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Robust time–frequency representation based on the signal normalization and concentration measures

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ABSTRACT

An efficient procedure for obtaining time–frequency representations under high influence of impulsive noise is proposed in this paper. The procedure uses the fast Fourier transform based algorithm instead of sorting procedures common in the case of various robust time–frequency representations proposed recently. Concentration measure is used to select a free parameter of the transform.

1. Introduction

Spectral analysis of nonstationary signals with the high-frequency content corrupted by an impulsive noise has become a very interesting research topic in the past decade. Several filters robust in both time and frequency domains have been proposed [1–6]. Commonly, all these techniques require consuming sorting or iterative procedures for each instant, or for all frequencies, or even for each point in the time–frequency (TF) plane. In this paper, we propose an alternative simple strategy based on the signal normalization. Signal normalization is used as a processing tool for signals corrupted by impulse noise [7–11]. Normalization strategies are already considered in the TF analysis. The fractional lower order technique is introduced in [12]. In addition, a similar technique has been proposed for the TF analysis from the quantum mechanics perspective [13], and used as instantaneous frequency estimator in [14]. Here, we consider an alternative normalization strategy inspired mainly by the problem of inverse filtering in the digital image processing [15]. The proposed normalization strategy has a free parameter. A technique for selecting this parameter, based on the concentration measure for obtaining TF representation (image) of nonstationary signals, is proposed as well.

The paper is organized as follows. Some of the existing techniques for TF imaging of nonstationary signals corrupted by impulsive noise are reviewed in Section 2. The proposed technique is described in Section 3. Numerical examples are given in Section 4, while Section 5 concludes the paper.

2. Spectral analysis of signals corrupted by impulsive noise

Consider a noisy frequency modulated (FM) signal

$$x(t) = A\exp(j\phi(t)) + \nu(t), \quad (1)$$

where $\nu(t)$ is a white noise that can be impulsive and/or heavy tailed. Under these terms it is assumed that the

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noise can take values whose amplitudes are of a significantly higher magnitude than the signal magnitude $|A|$. Common techniques for removing impulsive noise, such as median-, L- or myriad-based filters, are not efficient in FM signal filtering [15,2] since they have low pass characteristics. Their application removes high frequency components from the signal. Therefore, research for alternative techniques becomes a hot issue recently.

Particularly useful techniques are weighted median and myriad filters admitting negative weights [1,2]. They are designed with the same logic as classical linear weighted filters. However, they require a sorting or an iterative procedure to be performed for each instant of the signal. This group of filters is sometimes referred to as the robust filters of FM signals in the time domain.

Another technique is based on the robust DFT evaluation [5,16]. The robust DFT is calculated for each frequency using nonlinear techniques eliminating impulses and providing estimate of the standard DFT of the FM signal. Again these techniques assume sorting or iterative procedures meaning that they are significantly more demanding than the standard (fast) DFT based techniques.

Similar problems arise in the case of the TF representations. There are several alternatives how to evaluate the TF representations of signals corrupted by impulsive noise [3,12,17–19]. One possibility is to calculate the robust short time Fourier transform (STFT) for each point (pixel) in the TF plane based on the robust DFT algorithms and to use it for obtaining other robust TF representations [18]. Namely, the higher-order TF representations can be realized by using the STFT calculated in the initial stage, avoiding undesired effect such as the cross-terms. Also, evaluation of the robust TF representation by filtering auto-correlations of the signal is more sensitive to impulse noise errors than the signal (or modulated signal) itself [18].

In general, a robust STFT can be expressed as

$$STFT_R(t,\omega) = \mathrm{R}\{x(t+n\Delta t)\exp(-j\omega n\Delta t)|n\in[-N/2,N/2)\}, \quad (2)$$

where R{} is the robust operator applied on the modulated signal sequence, Δt is the sampling period, while N is the number of samples in the considered window. In the following we are presenting some of the robust STFT forms.

The robust STFT evaluated using the marginal-median approach is defined as

$$\begin{aligned} STFT_{|e|}(t,\omega) = &\operatorname{median}\{\mathrm{Re}\{x(t+n\Delta T)\\ &\times\exp(-j\omega(n\Delta t))|n\in[-N/2,N/2)\}\\ &+j\operatorname{median}\{\mathrm{Im}\{x(t+n\Delta T)\\ &\times\exp(-j\omega(n\Delta t))|n\in[-N/2,N/2)\}. \end{aligned} \quad (3)$$

To calculate the STFT given by (3), one has to preform the sorting procedure for each (t,ω) pair.

