An Analysis of Quantization Influence on Optimal Detection of Multiplicative Watermark

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***Abstract* - Analysis of quantization effects on watermarked coefficients and watermark is considered. A criterion for selection of coefficients suitable for watermarking in the presence of quantization is derived. The effects of quantization are further analyzed in terms of the probability of detectable watermark and the probability of zero-quantized watermarked coefficients. This analysis is used to define an image watermarking procedure that provides robustness to an arbitrary JPEG compression/quantization degree. Theoretical results are illustrated by examples.**

***Keywords* - Image watermarking; JPEG compression; Optimal detection**

I. INTRODUCTION

Digital watermarking has been developed for the purpose of digital multimedia data protection [1], [2]. Watermarking is usually based on secret signal embedding into multimedia data. Watermark embedding can be done in the time/spatial [3] or in the transform domain (DFT, DCT, DWT, or time-frequency domain using time-varying mask) [4]-[8]. As the one of particular interest in the presence of JPEG compression, the 8x8 block-based DCT domain has been considered [9], [11]. Among different requirements imposed by watermarking applications, the robustness to different attacks is probably one of the most important besides the watermark imperceptibility. The robustness mostly depends on the watermark strength and watermarking coefficients selection that is usually done empirically. In this paper, we focus on the JPEG quantization effects as a very common and almost inevitable attack in image processing. Hence, the idea is to perform detailed analysis of quantization effects in order to derive the criterion for selection of coefficients suitable for watermarking in the presence of quantization. This principle can be further extended to any attack that can be analytically modeled, such as the noise probability density function.

The influence of quantization on watermarked DCT coefficients and watermark itself has been initially investigated in [11]. Therein, the coefficients selection criterion was derived, but the analysis assumed already quantized DCT coefficients. It means that the quantization error was not considered and the negative quantization effects were neglected at the cost of lower image quality (coefficients were quantized before watermark embedding). This work represents an extension of the approach proposed in [11]. The influence of quantization error to watermark and watermarked coefficients is considered. The analysis focuses to the estimation of detectable watermark amount under quantization. Furthermore, the probability of zero-quantized watermarked coefficients is derived. Based on this analysis a watermark, created as a pseudo random sequence, is embedded by using the multiplicative procedure. The efficient watermark detection is provided, where the optimal detector form is obtained from the statistical characteristics of selected DCT coefficients [12]-[14]. The theoretical considerations are illustrated by the examples.

II. WATERMARKING PROCEDURE

Quantization can be considered as a part of the standard compression algorithms. As a common case, the JPEG quantization applied on the DCT coefficients in the 8x8 blocks is considered [7]-[11]. In watermarking, the quantization can affect the efficiency of watermark detection, since the watermarked coefficients could be significantly altered. Thus, the analysis of quantization influence on watermarked coefficients is provided in this Section.

A. Selection of watermarking coefficients

Let us consider the multiplicative procedure for watermark embedding given in form:

$$I_w(i,j) = I(i,j) + \alpha \left| I(i,j) \right| w, \tag{1}$$

where parameter α controls the watermark strength, while $I(i,j)$ represents image coefficient at the (i,j) position selected for watermarking. In the presence of quantization, the coefficients that are less important from the perceptual point of view could be quantized to zero value. If we consider such coefficients in watermarking, these would be useless in watermark detection (they do not contribute to the detector response). Thus, in order to avoid rounding to zero-value, the watermarking coefficients should be selected according to the following condition:

$$\left| I(i,j) \right| - \left| W \right| \geq \frac{Q(i,j)}{2} \quad , \tag{2}$$

where $W = \alpha w \left| I \right|$, while the quantization matrix $Q(i,j)$ defines the quantization step. Here, we considered worst case scenario where the image coefficients I and W are of opposite signs. Note that a certain image coefficient $I(i,j)$, is quantized to the value $K(i,j)Q(i,j)$, where $K(i,j) = round(I(i,j)/Q(i,j))$. Since the lowest value of $I(i,j)$ that is quantized to $K(i,j)Q(i,j)$ is: $|I(i,j)| = |K(i,j)| Q(i,j) \text{-} Q(i,j)/2$, the above relation becomes:

$$|W| < (|K(i,j)|-1)Q(i,j)\,. \qquad (3)$$

In order to satisfy (3), it is obvious that $|K(i,j)| \geq 2$ holds.

