^*(n_2,i_1) \times R(i_1,n_1,n_2).$$

Similar to the A_2 calculation case, define the $N \times N$ auxiliary matrix $\mathbf{t^2_{k_1k_2}}$ as

$$\mathbf{t^2_{k_1k_2}} = \begin{bmatrix} R(1,k_1,k_2) & 0 & \cdots & 0 \\ 0 & R(2,k_1,k_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R(N,k_1,k_2) \end{bmatrix}$$

and the matrix $\mathbf{T_2}$ as

$$\mathbf{T_2} = \begin{bmatrix} \mathbf{t^2_{11}} & \mathbf{t^2_{12}} & \cdots & \mathbf{t^2_{1N}} \\ \mathbf{t^2_{21}} & \mathbf{t^2_{22}} & \cdots & \mathbf{t^2_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{t^2_{N1}} & \mathbf{t^2_{N2}} & \cdots & \mathbf{t^2_{NN}} \end{bmatrix}_{\mathbf{N^2 \times N^2}}.$$

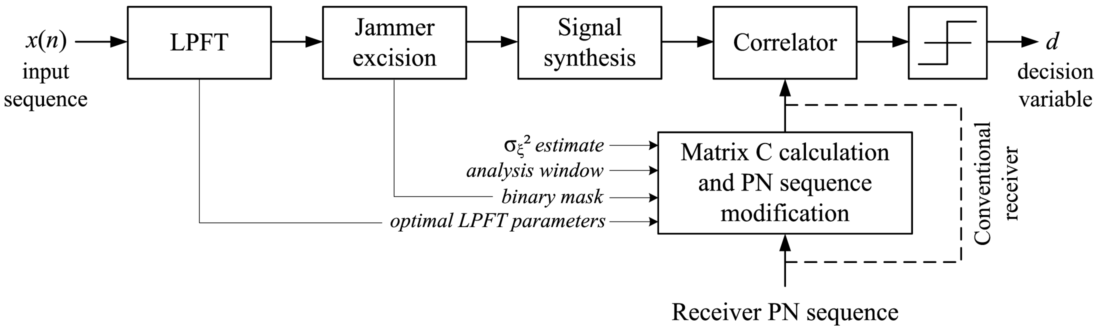

Fig. 1. Block diagram of the optimal LPFT receiver. The conventional receiver is obtained when no modification of the receiver PN sequence is performed (dashed line).

Thus we obtain

$$A_4 = \frac{1}{N^2}\mathbf{C_1^H T_2 C_1}. \tag{27}$$

Now that we have completed the calculation of the first component of $E\left[d^2\right]$ (recall (18) and (19)), we can return to calculate the second one, which will be denoted as D. We have

$$\begin{aligned} D &= \frac{1}{N^2}\sum_{\substack{n_1,k_1,i_1,\\ n_2,k_2,i_2=1}}^{N} E\left[\Xi\left(n_1,k_1\right)\Xi^*\left(n_2,k_2\right)\right] \\ &\times E\left[p\left(i_1\right)p^*\left(i_2\right)\right]W_B\left(k_1,n_1\right) \\ &\times C\left(n_1,i_1\right)W_B^*\left(k_2,n_2\right)C^*\left(n_2,i_2\right) \\ &= \frac{1}{N^2}\sum_{i_1,n_1,n_2=1}^{N} C\left(n_1,i_1\right)C^*\left(n_2,i_1\right) \\ &\times \sum_{k_1=1}^{N}\sum_{k_2=1}^{N} E\left[\Xi(n_1,k_1)\,\Xi^*(n_2,k_2)\right] \\ &\times W_B(k_1,n_1)W_B^*(k_2,n_2) \end{aligned}$$

which comes to the A_1 calculation case, i.e.,

$$D = \sigma_\xi^2 A_1 = \frac{\sigma_\xi^2}{N^2}\mathbf{C_1^H T_1 C_1}. \tag{28}$$

