SIGNAL CONTENT ESTIMATION BASED ON THE SHORT-TERM TIME-FREQUENCY RÉNYI ENTROPY OF THE S-METHOD TIME-FREQUENCY DISTRIBUTION

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ABSTRACT

A key characteristic of a nonstationary signal, when analyzed in the time-frequency domain, is the signal complexity, quantified as the number of components in the signal. This paper describes a method for the estimation of this number of components of a signal using the short-term Rényi entropy of its time-frequency distribution (TFD). We focus on the characteristics of TFDs that make them suitable for such a task. The performance of the proposed algorithm is studied with respect to the parameters of the S-method TFD, which combines the virtues of both the spectrogram and the Wigner-Ville distribution. Once the optimal parameters of the TFD have been determined, the applicability of the method in the analysis of signals in low SNRs and real life signals is assessed.

***Index Terms*—** Component, nonstationary signals, time-frequency distributions, Rényi entropy, complexity.

1. INTRODUCTION

Nonstationary signals are characterized by their components instantaneous amplitude, the instantaneous frequency and bandwidth. Time-frequency distributions (TFDs) are an efficient and complete way to present all important information about nonstationary signals [1]. They are two-variable functions of the signal $s(t)$, $C_s(t, f)$, describing the changes of the frequency content of the signal in the joint time-frequency plane, and distributing the signal energy, E_s, in the (t, f) plane as:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_s(t, f)\; dtdf = E_s. \quad (1)$$

For unit energy signals, the TFD $C_s(t, f)$, due to its nonpositivity, can be interpreted as a pseudo-probability density function [2]. This allows to apply information measures to TFDs, as they have been used in information theory.
In [3], the generalized Rényi entropy (RE) of order α

$$H_\alpha(C_s) := \frac{1}{1-\alpha}\log_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_s^\alpha(t, f)\; dt\; df \quad (2)$$

has been shown to be a valuable indicator of the signal complexity, where $C_s(t, f)$ must be normalized with respect to the signal energy [4].
The information on the number of components present in the signal can be obtained from the entropy of the signal TFD, since signals of high complexity are composed of many elementary components. Consider, for example, an ideal TFD, $C_{s_1}(t, f)$, of a unit energy signal $s_1(t)$. The addition of a second component to the signal, $s_2(t)$, by shifting the first component in time and/or frequency (under the condition of nonoverlapping components in the (t, f) plane and by maintaining the TFD energy unitary), brings an increase of one bit in the RE of the TFD of the two component signal, when compared to the RE of a single component signal [4]:

$$H_\alpha(C_{s_1(t)+s_2(t)}(t, f)) = H_\alpha(C_{s_1(t)}(t, f)) + 1. \quad (3)$$

Thus, the number of components can be determined as [5]

$$n = 2^{H_\alpha(C_{s_1(t)+s_2(t)}(t,f)) - H_\alpha(C_{s_1(t)}(t,f))}. \quad (4)$$

Eq. (4) holds because the RE is invariant to time and frequency shifts of the signal [4]. By computing the third order RE (all the results presented in this paper correspond to the third order RE) of an n component signal $\sum_{i=1}^{n} s_i(t)$, for which all components have same time and frequency supports and the same FM, and by calculating the RE of an arbitrarily chosen component of the signal, say $s_m(t)$, the number of components in this multicomponent signal is:

$$n = \sqrt{\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C^3_{s_m(t)}(t, f)dtdf}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C^3_{\sum_{i=1}^{n} s_i(t)}(t, f)dtdf}}. \quad (5)$$

As shown in [6], Eq. (4) does not hold when the components exhibit different time and frequency supports in the (t, f) plane, and if one of the components is not known in advance. This means that, in general, the global RE can not be used as an estimator of the number of components in a signal. Section 2 presents a method, based on the short-term RE, that overcomes those constrains. The conditions under which a TFD assures an accurate performance of the algorithm are discussed, including the selection of the optimal set of TFD parameters. Examples, including monocomponent, multicomponent, noisy, and real-life signals are presented in Section 3. The conclusion is given in Section 4.