Minimal value of $|K(i,j)|$ is denoted as floor value K_f. Thus, $K_f=2$ represents a criterion for selection of coefficients for watermarking in the presence of quantization.

B. Statistical model of selected watermarking coefficients and corresponding optimal detector form

The optimal watermark detector is usually obtained by using the statistical model of the coefficients selected for watermarking. As a model for the probability density function (pdf) of DCT coefficients, different functions were used in the literature: Gaussian, Generalized Gaussian (GGF) [7], [9], Laplacian function [15], etc. However, if the coefficients are selected according to the criterion $K_f=2$, the pdf is significantly altered [16], as illustrated in Fig.1. Based on the GGF, the pdf of these coefficients can be approximately modeled as [14]:

$$p(Iw) \simeq \frac{(\frac{Iw}{a})^{2n}}{1+(\frac{Iw}{a})^{2n}} \exp(-abs(\frac{Iw}{a})^{2\gamma})\,, \qquad (4)$$

where parameter a defines the position of pdf maxima, and the parameter n controls the rate of decay between maximum and the origin. Parameter γ can take values ½, 1, and 2.

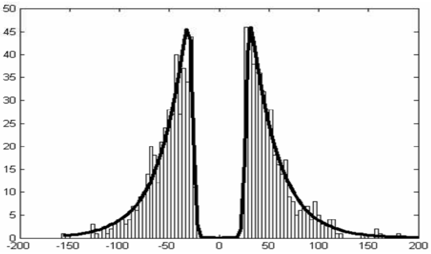


Figure 1. Histogram of the DCT coefficients and approximated pdf

Now, based on the watermarking coefficients pdf, we can define the optimal detector form as follows [14], [17]:

$$D = \sum_{i=1}^{L} -w_i \frac{p'(I_w)}{p(I_w)}\,, \qquad (5)$$

where L is the number of watermarked coefficients, while $p(I_{w_i})$ and $p'(I_{w_i})$ represents the coefficients pdf and its first derivative, respectively. According to (4) and (5), for $\gamma=1$, the optimal detector form is defined as:

$$D_{opt} = \sum_{i=1}^{K} w_i (I_{w_i} - \frac{na^2}{I_{w_i}(1+(\frac{I_{w_i}}{a})^{2n})})\,. \qquad (6)$$

III. ANALYSIS OF QUANTIZATION EFFECTS

The watermark is often created as a pseudo-random sequence, e.g., Gaussian sequence. In the presence of quantization only a certain amount of watermark will be detectable due to the rounding operation. Here, we distinguish two critical cases: 1) original and watermarked coefficient are quantized to the same value, or in other words, the quantization neutralized watermark sample belonging to the observed coefficient; 2) watermarked coefficient is quantized to zero (quantization destroyed the watermarked coefficient).

A. Probability of detectable watermark

In the sequel, we provide the analysis of quantization error to watermark detectability. The watermark scaling factor $\alpha|I(i,j)|$ is approximated as $\alpha\cdot K(i,j)Q(i,j)$. The difference between original and quantized coefficient is the quantization error: $e(i,j) = I(i,j) - K(i,j)Q(i,j)$, $e \in (-Q/2, Q/2)$ (from this point forward we omit notation (i,j) to simplify expressions). An illustration of quantization error is given in Fig.2.

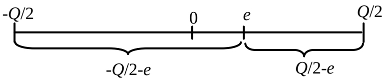


Figure 2. Illustration of the quantization error e

The quantization error influences watermark detectability. Namely, the watermark will not be detectable after quantization if the value $W=\alpha|K|Qw$ added to the coefficient I falls within the quantization interval $(-Q/2-e, Q/2-e)$ for $e>0$. The analogy holds for negative e. Therefore, the probability of detectable watermark in the presence of quantization can be calculated according to:

$$P(e) \simeq 1 - \frac{1}{Q} \int_{0}^{\frac{Q}{2}} (erf(\frac{\frac{Q}{2}-e}{\sqrt{2}\sigma}) - erf(\frac{-\frac{Q}{2}-e}{\sqrt{2}\sigma}))de \qquad (7)$$

and it actually represents the percentage of detectable watermark amount. Here, $\sigma \simeq \alpha\cdot|K|\cdot Q\cdot\sigma_\omega = \alpha\cdot|K|\cdot Q$, since the standard deviation of watermark is $\sigma_\omega=1$. After few mathematical operations, the approximate expression for probability of detectable watermark can be written as:

$$P = 1 + \frac{\sqrt{2}\alpha|K|}{\sqrt{\pi}} - erf(\frac{1}{\sqrt{2}\alpha|K|}) - \frac{1}{\sqrt{\pi}} * \sqrt{2}\alpha|K| * e^{\frac{-1}{(\sqrt{2}\alpha|K|)^2}}) \quad (8)$$

Probabilities of detectable watermark, for different values of α and $|K|$, are given in Table I.