The myriad-based STFT can be evaluated using the following iterative procedure:

$$STFT_\kappa^{(i)}(t,\omega) = \frac{\sum_{n=-N/2}^{N/2-1}\frac{x(t+n\Delta t)\exp(-j\omega n\Delta t)}{[\kappa^2+|x(t+n\Delta t)\exp(-j\omega n\Delta t)-STFT_\kappa^{(i-1)}(t,\omega)|^2]}}{\sum_{n=-N/2}^{N/2-1}\frac{1}{[\kappa^2+|x(t+n\Delta t)\exp(-j\omega n\Delta t)-STFT_\kappa^{(i-1)}(t,\omega)|^2]}}. \quad (4)$$

with a properly selected initial iteration.

Various robust TF forms can be obtained from the robust STFT as the initial signal representation. Here, we consider the S-method (SM) [20]

$$SM_R(t,\omega) = |STFT_R(t,\omega)|^2 + 2\mathrm{Re}\left\{\sum_{l=1}^{L} STFT_R(t,\omega+l\Delta\omega)STFT_R^*(t,\omega-l\Delta\omega)\right\},$$

where $2L+1$ is the frequency window length and $\Delta\omega$ is the difference between two consecutive samples on the frequency grid in the TF plane. For $L=0$, the robust SM is equal to the robust spectrogram (square magnitude of STFT), while for $L\to N/2$, the robust Wigner distribution (WD) is obtained. Relatively small $L\in[1,10]$ significantly improves concentration of the spectrogram but without cross-terms that appear in the WD. In the same way the SM can be extended to other higher-order TF representations, since all of them can be realized using the STFT [18]. Evaluation of the robust TF representation requires a computationally efficient form of the STFT robust to the impulsive/heavy tailed noise in the initial step. This implies an alternative to the sorting or iterative procedures in order to obtain both fast and accurate TF representations of signals corrupted by impulsive/heavy tailed noise.

3. Signal normalization

Consider the following normalized signal:

$$y(t) = \frac{x(t)}{|x(t)|}. \quad (5)$$

When $x(t)$ is a monocomponent non-noisy signal, (5) becomes $y(t)=\exp(j\phi(t))$ and has the same instantaneous frequency (IF) $\omega(t)=\phi'(t)$ as the original signal $x(t)$, without a proper information on the signal amplitude. However, usually the IF of a signal is more important feature. Based on the IF we can later estimate the signal amplitude. One particular strange situation is that this normalization for signal with varying amplitude will give signal with unit amplitude. In the case of a signal corrupted by a Gaussian noise, the analysis of resulting noise after normalization (5) is performed in [8,9]. For the signal-to-noise ratio $SNR>5$ dB we cannot expect big difference in the resulting SNR with respect to the signal $x(t)$. Samples that are not corrupted by impulsive noise are proportional to the original samples. Thus we have a similar situation like in the additive Gaussian noise case. Samples corrupted by impulses have unitary amplitude with random phase (probably $\pm\pi/2$ or 0). Variance of this noise is proportional to the percentage of impulses [21]. This noise has no impulsive characteristics.

However, the primary concern in the application of the signal normalization comes in the case of multicomponent signals. Consider a two component signal

$$x(t) = A_1\exp(j\phi_1(t)) + A_2\exp(j\phi_2(t)). \quad (6)$$

Amplitude of $x(t)$ is equal to

$$|x(t)| = \sqrt{A_1^2+A_2^2+2A_1A_2\cos(\phi_1(t)-\phi_2(t))}. \quad (7)$$

For example, for $A_1=A_2$ and $\phi_1(t)-\phi_2(t)=(2k+1)\pi$, where $k\in Z$, the obtained amplitude is equal to zero. The normalization in this case is even not possible or it is giving

