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A VIRTUAL INSTRUMENT FOR HIGHLY CONCENTRATED TIME-FREQUENCY DISTRIBUTIONS

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***Abstract* – A virtual instrument for highly concentrated time-frequency distributions is proposed in this paper. It provides efficient solutions for the analysis of signals with highly non-stationary instantaneous frequency. The time-frequency distributions of different orders are considered. Thus, due to the variations of signal phase function, the order of the distribution can be increased until a satisfactory concentration is achieved. The instrument could be used for both monocomponent and multicomponent signals. Application of this instrument is demonstrated on the examples.**

***Keywords* – time-frequency distributions, complex-lag argument, virtual instrument**

1. INTRODUCTION

Time-frequency analysis has become very popular and attractive research area in the last few decades. Consequently, various time-frequency distributions have been proposed [1]-[4]. They are used in numerous practical applications in the areas of biomedical signal analysis, radar imaging, geosciences, communications, speech and image processing, etc. Depending on signal phase function, different distributions could be used: linear (spectrogram), quadratic distributions (the Wigner distribution, the S-method, the distributions from the Cohen class [1],[5]), and higher order time-frequency distributions (polynomial distributions, distributions with complex-lag argument, etc. [6]-[10]).

Various software and hardware solutions for realizations of time-frequency distributions have been proposed [11]-[13]. In [11] the virtual instrument based on the short time Fourier transform and S-method have been proposed. In this paper we propose a virtual instrument that includes highly concentrated time-frequency distributions. It provides an efficient instantaneous frequency analysis for a large class of signals. The proposed instrument includes the following time-frequency distribution: the spectrogram, the Wigner distribution, the S-method, the fourth and the sixth order complex-lag time-frequency distributions. One may use this instrument either to calculate each distribution separately or to compare the results obtained by using different distributions. The proposed instrument could be useful for researchers and practitioners working in this field.

The paper is organized as follows. The review of time-frequency distributions considered within the virtual instrument is given in Section 2. Section 3 describes the outlook of the instrument, while its application for the analysis of monocomponent and multicomponent signals are presented through the examples in Section 4. Concluding remarks are given in Section 5.

2. THEORETICAL BACKGROUNG FOR VIRTUAL INSTRUMENT

A short review of time-frequency distributions that are implemented in the proposed virtual instrument is provided in this Section.

Spectrogram

The spectrogram (square module of the Short Time Fourier Transform - STFT) is the simplest and most commonly used time-frequency representation. It is defined as:

$$SPEC(n,k) = \left|STFT(n,k)\right|^2 = \left|\sum_{m=-N_w/2}^{N_w/2-1} x(n+m)w(m)e^{-j2\pi mk/N_w}\right|^2, \quad (1)$$

where $w(m)$ is a window function, n and k are the discrete time and frequency variables, while N_w is the signal length. Due to its simplicity, the STFT can be used as a base for realization of other time-frequency distributions. In the case of multicomponent signal, the STFT does not produce cross-terms and it is equal to the sum of individual STFTs. However, the main drawback of this representation is the trade off between time and frequency resolution. If narrow window is used the time resolution is good, while for a wide window good frequency resolution is provided. In order to improve the time-frequency resolution, the quadratic time-frequency distributions have been introduced.

Wigner distribution

The Wigner distribution is the most commonly used quadratic time-frequency distributions. The pseudo form of the Wigner distribution, in the discrete domain, is defined as:

$$WD(n,k)=2\sum_{m=-N_w/2}^{N_w/2} x(n+m)x^*(n-m)e^{-j2\pi 2mk/N_w} \quad (2)$$

Unlike the STFT, the Wigner distribution does not depend on the window width and may achieve good resolutions in both time and frequency directions. In the case of linear frequency modulated signals, Wigner distribution produces an ideal concentration along the instantaneous frequency. However, for a multicomponent signal $x(n)=\sum_q x_q(n)$, the Wigner distribution produces a large amount of cross-terms. In order to reduce the number of cross-terms, but to preserve the same concentration of auto-terms as in the case of Wigner distribution, the S-method has been introduced [5].

S-method

In the discrete form the S-method has been defined as [5]:

$$SM(n,k)=\sum_{l=-L}^{L} P(l)STFT(n,k+l)STFT^*(n,k-l), \quad (3)$$

where $P(l)$ is a finite frequency domain window with the length $2L+1$. As a special cases spectrogram and Wigner distribution are obtained for $L=0$, and $L=N/2$, respectively. Thus, the S-method improves the concentration of the spectrogram toward the quality of the Wigner distribution. Also, as long as the width of window P is smaller than the minimal distance between the signal components, the S-method is cross-terms free. Note that, unlike the Wigner distribution, the S-method does not require oversampling of the signal.