The total probability (the detectable watermark amount for all $|K|$) is calculated as:

$$P_u = \frac{1}{N} \sum_{|K|=1}^{\max(|K|)} P(|K|,\alpha)\cdot n(|K|) \qquad (9)$$

where $P(|K|,\alpha)$ represents the probability for particular $|K|$, while $n(|K|)$ is the number of coefficients with same $|K|$:

$$N = \sum_{|K|} n(|K|). \qquad (10)$$

In this way, the total amount of detectable watermark can be determined in advance depending on the coefficients chosen for watermarking (depending on *K*) and the watermark embedding strength.

TABLE I. PROBABILITIES OF DETECTABLE WATERMARK AMOUNT

α |K|	0.09	0.15
1	7%	11%
2	14.4%	24%
3	21.5%	35.5%
4	28.7%	45.5%
5	35.5%	53.5%
6	41.7%	59.8%
7	47.2%	64.5%
10	59.8%	74.3%
15	71.7%	82.5%
20	78.4%	86.8%
30	85.4%	91.2%
40	89%	93.4%

B. *Probability of zero-quantized watermarked coefficients*

Another critical case, from the watermark detection standpoint, appears when the watermarked coefficients are quantized to zero as a consequence of watermark embedding. Therefore, in order to derive the probability of zero-quantized watermarked coefficient, let us firstly define the properties of watermark. According to the watermark embedding procedure (1), the zero-quantized watermarked coefficients appear when:

$$(|K|+r)Q-\alpha(|K|+r)Q|w|<\frac{Q}{2}\,, \tag{11}$$

where $e=rQ$. Consequently, to avoid zero-quantized watermarked coefficients, the watermark should satisfy the following condition:

$$|w|<\frac{1}{\alpha}\left(1-\frac{1}{2(|K|+r)}\right) \tag{12}$$

Further, the probability that the image coefficient will be rounded to zero-value, due to watermark embedding and quantization, can be calculated according to:

$$P(|K|,\alpha)=\int_{-1/2}^{1/2-\delta} erf\left(\frac{1}{2\alpha}\left(1-\frac{1}{2(|K|+r)}\right)\right)dr \tag{13}$$

Since the quantization error *e* cannot obtain value ½, for the upper integral limit, parameter δ (with small value for example 0.001) is used. Note that according to (11), P(1,0.09)=4.08% while already for |*K*|=2 holds P(2,0.09)=0.01%. Similarly, P(1,0.15)=7.5%, while P(2,0.15)=0.01%. Therefore, the probability that the coefficients selected using |*K*|≥2 will be zero-quantized is very low, which is in accordance with analysis performed in Section II.A, and these can be considered for watermarking.

Having in mind previous consideration and Table I it is obvious that higher |*K*| will assure higher probability of detectable watermark and lower probability of zero-quantized coefficients. However, using only higher values of |*K*| means that only the strongest DCT coefficients (but the DC) will be watermarked. The number of coefficients having high |*K*| is relatively small, and consequently, correlation based watermark detector might fail, unless we increase the watermark embedding strength which will then produce low PSNR and image degradation. Note that proposed analysis of quantization effects can be used as a model for other watermarking attacks that could be mathematically modeled. For example, in the case of noise, its pdf could be approximated by a certain model, which can be further used to analyze the influence to watermark detection.

IV. EXAMPLES

Example 1: In the following example, the total probability of detectable watermark is experimentally evaluated and compared with the theoretical results. Note that the purpose of this example is just to verify empirically the expression for total probability of detectable watermark (9). The watermark is created as a Gaussian sequence. The DCT coefficients (but the DC) from the 8x8 blocks of the Lena image, satisfying 3<|*K*|<8, are considered. Watermark is embedded according to (1), with α=0.15. The experimental quantization matrix Q_{50} (quality factor *QF*=50) is used.