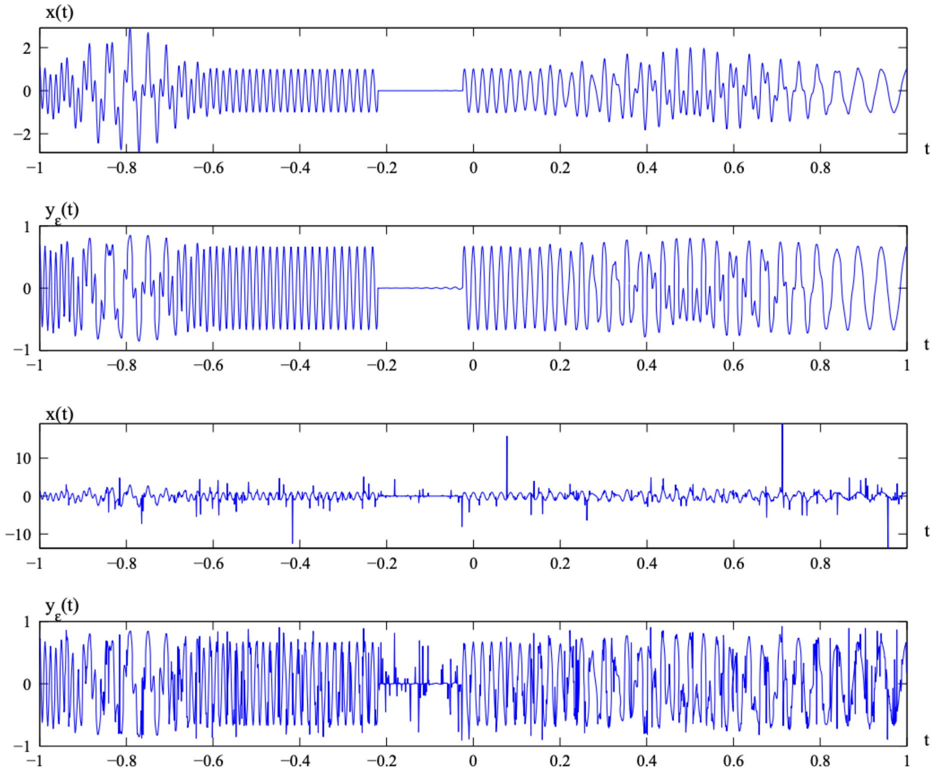

Fig. 1. A multicomponent signal with a zero-amplitude interval (first row) and its normalized version (second row). The signal with a strong impulsive noise (third row) and its normalized version (fourth row).

unexpected results such as significant increase of the variance of additive noise.

A similar strategy has already been used in the TF analysis with lower order distributions to reduce the noise in intervals where the signal spectral variations are not high [14]. For signals corrupted by impulsive noise, the fractional lower order distributions are proposed in [12]. Under the fractional lower order it was considered as a signal

$$y^{(a)}(t) = \frac{x(t)}{|x(t)|^a}, \quad (8)$$

where $a \in [0, 1]$. However, form (8) is not addressing the main problems associated with (7) what can cause the same undesired effects in the TF analysis. Also, as it will be shown later, criterion of maximal concentration in the TF plane for signals with some amount of impulsive noise produces $a = 1$ as the most robust value almost in all trials.

Therefore, in this paper we propose an alternative normalization function given as

$$y_\varepsilon(t) = \frac{x(t)}{\sqrt{|x(t)|^2 + \varepsilon^2}}, \quad (9)$$

where ε is the transform free parameter. For $\varepsilon \to 0$, (9) is proportional to $y^{(a)}(t)$, $a \to 1$, while, for $\varepsilon \to \infty$, the produced signal (or transform) is proportional to the original signal $x(t)$ (or transform). The parameter ε is used to preserve the phase variations for multicomponent signals as well as to enable dealing with situations when the signal amplitude tends to zero, $|x(t)| \cong 0$ (see Fig. 1). It will be shown later that the proposed normalization strategy offers better set of possible transforms than the fractional lower order technique.

Remark. A similar idea is exploited in [15] to design inverse filters for image deblurring.

Once the normalization strategy has been proposed, we can calculate the standard TF representations for the normalized signal. In order to perform that task, we will start from the robust STFT

$$STFT_\varepsilon(t, \omega) = \sum_{n=-N/2}^{N/2-1} y_\varepsilon(t + n\Delta t) \exp(-j\omega n \Delta t) \quad (10)$$

and improve its concentration toward the higher order TF representations using the robust SM:

$$SM_\varepsilon(t, \omega) = |STFT_\varepsilon(t, \omega)|^2 + 2\mathrm{Re}\left\{ \sum_{l=1}^{L} STFT_\varepsilon(t, \omega + l\Delta\omega) STFT_\varepsilon^*(t, \omega - l\Delta\omega) \right\}.$$

In this way, the robust STFT is evaluated using the FFT algorithms, while the SM is evaluated using the existing scheme that is rather efficient for small L (common in practice). Note that the same technique can be performed for the fractional lower order signal.

The remaining question is related to selection of ε since quite different results could be obtained for various values of ε. Small values of ε produce results that are robust to impulse noise but, at the same time, it could produce unrealistic image with respect to amplitudes of signal components as well as possible effects related to increasing overall Gaussian noise influence in some regions in the TF plane for the moderate level of input noise. Large values of ε would produce results similar to the standard TF representations.