Time-frequency distributions with complex-lag argument

In order to improve the concentration in the case of signals with highly nonstationary phase function, the time-frequency distributions with complex-lag argument have been introduced [7]-[10]. A discrete form of the N-th order time-frequency distribution with complex-lag argument has been defined as [10]:

$$GCD_N(n,k)=\sum_{m=-N_w/2}^{N_w/2-1} x(n+m)x^*(n-m)c(n,m)e^{-j\frac{2\pi}{N_w}Nmk}, \quad (4)$$

The complex-lag concentration function $c(n,m)$ is defined as follows:

$$c(n,m)=\prod_{p=1}^{N/2-1} x^{w^*_{N,p}}\left(n+m\frac{w_{N,p}}{N}\right)x^{-w^*_{N,p}}\left(n-m\frac{w_{N,p}}{N}\right), \quad (5)$$

where the terms $w_{N,k}=e^{j2\pi k/N}$ (N=2,4,6,…) represent the roots on the unit circle, while N is the order of the distribution. By appropriate choice of distribution order an arbitrary high concentration could be achieved.

In practical realization, the values of the signal along the complex axis are not available. Thus, they should be determined based on the values of the signal with real argument. The signal values with complex-lag argument could be obtained by using the analytical extension of the values with real argument, as follows:

$$x(n\pm w_{N,p}m)=\frac{1}{N_w}\sum_{k=-N_w/2}^{N_w/2-1} X(k)e^{j\frac{2\pi}{N}(n\pm w_{N,p}m)k} \quad (6)$$

Equation (4) could be written in the form:

$$GCD_N(n,k)=\frac{N}{2}WD(n,\frac{N}{2}k)*_k C(n,k). \quad (7)$$

where $*_k$ denotes the convolution in frequency domain. This form provides an efficient realization for monocomponent signals. However, in order to provide cross-terms free representation for multicomponent signals, the components are firstly separated by using STFT and then the analytical extension is calculated separately for each component as follows [10]:

$$x(n\pm m\frac{w_{N,p}}{N})_q=\sum_{k=k_q-W_q}^{k_q+W_q} STFT(n,k+k_q(n))e^{j(n\pm m\frac{w_{N,p}}{N})k}, \quad (8)$$

where $k_q(n)=\arg\left\{\max_k STFT(n,k)\right\}$ represents the position of q-th component maximum. Furthermore, the concentration function: (for the q-th signal component) is calculated as: $c(n,m)_q=c_r(n,m)_q\cdot c_i(n,m)_q$, where:

$$\begin{aligned} c_r(n,m)_q&=\prod_{p=1}^{N/2-1} c_{r_p}(n,m)_q=\prod_{p=1}^{N/2-1} e^{jw_{r_p}\,angle(x(n+m\frac{w_{N,p}}{N})_q\cdot x^*(n-m\frac{w_{N,p}}{N})_q)} \\ c_i(n,m)_q&=\prod_{p=1}^{N/2-1} c_{i_p}(n,m)_q=\prod_{p=1}^{N/2-1} e^{-jw_{i_p}\ln\left|x(n+m\frac{w_{N,p}}{N})_q\cdot x^*(n-m\frac{w_{N,p}}{N})_q\right|} \end{aligned} \quad (9)$$

The time-frequency representations of the concentration functions are obtained as:

$$C_r(n,k)=FT_m\left\{\sum_{q=1}^{Q} c_r(n,m)_q\right\},\; C_i(n,k)=FT_m\left\{\sum_{q=1}^{Q} c_i(n,m)_q\right\}. \quad (10)$$

The concentration functions $C_r(n,k)$ and $C_i(n,k)$ are further convolved in the frequency domain and the resulting concentration function is obtained:

$$C(n,k)=\sum_{i=-L}^{L} P(i)C_r(n,k+i)C_i(n,k-i). \quad (11)$$

By using the S-method instead of the Wigner distribution in (7), the cross terms free form of GCD is defined as [10]:

$$MGCD_N(n,k)=\sum_{l=-L}^{L} P(l)SM(n,k+l)C(n,k-l). \quad (12)$$

Note that the cross-terms free distribution is obtained when the size of window P is less than the minimal distance between the auto-terms.