Finally, the SNR at the output of the receiver correlator is

$$\begin{aligned} \mathrm{SNR}_{out} &= \frac{A_3}{A+D-A_3} \\ &= \frac{\mathbf{C_1^H S_1 S_1^H C_1}}{\mathbf{C_1^H}\left((1+\sigma_\xi^2)\mathbf{T_1}+\mathbf{S_2 S_3^T}-2\mathbf{T_2}\right)\mathbf{C_1}} \\ &= \frac{\mathbf{C_1^H Z C_1}}{\mathbf{C_1^H Y C_1}} \end{aligned} \tag{29}$$

where $\mathbf{Z}$ and $\mathbf{Y}$ are the $N^2 \times N^2$ matrices defined by

$$\begin{aligned} \mathbf{Z} &= \mathbf{S_1 S_1^H} \\ \mathbf{Y} &= \left(1+\sigma_\xi^2\right)\mathbf{T_1}+\mathbf{S_2 S_3^T}-2\mathbf{T_2}. \end{aligned} \tag{30}$$

The final form of SNR_{out} in (29) can be considered to be a Rayleigh quotient. Matrices $\mathbf{T_1}$, $\mathbf{S_2 S_3^T}$ and $\mathbf{T_2}$ are Hermitian, which implies that the matrices $\mathbf{Y}$ and $\mathbf{Y}^{\frac{1}{2}}$ are also Hermitian. This fact allows us to define the vector $\hat{\mathbf{C}}_\mathbf{1} = \mathbf{Y}^{\frac{1}{2}}\mathbf{C_1}$ and to rewrite (29) as follows:

$$\mathrm{SNR}_{out} = \frac{\hat{\mathbf{C}}_\mathbf{1}^\mathbf{H}\left[\left(\mathbf{Y}^{-\frac{1}{2}}\right)^\mathbf{H}\ \mathbf{Z}\,\mathbf{Y}^{-\frac{1}{2}}\right]\hat{\mathbf{C}}_\mathbf{1}}{\hat{\mathbf{C}}_\mathbf{1}^\mathbf{H}\hat{\mathbf{C}}_\mathbf{1}}. \tag{31}$$

The Rayleigh quotient states that SNR_{out} reaches its maximum when $\hat{\mathbf{C}}_\mathbf{1}$ is an eigenvector of the $N^2 \times N^2$ matrix $\left(\mathbf{Y}^{-\frac{1}{2}}\right)^\mathbf{H}\ \mathbf{Z}\,\mathbf{Y}^{-\frac{1}{2}}$ corresponding to its largest eigenvalue. We will denote such a vector as $\hat{\mathbf{C}}_{\mathbf{1}\,\max}$. The optimal correlator is then obtained by rearranging the $N^2 \times 1$ optimal vector

$$\mathbf{C}_{\mathbf{1}\ \mathrm{opt}} = \mathbf{Y}^{-\frac{1}{2}}\ \hat{\mathbf{C}}_{\mathbf{1}\,\max} \tag{32}$$

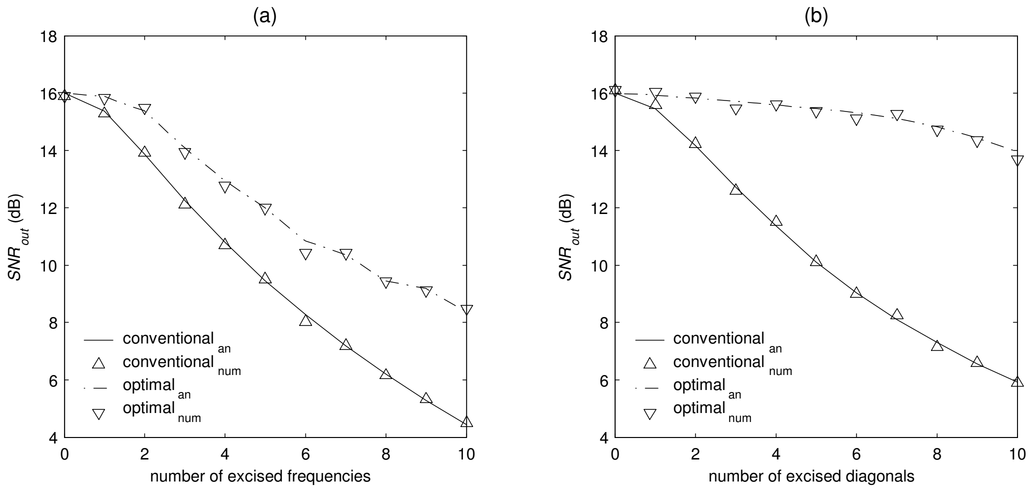

Fig. 2. Output SNR for the conventional (suboptimal) and the optimal receiver under the binary excision of the LPFT. Curves represent analytical results according to (31). Triangles represent numerical values according to (10) over 20000 realizations.