The common technique for selecting the transform free parameter in the TF representation is based on the concentration measures. Some of these measures, based on the ratio of the TF representation higher order norms, are proposed in [22]. Measures are reviewed and compared in [23], where a possibility of lower order norms is proposed as well. Here, we consider the following measure:

$$M(\varepsilon)=\left(\sum_n\sum_m |SM_\varepsilon(n\Delta t, m\Delta\omega)|^{1/p}\right)^p, \tag{11}$$

where $p>1$. Theoretically, for $p\to\infty$, the term $|SM_\varepsilon(n\Delta t, m\Delta\omega)|^{1/p}$ is equal to one if $|SM_\varepsilon(n\Delta t, m\Delta\omega)|\neq 0$, while, for $|SM_\varepsilon(n\Delta t, m\Delta\omega)|=0$, it is equal to 0. Therefore, the minimal value of $M(\varepsilon)$ corresponds to the TF representation that is best concentrated in the TF plane (with the smallest spread). In order to compare various TF representations in a fair manner one has to first normalize them to $\sum_n\sum_m SM_\varepsilon(n\Delta t, m\Delta\omega)=1$, while, in order to avoid effects of noise and quantization, he has to adopt a moderate value of p (about $p=2$, corresponding to the norm one in the STFT [23]).

However, the concentration measure given by (11) does not work properly in the case of signals corrupted by an impulsive noise. Namely, impulses are highly concentrated in the TF plane. Therefore, if the TF representation does not eliminate them, the concentration measure could indicate an excellent concentration in the TF representation. Impulses appear in the signal for certain instant and they corrupt all frequencies of the TF representation for that instant. This property can be used in order to design alternative concentration measure that is able to produce large values for TF representation influenced by the impulsive noise. Therefore, we are proposing the following slight modification of the previous concentration measure:

$$\tilde{M}(\varepsilon)=\sum_n\left[\sum_m |SM_\varepsilon(n\Delta t, m\Delta\omega)|^{1/p}\right]^p. \tag{12}$$

In this way we are measuring concentration for each instant. If all TF representation values in that instant are large, the obtained $\tilde{M}(\varepsilon)$ will be small, meaning weak concentration.

4. Numerical examples and statistical study

4.1. Numerical example

Consider the sum of a linear FM, sinusoidal FM and short linear FM signals

$$x(t)=\frac{\sqrt{2}}{2}\exp(j48\pi t^2+j128\pi t)+\exp(j36\sin(2\pi t)-j104\pi t)$$
$$+\exp(j64\pi t^2+j36\pi t)\exp(-25t^2), \tag{13}$$

where $t\in[-1,1)$. The signal $x(t)$ is sampled with the sampling rate equal to $\Delta t=1/256$, while, in order to calculate the STFT and the SM, we used $N=256$ (window length used in the calculation of the STFT) and $L=8$. The SM of the non-noisy signal has been shown in Fig. 2a for reference, while the SM calculated for the signal $x(t)$ corrupted by the impulsive noise

$$\nu(t)=0.3\left[\frac{\nu_1(t)}{\nu_2(t)}+j\frac{\nu_3(t)}{\nu_4(t)}\right], \tag{14}$$

where $\nu_i(t)$, $i=1,\dots,4$ are independent Gaussian white noise processes, $E\{\nu_i(t)\}=0$ and $E\{\nu_i(t')\nu_j(t'')\}=\delta(i-j)\delta(t'-t'')$, is depicted in Fig. 2b. The considered form of noise is impulsive/heavy tailed with the Cauchy distribution. From Fig. 2b the signal components cannot be recognized. However, the proposed technique whose representation is shown in Fig. 2c gives an excellent accuracy. Its concentration measure $\tilde{M}(\varepsilon)$ is given in Fig. 2d, while zoomed part of that measure is shown in Fig. 2e. As it can be noticed, the optimal value of ε is $\varepsilon=0.4$. From the concentration measure $M(\varepsilon)$ given in Fig. 2f we can see its decrease for large ε. It means that we should carefully use this kind of concentration measure for large ε since it could point to better concentration than for a small ε due to impulse noise influence. Therefore, in order to avoid this problem, we have used $\tilde{M}(\varepsilon)$.

The second experiment was a visual comparison of the proposed technique with the fractional lower order TF representations from [12]. For a fair comparison we varied a in the range $a\in[0,1]$. Obtained results for characteristic TF representation are depicted in Fig. 3. It can be seen that the best concentration is achieved for $a=1$. The similar results have been obtained for almost all experiments that we have conducted, i.e., from a set of possible TF representations for any impulsive or heavy tailed noise environment we are obtaining that the best concentration, among all considered fractional lower order TF representations, is with $a=1$. Therefore, we can conclude that the technique for adaptation of the TF representations we are proposing here is better than the fractional lower order transform since it is offering more possibilities for selection of adaptive TF representation. In addition, we can observe the main difficulty in the case of the transforms with $a=1$ or in our case with $\varepsilon=0$ when concentration of components is not equal within the signal duration and when we can have component that is not represented in some interval. For example, linear FM signals cannot be observed for $t\in[0.2,0.5]$, while the sinusoidal FM signal in this interval is rather strong.