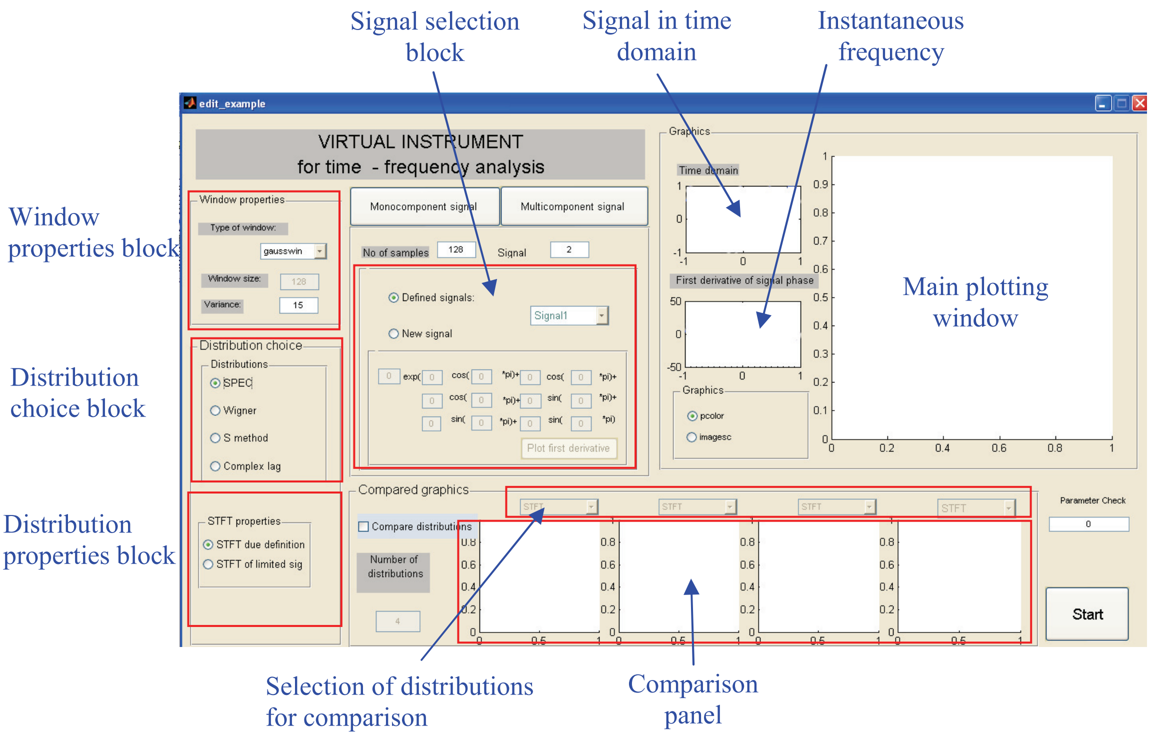

Fig. 1 - Virtual instrument for highly concentrated time-frequency distributions

3. A VIRTUAL INSTRUMENT

The user-friendly virtual instrument (VI) for the time-frequency signal analysis is presented in this Section. Its outlook is shown in Fig. 1. The proposed virtual instrument includes the following time-frequency distributions: the Spectrogram, the Wigner distribution, the S-method and the complex-lag distributions of the fourth and the sixth order. These distributions are realized according to the theory presented in Section 2. The realizations for both monocomponent and multicomponent signals are provided. A user may choose some of the offered signals that are suitable to emphasize the main characteristics of the implemented distributions. Otherwise, for an arbitrary signal with nonlinear phase function in the form:

$$f(t) = A\, e^{a_1 \cos(b_1 \pi t) + a_2 \cos(b_2 \pi t) + a_3 \cos(b_3 \pi t)} \times e^{a_4 \sin(b_4 \pi t) + a_5 \sin(b_5 \pi t) + a_6 \sin(b_6 \pi t)}, \tag{13}$$

the users should choose the parameters A, a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6. Thus, the proposed instrument could provide analysis for a large class of signals that we can met in real applications. The signal and its instantaneous frequency are automatically plotted after signal selection (Fig. 1).