2. THE SHORT-TERM TIME-FREQUENCY RÉNYI ENTROPY: A METHOD FOR THE LOCAL COMPONENT DETECTION

The algorithm used in this paper first requires annulling the TFD, $C_s(t,f)$, of the analyzed signal (see Fig. 1(a)), outside the time interval Δt, to obtain a TFD of the form

$$C_{s_p}(t,f) = \begin{cases} C_s(t,f) & p - t_0 < t < p + t_0, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where $\Delta t = 2t_0$. The aim of the TFD annulation outside the short time interval Δt (in this paper $\Delta t = 45$ time samples) is necessary in order to meet the condition in Eq. (4). In fact, independently from their time duration and frequency modulations, different components locally have same time durations and similar bandwidths, as shown in Fig. 1(c). The subtrahend in the exponent of Eq. (4) is obtained as a time slice of length Δt of a synthetic reference signal TFD (Fig. 1(b), (d)). The selected synthetic reference signal is a cosine signal of arbitrary amplitude and arbitrary constant frequency. By using Eq. (6) for different time instants p, different time slices of the TFD are isolated, with their respective RE values. By comparing the obtained values of the short-term RE of the multicomponent signal and the reference signal, a function of the instantaneous number of components of the analyzed signal, $n(p)$, is obtained. Complete steps of the algorithm are given in the flowchart in Fig. 2.

Since many engineering applications require the instantaneous number of components to be an integer [7, 8], a thresholding of $n(p)$ has been introduced. Our simulations have shown that 0.1 is an adequately sensitive threshold for detection of the first component, while *round*$(n(p))$ value for each next component is sufficiently robust to reduce the influence of cross-terms between the components. Indeed, the quantized instantaneous number of components also allows to estimate the total number of components by adding the number of rising edges to the initial number of components.

The selection of an appropriate TFD is a key step for the algorithm accurate performance. The global RE is cross-terms invariant, since, due to their oscillatory nature, the cross-terms are annulated under the integration over the entire (t,f) plane for odd values of the parameter α [4]. On the other hand, when the integration is performed over a short time slice, as per Eq. (4), the presence of cross-terms will significantly affect the value of $n(p)$. Therefore, the algorithm requires a TFD with reduced cross-terms, having good energy concentration of components, and maintaining the local entropy invariance to the signal IF.

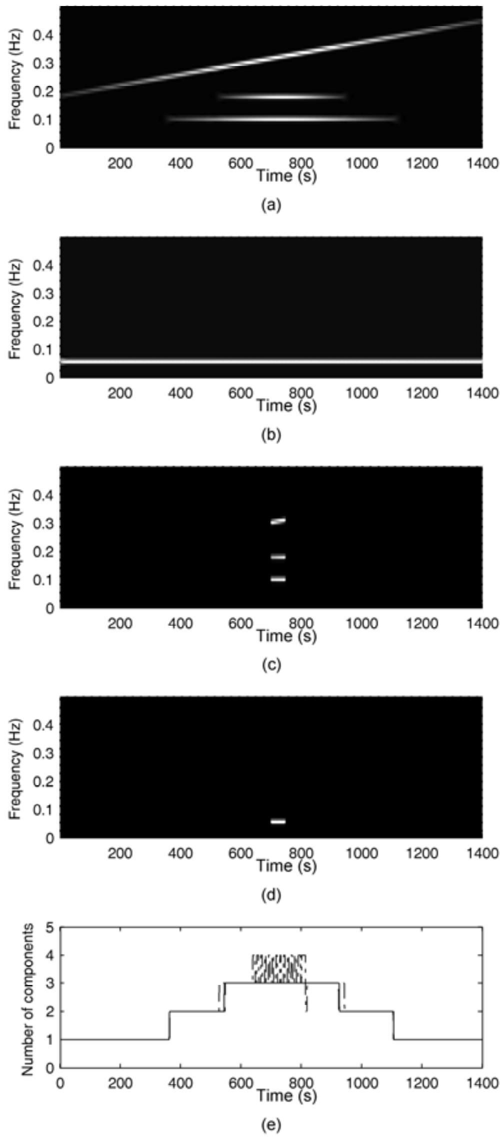

Fig. 1. (a) The S-method TFD of a three component signal $L = 1$, (b) the S-method TFD of the reference signal, $L = 1$, (c) time slice around $p = 775$ s of the three component signal, (d) time slice around $p = 775$ s of the reference signal, (e) the number of components in time for: $L = 1$ (solid), $L = 2$ (dashed), and $L = 3$ (dotted), and for $L = 4$ (dotted-dashed).