The proposed technique has also been tested for more challenging mixed Gaussian and impulse noise environment

$$\nu(t)=0.3\left[\frac{\nu_1(t)}{\nu_2(t)}+j\frac{\nu_3(t)}{\nu_4(t)}\right]+0.7[\nu_5(t)+j\nu_6(t)], \tag{15}$$

where $\nu_i(t)$, $i=1,\dots,6$ are independent Gaussian white noise processes, $E\{\nu_i(t)\}=0$ and $E\{\nu_i(t')\nu_j(t'')\}=\delta(i-j)\delta(t'-t'')$. The SM obtained with the proposed procedure and its concentration measure are depicted in Fig. 4. It can be seen that the proposed technique is useful even for this kind of noise. Namely, all three components can be easily observed in the TF

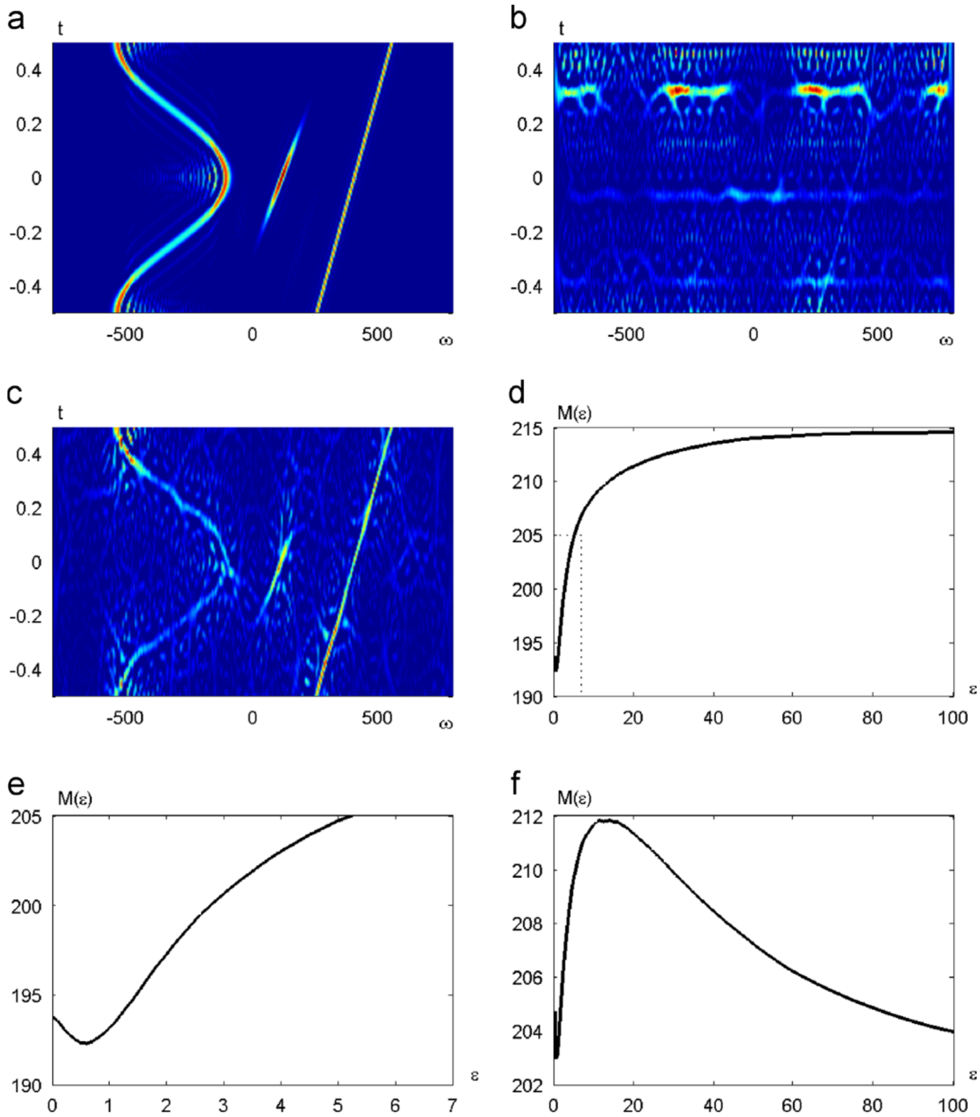

Fig. 2. Signal corrupted with Cauchy noise: (a) the SM of non-noisy signal; (b) the SM of corrupted signal; (c) the robust SM obtained with the proposed approach; (d) concentration measure $\tilde{M}(\varepsilon)$; (e) zoomed concentration measure $\tilde{M}(\varepsilon)$; and (f) concentration measure $M(\varepsilon)$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

plane and their parameters can be estimated from this representation.

