Entropy Based Extraction and Classification of Frequency Hopping Signals from Their TFDs

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***Abstract*—This paper presents a method for extraction of frequency hopping signals, for the case of variable number of transmitters with aperiodic frequency hops. The mixture of signals generated by different transmitters are represented in the time-frequency domain, by time-frequency distributions belonging to the Quadratic class. Different components are extracted from the time-frequency distribution of the mixture, and classified among the various transmitting sources, using the information on the instantaneous number of components, hopping instants, and the total number of components estimated by using different orders of the time-frequency Rényi entropy.**

***Keywords*—Time-frequency distributions; Frequency hopping signals; Component separation**

I. Introduction

Frequency hopping signals (FHS) belong to a class of signals widely used in wireless communications, and they are obtained by switching the frequency of an exponential tone $e^{j2\pi f_k t}$ at hopping instants, generated by the transmitter, according to a hopping pattern known only by the transmitter and the receiver [1]. These nonstationary signals are characterized by several parameters such as the instantaneous frequency f_k between two hopping instants, the number of hopping instants, the total number of transmitters, etc... In the case of unknown hopping patterns (signal interception purposes, or initial synchronization) the estimation of the hopping moments is key for the identification of the different signal components whose identification and separation is the goal in many applications [1]–[4]. The characteristic features of FHS are well emphasized by their representation in the time-frequency plane. Time-frequency representations (summarized in Section II) allow to recover information on the local and total number of components that are present in a signal by the application of the Short-term Rényi entropy [5]. Section III presents a method for identification of the hopping instants in the case of aperiodic hop timing and variable number of transmitters, based on the application of different orders of the Short-term Rényi entropy on the time-frequency distributions (TFDs) of the signal mixture. Using this information, different components are extracted and classified with the algorithm presented in Section IV. Conclusion is given in Section V.

II. Theory of Quadratic class of Time-frequency distributions

TFDs are two variable functions, $C_s(t,f)$, defined over the two-dimensional (t,f) space [6]. A monocomponent signal is defined in its analytic form as [7]:

$$s(t) = a(t)e^{j\phi(t)}, \tag{1}$$

where $a(t)$ is the instantaneous signal amplitude, and the signal instantaneous frequency (IF) is defined as the time derivative of its instantaneous phase $\phi(t)$ [7]

$$f_i(t) = \frac{\phi'(t)}{2\pi}. \tag{2}$$

For a single tone signal $s(t)$ with frequency f_0, the ideal TFD represents a series of Dirac functions in the (t,f) plane, tracking the signal IF [7]:

$$I_s(t,f) = \delta(f - f_0).$$

Such an ideal representation is achieved by the Wigner-Ville distribution (WVD) defined as

$$\begin{aligned} W_s(t,f) &= \mathcal{F}_{\tau\to f}\{s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2})\} \\ &= \int_{-\infty}^{\infty} s(t+\frac{\tau}{2})s^*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau. \end{aligned} \tag{3}$$

However, due to the nonlinearity of $W_s(t,f)$, cross-terms appear between any pair of signal points in the (t,f) plane. Also, nonlinear IF causes unwanted artifacts [8]. Both effects result in degraded representation quality.
To avoid this limitation of the WVD, a class of Quadratic distributions has been introduced that filters out the interferences generated in the WVD in both time and frequency direction [7]:

$$C_s(t,f) = \gamma(t,f) *_t *_f W_s(t,f), \tag{4}$$

where $\gamma(t,f)$ is called the time-frequency kernel filter, and the double asterisk denotes a double convolution in t and f. Quadratic distributions with separable time-frequency kernel are particularly efficient in the task of reducing interferences, maintaining at the same time an acceptable auto-terms concentration [7]. The Smoothed-Pseudo WVD (SPWVD) is formulated with a simple kernel, being the product of the time and lag window functions [9], which in the time-frequency domain takes the form

$$\gamma(t, f) = g(t)\; H(f). \tag{5}$$

In this paper signals are represented in the (t, f) plane by the SPWVD with time and lag *hamming* windows.

III. Rényi entropy based estimation of the number of components, transmitters, and hopping instants

The generalized Rényi entropy of order α

$$H_\alpha(C_s) := \frac{1}{1-\alpha}\log_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_s^\alpha(t, f)\; dt\; df, \tag{6}$$

widely used as measure of signal complexity, exhibits a counting property, but only in the case of signals with multiple components of equal time-frequency supports and equal amplitudes [10]. Another limitation of the Rényi entropy counting property is that one of the signal components must be available to the user as a reference for the estimation.