A natural choice of TFD for this purpose would be the spectrogram which doesn't contain cross-terms when signal components do not overlap in the (t,f) plane. But it is also characterized by poor energy concentration, and it suffers from significant dependency of the local component bandwidth on the component IF (signals with fast changes of the IF present locally larger bandwidths in the (t,f) plane), leading to wrong estimations (see Fig. 3). The dependency of the local component bandwidth on the IF can be reduced by improving the auto-terms concentration in the (t,f) plane: a computationally simple method, referred to as the S-method, produces the same auto-terms as in the Wigner distribution, but without cross-terms. Using a frequency window $P(l)$ of finite duration, the S-method is obtained as [9]:

$$SM(n,k) = \sum_{l=-L}^{L} P(l) STFT(n,k+l) STFT^{*}(n,k-l)$$

where STFT is the short time Fourier transform of the signal. The S-method can be seen as a distribution that combines good properties of the spectrogram and the Wigner distribution. The convolution along the frequency axis improves the auto-terms concentration, but it should be performed only over the same auto-term, avoiding different signal components being convolved. For an efficient performance of the algorithm, the window $P(l)$ should have short time duration in order to capture the fast variations of the IF (the recommended duration is $N/10$ where N is the duration of the signal). Fig. 1(e) shows the performance of the algorithm on a three component signal for the values of $L = 1, 2, 3, 4$. For $L = 1, 2$, and 3, the curves that represent the local number of components overlap over the entire measurement time. By increasing the value of L, the cross-terms start to affect the algorithm result (Fig. 1(e)). Since for the proposed algorithm the suppression of the cross-terms is essential, small values of L ($L \leq 3$) must be used. Fig. 3 shows that the S-method with $L = 1$ reduces the influence of the component IF on the instantaneous bandwidth, and it gives a correct component content estimate. So, the value of 1 can be recommended as the optimal value of the parameter L in the S-method for most applications as in examples of Section 3.

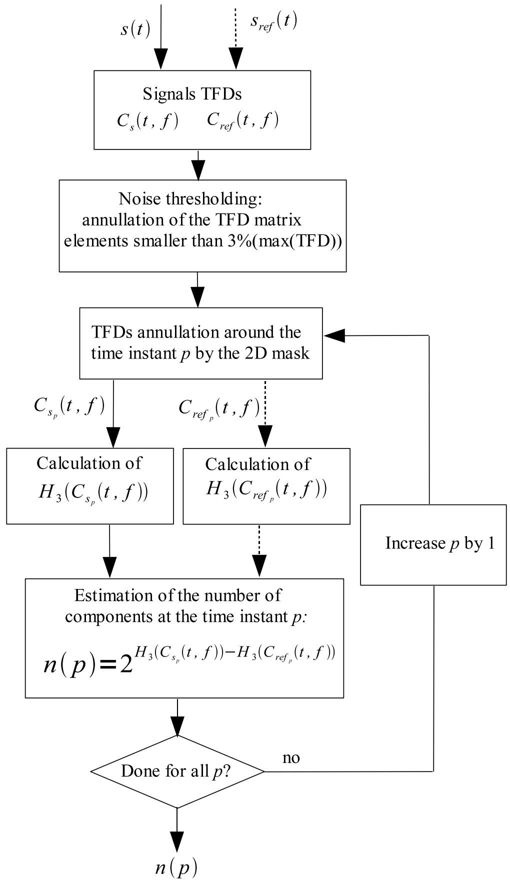

Fig. 2. Flowchart of the proposed algorithm.

3. RESULTS: NOISE AND REAL DATA

To validate the performance of the algorithm, a three component signal with different (linear and parabolic) FMs, embedded in additive white noise (Fig. 4(a)), is considered. Fig. 4(b) shows the obtained results for different SNRs (15 dB solid, 10 dB dotted, and 5 dB dashed line), proving the robustness of the method to additive white noise. Similar estimates have been obtained for different noise environments.
A real test signal, a bat emitted sound representing a natural sonar is shown in Fig. 5(a). As it can be seen in Fig. 5 (b), the proposed algorithm applied to the S-method presents high sensitivity to low energy components, even when they overlap with higher energy components for very short time intervals.