Note: All TF representations in Figs. 2–4 are given with the same MATLAB colormap in order to show results in a fair and comparable manner.

Based on numerous experiments that we have conducted it can be concluded that the value of $\varepsilon \approx 0.5$ produces results of good quality, meaning that search for optimal ε can be performed in a rather narrow interval around $\varepsilon \approx 0.5$. In addition, instead of the direct search for ε an iterative procedure can be performed starting with $\varepsilon = 0$. However, this topic will not be considered here.

4.2. Comparison of the robust TF representations

In this section, the proposed approach is compared with marginal-median SM and myriad SM using numerical measure rather than visual comparison. As the quality measure of the TF representation $SM_2(n\Delta t, m\Delta\omega)$ we considered the Pearson correlation coefficient [24] given as

$$\rho_{SM_1,SM_2} = \frac{E\{[SM_1(n\Delta t, m\Delta\omega) - \mu_{SM_1}][SM_2(n\Delta t, m\Delta\omega) - \mu_{SM_2}]\}}{\sqrt{\text{var}_{SM_1}\ \text{var}_{SM_2}}}$$

where μ_{SM_i} and var_{SM_i}, $i = 1, 2$ are mean and variance of the SM calculated over entire TF plane, respectively and the reference TF representation $SM_1(n\Delta t, m\Delta\omega)$ is the standard SM of the non-noisy signal. The Pearson correlation coefficient is the similarity measure of two statistics or random variables. Values of ρ_{SM_1,SM_2} close to 1 indicate larger similarity between non-noisy signal TF representation ($SM_1(n\Delta t, m\Delta\omega)$) and considered robust TF transform ($SM_2(n\Delta t, m\Delta\omega)$) than those for smaller ρ_{SM_1,SM_2}. The experiment is performed for various

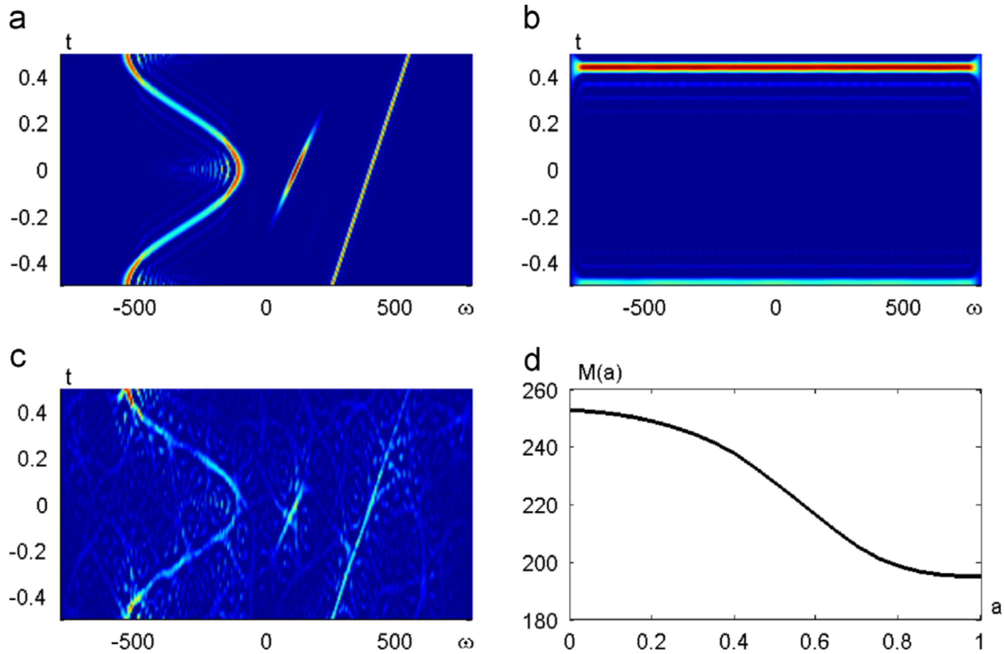

Fig. 3. Signal corrupted with Cauchy noise: (a) the SM of non-noisy signal; (b) the SM of corrupted signal; (c) the adaptive robust SM for adaptive fractional-lower order signal; and (d) concentration measure $\tilde{M}(a)$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

mixed Gaussian and impulse noise environments

$$\nu(t) = \alpha\left[\frac{\nu_1(t)}{\nu_2(t)} + j\frac{\nu_3(t)}{\nu_4(t)}\right] + \sigma[\nu_5(t) + j\nu_6(t)], \tag{16}$$