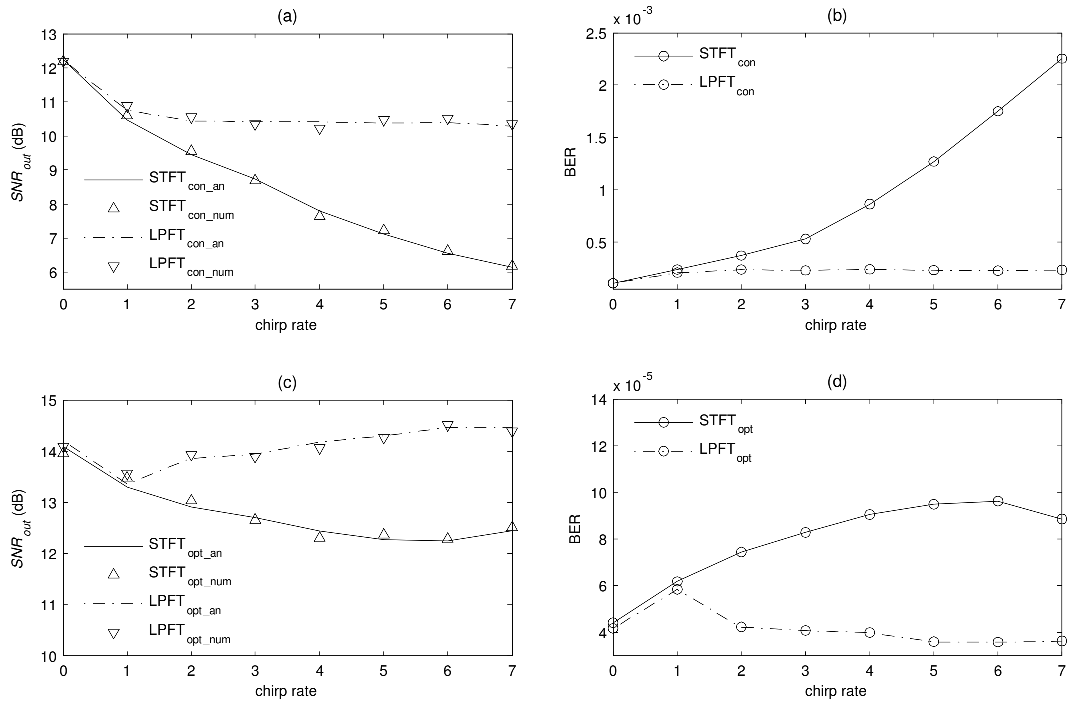

Fig. 3. Linear FM interference case. *Left side*: Output SNR for (a) conventional and (c) optimal receivers. Curves represent analytical results according to (31). Triangles represent numerical values according to (10) over 20000 realizations. *Right side*: Numerical BER for (b) conventional and (d) optimal receivers.

back into the $N \times N$ matrix $\mathbf{C}$ according to (16).

The equations (29)-(32) and the previous analysis show that the optimal LPFT receiver depends on the analysis window employed in the LPFT calculation, optimal LPFT parameters obtained in the jammer concentration procedure, the binary excision mask and the added white noise variance σ_ξ^2. The optimal LPFT receiver block diagram is shown in Fig. 1. A variance σ_ξ^2 estimation procedure is presented in [14]. The conventional LPFT receiver is simply obtained by bypassing the PN sequence modification block (dashed-line).

xample 1: To compare the conventional and the optimal LPFT receiver, we calculated the corresponding SNR_{out} values for two different excision cases. In the first case, the binary mask cancels frequency slices in the middle of both the positive and negative part of the spectrum (stationary jammer case), from 0 up to 10 slices. We begin with the central frequency, and then cancel one adjacent upper frequency, one adjacent lower frequency and so forth. In the second case, the binary mask cancels diagonal slices of both the positive and negative part of the spectrum (linear FM jammer case), from 0 up to 10 slices. We begin with the main diagonal, and then cancel one adjacent upper diagonal, one adjacent lower diagonal and so forth. In both cases, the length of the PN sequence was set to 32, the Hanning window characterized by 32 samples was employed as the analysis window and $\sigma_\xi^2 = 2$. In addition, numerical SNR values were calculated according to (10) over 20000 realizations of the decision variable d.