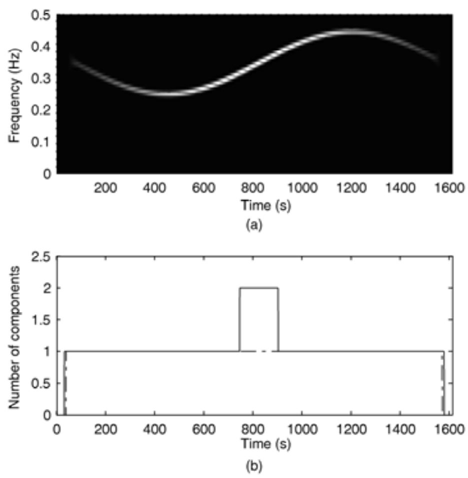

Fig. 3. (a) The S-method TFD of a monocomponent signal for $L = 1$, (b) the number of components in time for the spectrogram $L = 0$ (solid), and for the S-method, $L = 1$ (dotted-dashed).

4. CONCLUSION

The algorithm presented in this paper efficiently estimates the local number of components from a signal's TFD, exploiting the fact that the short-term RE is invariant to different structures of the signal components in the (t, f) plane. Tests on noisy and real-life signals show that the S-method TFD with small values of the parameter L, namely $L = 1$, is a good choice of TFD as it presents reduced interferences and the local bandwidth invariance to the component IF. The algorithm

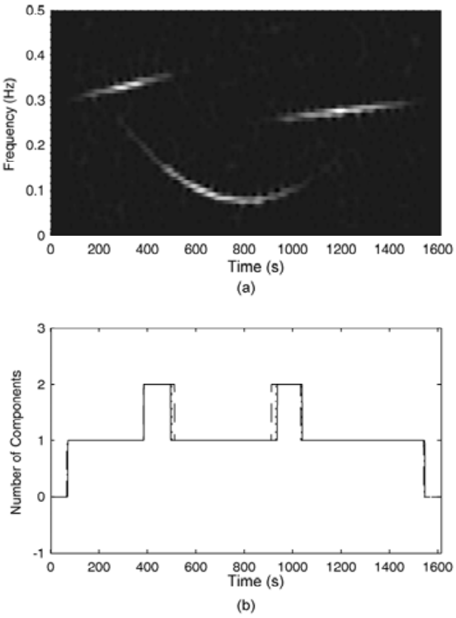

Fig. 4. (a) The S-method TFD of a multicomponent signal embedded in additive white noise with $SNR = 5\ dB$. (b) the local number of components: 15 dB (solid), 10 dB (dotted), and 5 dB (dashed).

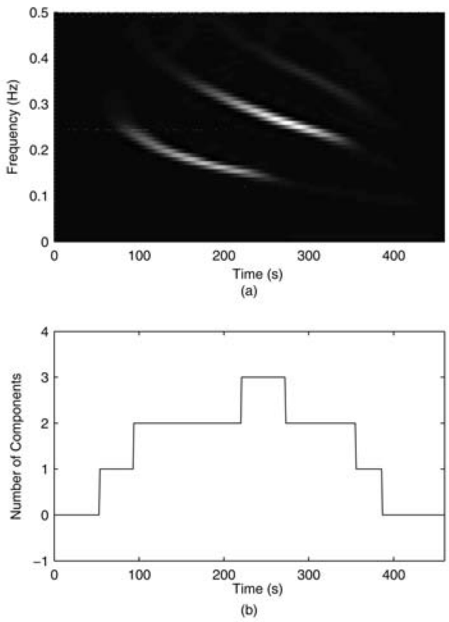

Fig. 5. (a) The S-method TFD of a bat echo location signal. (b) the local number of components.

can be used in various engineering applications where reliable information on the number of components present in the signal is required, e.g. in [7, 8].

5. ACKNOWLEDGMENTS

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