where $E\{\nu_i(t)\} = 0$ and $E\{\nu_i(t')\nu_j(t'')\} = \delta(i-j)\delta(t'-t'')$, $\alpha \in [0,1]$ and $\sigma \in [0,1]$. The results of the experiment are presented in Fig. 5. For brevity reasons, only two sets of the results are shown: those for Gaussian noise with $\alpha = 0$ and $\sigma \in [0,1]$ (shown by solid lines) and those for mixed Gaussian and impulse noise $\alpha = 0.3$ and $\sigma \in [0,1]$ (shown by dashed lines). Lines with square, hexagon and circle markers correspond to the proposed approach, marginal-median SM and myriad SM, respectively. For both cases, the proposed SM outperforms the marginal-median SM and the myriad SM meaning that it is closest to the SM of the non-noisy signal. This justifying its usage in the robust TF framework.

4.3. IF estimation

The robust SMs, obtained from the STFT evaluated with the proposed adaptive normalization strategy, marginal-median STFT and myriad STFT, are compared in the IF estimation. The myriad-based SM is evaluated for $\kappa = 1$ and, in order to ensure faster convergence of the myriad STFT, the marginal median-based STFT is used in the initial iteration.

The considered signal was the linear FM one

$$x(t) = \exp(j\phi(t)) = \exp(j48\pi t^2)$$

with the IF $\omega(t) = \phi'(t) = 96\pi t$. In order to compare the proposed TF representation with the marginal median and myriad based SMs, we have calculated the MSE of the obtained IF estimates. The considered noise environment was impulsive with pulses having the random phase and the magnitude that is five times larger than the signal magnitude. Also, the mixture of these impulses with a Gaussian noise is considered. We have performed the Monte-Carlo testing of the obtained results with 300 trials for each percentage of the additive impulsive noise added to the signal in the range of [0%,50%]. The obtained results are depicted with dotted lines in Fig. 6. Results of experiments performed with a mixture of the Gaussian and the impulsive noise are given with solid lines in the same figure. In both cases, the proposed technique produces better results than the marginal median based technique while it is slightly worse than the myriad based estimator.

4.4. Calculation complexity

The detailed calculation complexity analysis of the SM is carried out in [20,25]. By knowing that the STFT can be realized recursively, it can be show that the calculation of the SM in a single instant requires $O(NL)$ operations. Having this in mind, the complexity of the proposed technique is $O(N_\varepsilon NL)$ floating point operations, where N_ε is the number of ε-values in which $\tilde{M}(\varepsilon)$ is calculated in order to obtain optimal ε. The sorting is the most demanding in the calculation of the marginal-median SM and it requires $O(N^2 \log N)$ comparison operations. Since $N \ll L$ and $N > N_\varepsilon$, the marginal median SM has significantly larger complexity than the proposed technique. Finally, an additional iterative procedure, with N_I iterations, in the myriad SM requires $O(N_I N^2)$ floating point operations, which is again significantly larger than $O(N_\varepsilon NL)$.

To make comparison more concrete, it can be shown that the proposed approach with N=256 frequency bins and L=8, evaluated using $N_\varepsilon = 21$ different ε values