Recently, the Short-term Rényi entropy [5] has been introduced in order to overcome the limitations of the global Rényi entropy as an estimator of the local number of signal components.

The number of components that are present around an instant t_0 will be obtained by comparing the Rényi entropy of the TFD $C_{s_{t_0}}(t, f)$, inside a short time interval around the instant t_0, with the Rényi entropy of a TFD of an arbitrarily chosen reference cosine signal within the same time interval. The local number of components will thus be [5]:

$$n_\alpha(t_0) = 2^{H_\alpha(C_{s_{t_0}}(t,f)) - H_\alpha(C_{ref_{t_0}}(t,f))}. \tag{7}$$

By computing Eq. (7) for each time instant t_0, a continuous function representing the local number of components is obtained.

It has been shown that spectrum smearing caused by the frequency hopping in FHS leads to local entropy increase [2]. However, variation of the value of the parameter α allows to control the effect of the spectral smearing of the Short-term Rényi entropy. In fact, smaller values of α will emphasize the entropy contribution of the smeared fading/rising component (since smaller values of α enhance the entropy sensitivity to low energy components), while larger values of α will minimize it.

By using the algorithm presented in [5], a FHS embedded in moderate additive white Gaussian noise (AWGN, i.e. a generally accepted model of noise in communication channels [1], with $SNR = 10\quad$dB) has been analyzed. Three sources are involved in the transmission: the first transmission starts at the time instant $t = 1$ s, has frequency hops at the time instants $t = 30$ s, $t = 130$ s, and $t = 180$ s, and ending at $t = 220$ s. The second transmitter is activated at $t = 60$ s, it has one frequency hop at $t = 90$ s, ending at $t = 300$ s. At the time instant $t = 260$ s, a third source transmits a short impulse, lasting 20 s. Meanwhile, in the case of interception or synchronization phase, these parameters are unknown.

Fig. 1(a) shows the SPWVD (with normalized frequency, and time and lag windows of duration $N/10$ s, where N is the duration of the signal) of the described signal. Fig. 1(b) shows the Short-term Rényi entropy of the 10-***th*** order. It can be noticed that the high value of the parameter α makes the entropy based estimated number of components insensitive to the frequency hops. By counting the rising edges of the local number of components $n_{10}(t)$, the total number of transmitters in the signal is obtained. On the other hand, by decreasing the value of the parameter α to 1, the estimated number of components $n_1(t)$ will present a pronounced sensitivity to frequency hops (Fig. 1(c)). By counting the rising edges of $n_1(t)$ the total number of components present in the signal is obtained. The function detecting the occurrence of a frequency hop (Fig. 1(d)) is thus obtained as

$$n_{hop}(t) = n_1(t) - n_{10}(t). \tag{8}$$

IV. Algorithms for component extraction and classification

A. Component extraction algorithm

After obtaining the local and total number of components, and time locations of frequency hops, by the combination of different orders of Short-term Rényi entropy estimates, the extraction of individual components can be performed. Since the total and local number of components (generated by one or more sources), is available from the functions $n_1(t)$ and $n_{10}(t)$, the algorithm allocates memory dynamically. The total allocated memory is determined by the rising edges of $n_1(t)$, while the momentarily used memory block is addressed according to the value of $n_{10}(t)$.

This dynamic strategy of memory usage avoids blind allocation, thus decreasing the algorithm execution time and preventing memory overflow. If dynamic allocation is not used and the number of components is not available, the result may be an incomplete extraction (whenever the number of existing components is larger than the arbitrarily predicted one [3]).

Algorithm steps

- **Step 1**: If $n_1(t) > 0$, the TFD maximum is located at (t, f_0), or else the next instant t is considered
- **Step 2**: The neighbouring bandwidth of the located maximum is extracted from the TFD and stored in one of the allocated memory blocks according to the counter of extracted components.
- **Step 3**: If a new component is found, according to $n_1(t)$, the counter of extracted components is increased by one, until the counter is equal to the total number of components calculated from $n_1(t)$.
- **Step 4**: The above steps are repeated $n_1(t)$ times in order to extract all components at an instant t. Next, t is increased by one.

Fig. 2 shows the extracted components of the signal analyzed in Fig. 1.