Furthermore, the virtual instrument is made to provide a selection of various parameters that are important for realization of considered distributions. Different types of the lag-window function (rectangular, Gaussian, Hanning, Hamming) and the corresponding properties (length and variance) can be chosen in *Window properties* block. In the down left side a desired distribution should be selected (*Distribution choice* block). Each distribution has its own parameter selection panel. After setting the parameters and pressing the Start button, the outline of the chosen time-frequency distribution is given in main plotting window. Additionally, a comparison of different distributions is provided (bottom panel *Compared graphics* in Fig 1.). Thus, it is possible to highlight the advantages of some distributions over the others in terms of their concentration along the instantaneous frequency. The proposed virtual instrument is implemented by using graphical user interface – GUI in Matlab 7.

4. EXAMPLES

4.1. Monocomponent signal

Consider the signal:

$$x(t) = e^{2j\sin(2\pi t) + \cos(5\pi t) + \sin(3\pi t)}. \tag{14}$$

The time domain signal representation and its first phase derivative are shown in the small plotting windows in Fig 2. The Gaussian window of width N_w=128 and variance 15 is used for each distribution. These are experimentally obtained optimal values for all considered distributions. The comparison of the spectrogram, the Wigner distribution, the complex-lag distribution of the fourth and the sixth order are shown in Fig 2. It can be seen that the distribution concentration for spectrogram and the Wigner distributions are very low. The fourth order complex-lag distribution provides significant improvement, while the sixth order distribution provides almost an ideal concentration along the instantaneous frequency. The sixth order GCD is also shown in the main plotting window.

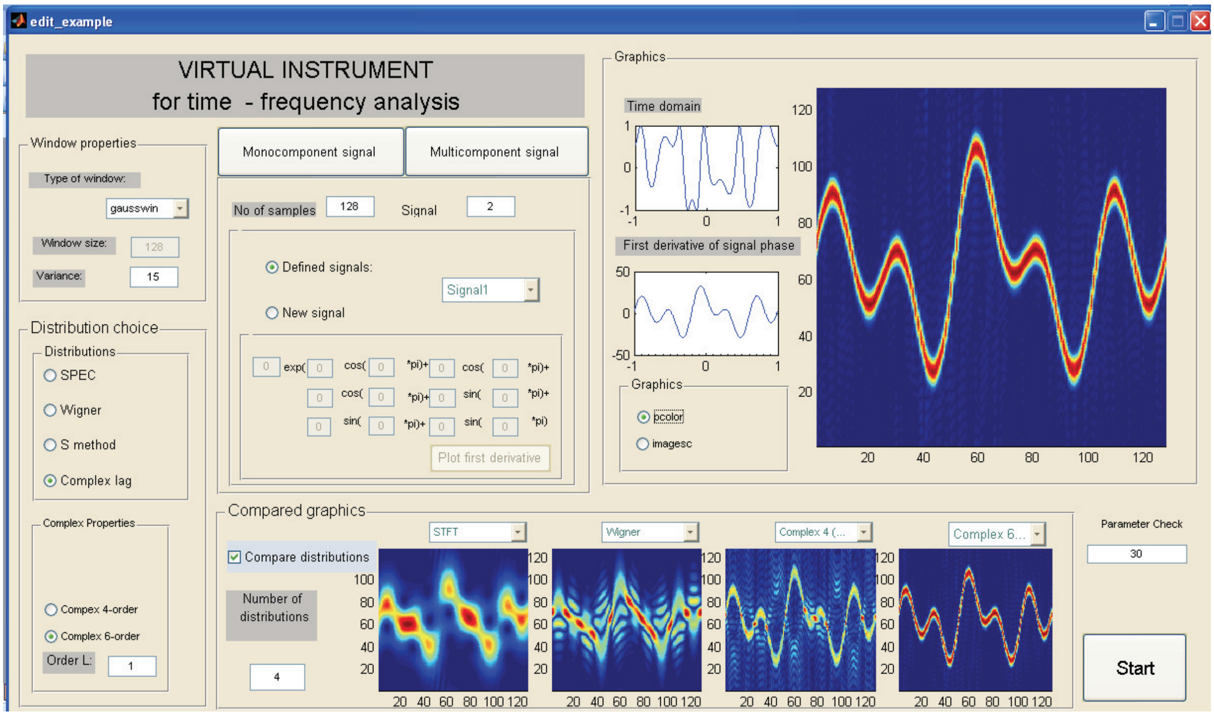

Fig. 2 – Comparison of spectrogram, Wigner distribution, fourth and sixth order complex-lag distributions for monocomponent signal