In order to compute the percentage of the detectable watermark, the quantized original and quantized watermarked coefficients are compared in the software simulations. The procedure is done for 1000 trials. Among the 1120 coefficients that satisfy 3<|*K*|<8, the average number of coefficients with detectable watermark is 593 i.e. 52.9%. In the considered sequence of the DCT coefficients, appear 448 with |*K*|=4, 311 with |*K*|=5, 215 with |*K*|=6, and 146 with |*K*|=7. According to (9) and corresponding probabilities for |*K*| (from Table I), the probability of detectable watermark amount is 52.94%, and it is almost the same as in the simulation algorithm.

Although the probability of detectable watermark is higher than 50%, the total number of coefficients is not sufficient to provide reliable watermark detection based on the correlation property. Thus, in the following example we include more coefficients to provide a reliable detection.

Example 2: DCT coefficients (from 8x8 blocks) satisfying 2<|*K*|<10 (for quantization matrix Q_{50}) are used for watermarking. Watermark is created as pseudo-random sequence, while α=0.15 is used in the embedding procedure. In this case, the total number of coefficients used for watermarking is 2045, probability of detectable watermark is 48,76%, which means that 997 samples can contribute to detection in the presence of quantization. This is approximately the minimal number of coefficients that can provide reliable detection and it is obtained experimentally.

The watermarking procedure is done for 100 watermarks (right keys), with PSNR≈44dB (average value for 100 watermarks). The original and watermarked images Lena are shown in Fig. 3. The detector form defined by (6) with *n*=8, is tested for right key (watermark) and 100 wrong trials. As a measure of detection quality, the following relation is used:

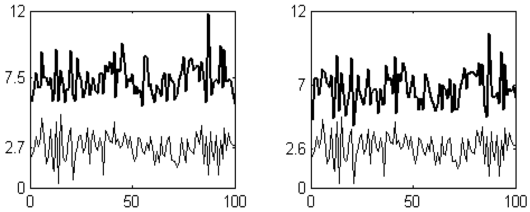
$$R=\frac{\overline{D}_{key}-\overline{D}_{wrong}}{\sqrt{\sigma^2_{key}+\sigma^2_{wrong}}}\,, \tag{14}$$

where $\overline{D}$ and σ^2 represents the mean values and the standard deviation of the detector responses, while notations *key* and *wrong* indicate the right key and wrong trials.

(a) (b)

Figure 3. a) Original image Lena, b) Watermarked image Lena

In order to test the optimal detector efficiency, it is compared with the standard correlator. The measures of detection quality are shown in Fig. 4 (before and after quantization attack).



(a) (b)

Figure 4. Measure *R* for optimal detector (thick line) and standard correlator (thin line) for Lena image coefficients satisfying K_f=2: a) before quantization, b) after quantization

It is obvious that the optimal detector for the proposed coefficients selection provides significantly better results compared to the standard correlation detector. Also, the quantization do not influence significantly the detectors responses if the coefficients are selected according to K_f=2. The slightly lower detector responses after the quantization appear as a consequence of lower amount of detectable watermark. Note that the robustness of the procedure is provided for all compression degrees defined by *QF*>50. The measures of detection quality *R* (average values for 100 trials) for different test images are given in Table II (*R* is related to the probability of detection error as: $P_{err} = 1/2 erfc(R/\sqrt{2})$ which means that we need at least *R*=4 for $P_{err}=10^{-5}$).

TABLE II. MEASURES OF DETECTION QUALITY R

Test images	**Optimal Detector**		**Standard Correlator**	
	Before quant.	**After quant.**	**Before quant.**	**After quant.**
Lena	8	7	3	2.6
Baboon	11	9.5	4.1	3.6
Barbara	8.7	7.7	3.4	3.1
Boat	8.9	8.1	3.4	3.2
Pepper	7.8	7.1	2.9	2.6

V. CONCLUSION

The influence of JPEG quantization attack to image watermarking has been considered. A criterion for section of coefficients that contribute in detection after quantization is derived. This coefficients selection criterion imposed a specific pdf of watermarked coefficients, used to define the optimal detector form. The probability of detectable watermark has been derived allowing us to assure in advance sufficient number of coefficients that contribute in detection. The proposed approach could be extended to include analysis for other attacks.

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