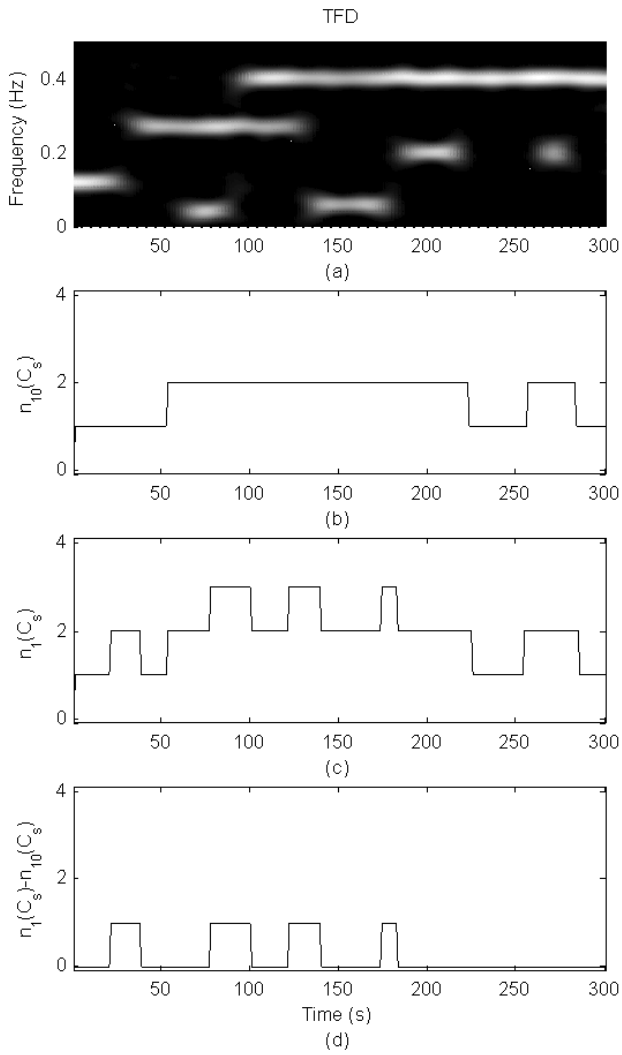

Fig. 1. TFD (SPWVD) of a multiple source FH signal (a), estimated number of components $n_{10}(t)$ (b), estimated number of components $n_1(t)$ (c), estimated hopping instants $n_{hop}(t)$.