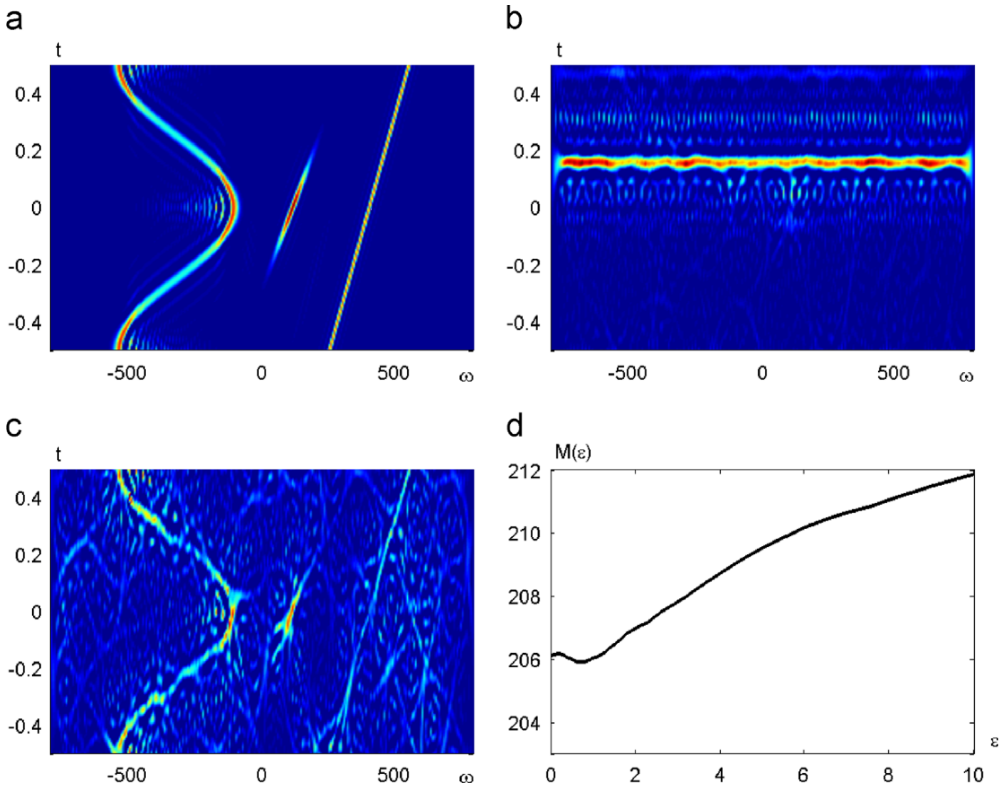

Fig. 4. Signal corrupted with mixture of Gaussian and Cauchy noise: (a) the SM of non-noisy signal; (b) the SM of corrupted signal; (c) the robust SM obtained with the proposed approach; and (d) concentration measure $\tilde{M}(\varepsilon)$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

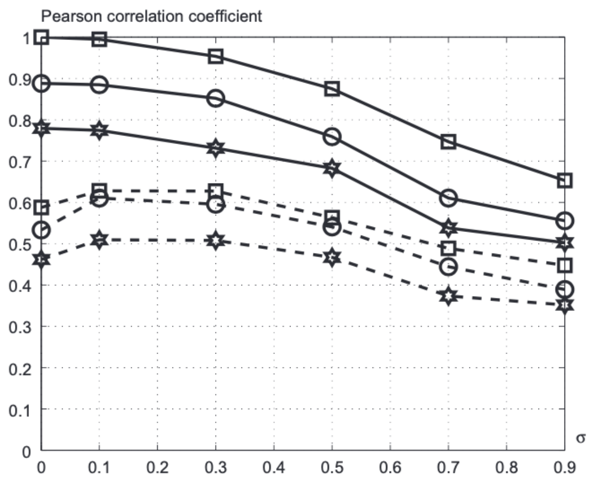

Fig. 5. Pearson correlation coefficient: solid line – Gaussian noise environment with $\sigma \in [0,1]$; and dashed line – mixed Gaussian and impulse environments with $\alpha = 0.3$ and $\sigma \in [0,1]$. Markers: square - proposed transform; hexagon – marginal median SM; and circle – myriad SM.

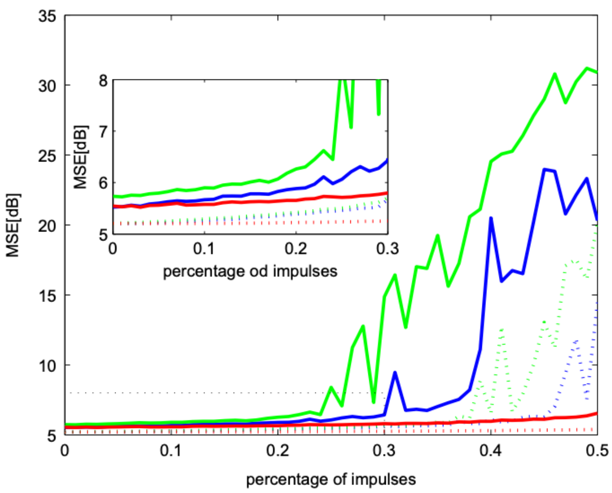

Fig. 6. MSEs of the IF estimates obtained by various robust SMs: dotted lines for impulsive noise environment; solid lines for mixture of Gaussian and impulsive noise. Green lines – marginal median realization; blue lines – proposed technique; and red lines – myriad technique. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

without any optimization has approximately 84.5% less calculations with respect to the marginal-median STFT-based SM and 96.1% less calculations than the myriad STFT-based counterpart.

5. Conclusion

An alternative signal normalization strategy for the time–frequency analysis of nonstationary signals has been proposed in this paper. The proposed technique gives more flexibility in design of time–frequency representations than the fractional lower order approach. In addition, it produces better results for the instantaneous frequency estimation than the time–frequency representations calculated based on the marginal median STFT, while it is slightly worse than the time–frequency representation calculated with the myriad filter-based robust strategy. However, the proposed technique is evaluated using the fast Fourier transform (FFT) algorithms and obtaining calculation measures, while the median and myriad counterparts are evaluated using extremely demanding sorting and iterative procedures.

Acknowledgments

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