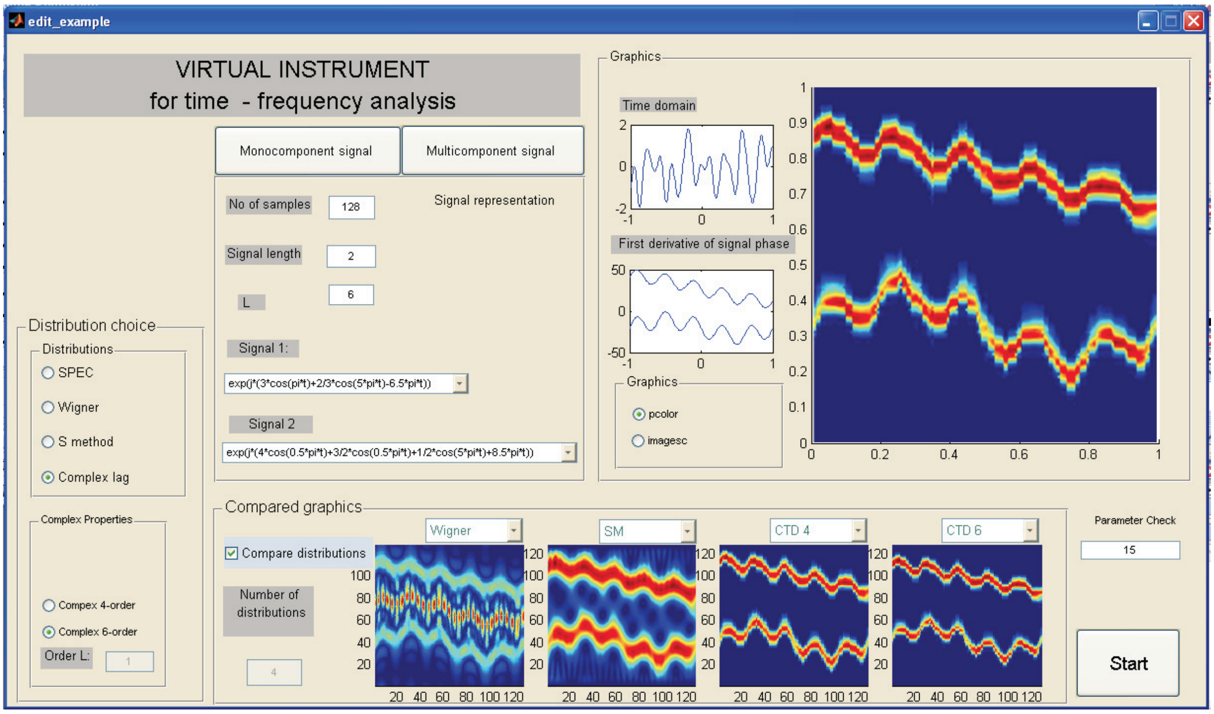

Fig. 3 – Comparison of Wigner distribution, S-method, fourth and sixth order complex-lag distributions for multicomponent signal

4.2. Multicomponent signal

In this example we observe a multicomponent signal in the form:

$$x(t) = e^{4j\cos(0.5\pi t)+\frac{2}{3}\cos(5\pi t)-6.5\pi t} + e^{4j\cos(\frac{1}{2}\pi t)+\frac{3}{2}\cos(\frac{1}{2}\pi t)+\frac{1}{2}\cos(5\pi t)+8.5\pi t}, \tag{15}$$

which consist of two non-overlapping signals, both with fast varying phase functions. The number of samples used in the STFT calculation is N_w=128, while the rectangular window with L=5 is used. A comparison of the Wigner distribution, the S-method, the fourth and the sixth order complex-lag distributions are given in Fig 3. Note that the Wigner distribution is useless since it contains a large amount of cross-terms. On the other side, the S-method is cross-terms free, but cannot provide good concentration due to the fast instantaneous frequency variations. The fourth order complex-lag distribution provides satisfying concentration that is further improved by using the sixth order GCD.

5. CONCLUSION

In this paper we propose a virtual instrument for highly concentrated time-frequency distributions. It represents a user-friendly tool that can serves to the researchers as well as to the practitioners in the area of time frequency signal analysis. This instrument could provide an efficient instantaneous frequency analysis even when it varies significantly within a few samples. It can be used for both monocomponent and multicomponent signals. This virtual instrument enables a comparison of various time-frequency distributions and allows the user to choose the most appropriate for a particular signal.

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