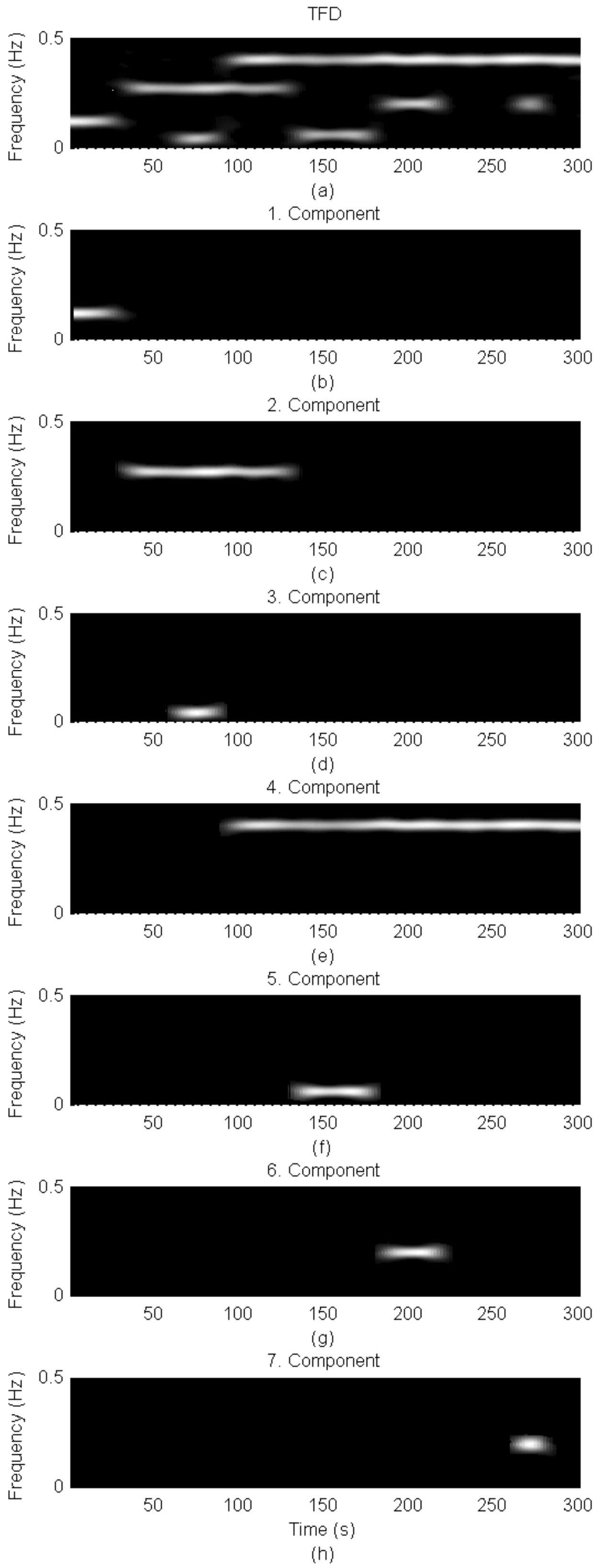

Fig. 2. Extracted components from the TFD of the mixture.

B. Transmitter based component classification

Different orders of the Short-term Rényi entropy estimates $n_1(t)$, and $n_{10}(t)$, give enough information to classify the components according to their source. The classification is performed as follows:

Algorithm steps

- **Step 1**: Find the time instant of the rising edge of the function $n_{hop}(t) = n_1(t) - n_{10}(t)$.
- **Step 2**: For that time instant locate the starting component as the one that wasn't already stored in one of the allocated memory blocks.
- **Step 3**: Of all other components at that time instant, according to $n_1(t)$, find the one with the instantaneous lowest maximum (meaning, the component is ending).
- **Step 4**: Add the components found in Step 2 and Step 3.

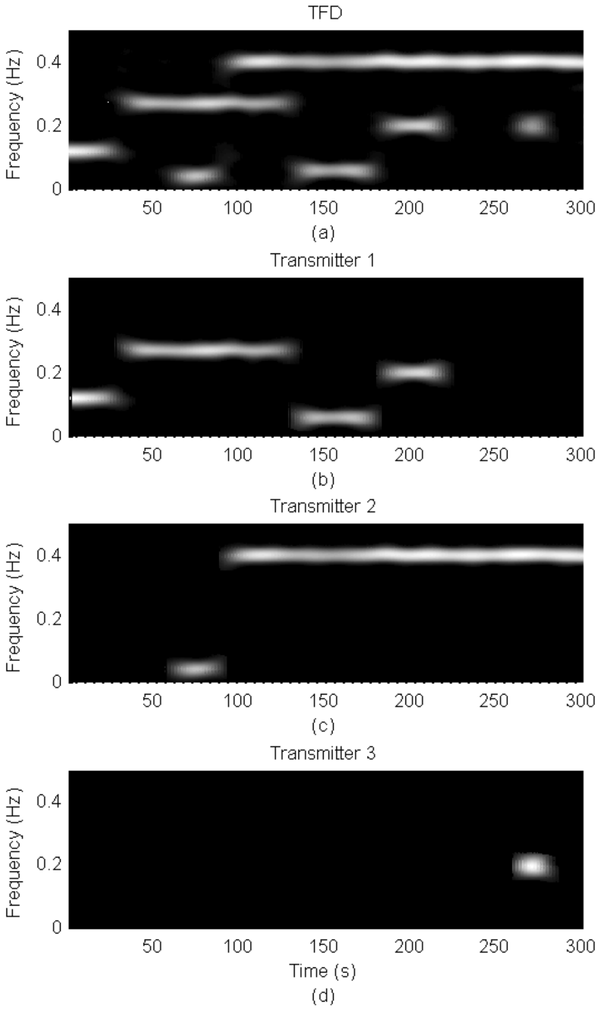

Fig. 3. Components classified with respect to different sources.

- **Step 5**: Repeat the steps for each rising edge of the function $n_{hop}(t) = n_1(t) - n_{10}(t)$.

Fig. 3 shows the components of the signal in Fig.1, classified with respect to different sources.

V. Conclusion

This paper presents a blind source separation technique applicable on FHSs. FHSs are represented using the Quadratic class of TFDs (SPWVD), and information on the local and global number of components are obtained by the calculation of different orders of the Short-term Rényi entropy. Based on these data, different components can be separated from the mixture, by a sequential peak extraction algorithm. Since the peak of each time slice of the TFD represents the IF of the component, this information can be used as a parameter for signal reconstruction. Also, relying on the information of frequency hopping occurrence, the components generated by the same source can be recovered.

The reported results, showing robustness of the algorithm in moderate noise conditions, are encouraging for further testing on different classes of wireless signals.

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