The obtained analytical SNR_{out} curves are shown in Fig. 2, along with the corresponding numerical values shown with triangles. In the first excision case, all the LPFT parameters in $\mathbf{\Theta}$ were set to 0, since this case corresponds to the stationary interference excision, i.e., the LPFT and STFT receiver coincide. In the second case, only ω_1 parameters were non-zero, since this case corresponds to a linear FM interference excision. Furthermore, ω_1 is constant and set to a predefined value at which the interference that diagonally sweeps the entire t-f plane (i.e., IF trajectory of the interference coincides with the main diagonal of the positive part of the LPFT spectrum) is optimally concentrated. With this setup, the LPFT also produces the same SNR values as the STFT receiver, which is quite straightforward to prove analytically. Nevertheless, for a given nonstationary interference, the LPFT will always produce better results, since it optimally concentrates the interference in the t-f plane and therefore excises less number of diagonals.

It is evident from Fig. 2 that the optimal receiver is less immune to the frequency slices excision than to the diagonal slices excision. This kind of difference stems from the fact that by removing a single frequency we permanently lose an information component at that frequency, while with removing a single diagonal a removed frequency bin at one time instant is still available at another one.

IV. Simulations

In the examples we present hereinafter, two types of simulated monocomponent interferences with variable parameters are considered, linear and sinusoidal FM interference. Analytically obtained SNR_{out} curves are numerically confirmed over 20000 realizations of the decision variable d, while BER values are computed over 20 million runs. As in the first example, the PN sequence length is $L = 32$, the Hanning window with 32 samples is used as the analysis window and $\sigma_\xi^2 = 2$. Besides, it should be acknowledged that, for each value of an interference parameter, the excision matrix $\mathbf{B}$ (and therefore $\mathbf{C_1}_{\text{opt}}$) is not determined for each run separately, but only once, i.e., for the first realization of the STFT and LPFT of the received signal. Therefore, for each value of the interference parameter, the optimal LPFT parameters are also calculated only in the first run and such parameters were used in all other runs. The STFT receiver is considered as the special case of the LPFT receiver.

xample 2: The aim of this example is to assess performances of the conventional and the optimal STFT and LPFT receivers when the desired signal is corrupted by a linear FM interference with a variable modulation index (chirp rate). The interference is characterized by JSR = 30dB[3] and its chirp rate varies

[3]The proposed method will produce satisfying results as long as a remaining jammer power, concentrated in its sidelobes, can be neglected. For the applied Han-

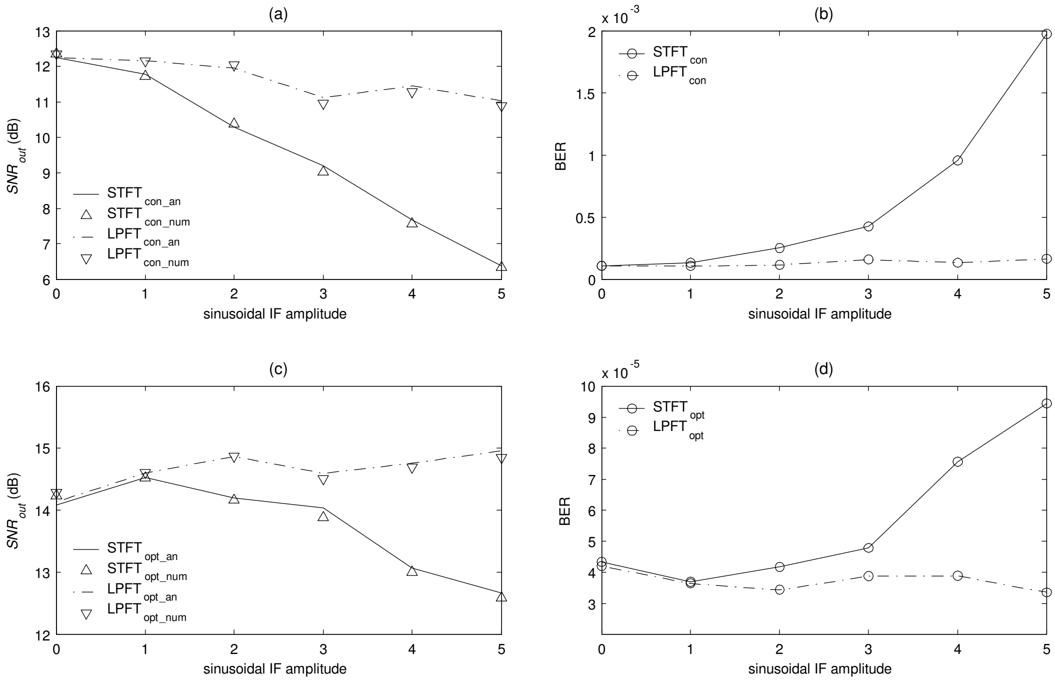

Fig. 4. Sinusoidal FM interference case. *Left side*: Output SNR for (a) conventional and (c) optimal receivers. Curves represent analytical results according to (31). Triangles represent numerical values according to (10) over 20000 realizations. *Right side*: Numerical BER for (b) conventional and (d) optimal receivers.

from 0 (a pure sinusoid in the middle of the spectrum) to a value at which the interference sweeps the entire spectrum, i.e., when we get the chirp with the IF values $f(t_1) = 0$ and $f(t_2) = f_{max}$, where $[t_1, t_2]$ is the time interval considered in the LPFT calculation and f_{max} is the highest frequency in the LPFT spectrum. Obtained SNR_{out} values versus chirp rate are shown in Fig. 3(a) and (c). The STFT and LPFT BER values versus chirp rate for both the conventional and the optimal case are shown in Fig. 3(b) and (d).

xample 3: In this example, performances of the conventional and the optimal STFT and LPFT receivers, when the desired signal is corrupted by a sinusoidal FM signal with a variable amplitude of an IF variation, are evaluated. The interference is characterized by JSR = 20dB and the amplitude of its sinusoidal IF law varies from 0 (a pure sinusoid in the middle of the spectrum) to a value at which the interference sweeps the entire frequency spectrum, i.e., when the lowest frequency in the LPFT spectrum corrupted by the interference is 0 and the highest corrupted frequency is f_{max}. Obtained SNR_{out} values versus amplitude of the sinusoidal IF law are shown in Fig. 4(a) and (c). The STFT and LPFT BER values versus amplitude of the sinusoidal IF law for both the conventional and the optimal case are shown in Fig. 4(b) and (d).

As Fig. 3 and 4 indicate, the optimal STFT and LPFT receivers exhibit significantly improved performances respectively compared to the conventional ones. In addition, LPFT_{opt} always produces the best results. It may seem unexpected that the SNR_{out} curves in the optimal LPFT case exhibit increasing trend (i.e., corresponding BER curves exhibit decreasing trend) as the interference parameter increases in both examples, but having in mind that with the optimal LPFT receiver the excised area in the t-f plane is approximately the same

ning window, this holds for the JSR up to approximately 40dB. For higher JSR values other windows should be used (see [7]).

for all interference parameter values [7], this ambiguity is partially resolved. Furthermore, for small parameter values, the interference is nearly stationary, which corresponds to the frequencies excision case from the first example. By increasing the parameter value we are approaching the diagonals excision case, which is characterized by better receiver performance (see Fig. 2).

V. Conclusion

This paper has presented an optimal local polynomial Fourier transform approach for a monocomponent jammer excision in DSSS communication systems. The purpose of the LPFT was to optimally concentrate the jammer in the t-f plane. The jammer excision was then obtained by removing its t-f signature via a binary mask. The binary mask inherently introduces a distortion of the received PN sequence, and therefore the decision making process at the output of the receiver correlator is suboptimal. The LPFT receiver was derived in matrix form and its optimization was performed, i.e., the receiver PN sequence was modified so as to maximize SNR_{out}. The conventional and the optimal LPFT receiver performances were evaluated by means of simulations carried out on the received signal corrupted by monocomponent linear and sinusoidal FM jammers with variable parameters. The optimal LPFT receiver exhibits significantly improved performance compared to the conventional one, which was verified by means of analytically obtained (and numerically confirmed) SNR_{out} values and numerically computed BER values. The STFT receiver was considered as the special case of the LPFT receiver and its performance was assessed simultaneously with the LPFT receiver performance.

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