Image Reconstruction from a Reduced Set of Pixels using a Simplified Gradient Algorithm

Isidora Stanković, Irena Orović, *Member, IEEE*, Srdjan Stanković, *Senior Member, IEEE*

Abstract - **A reconstruction of images in the DCT transformation domain based on the adaptive gradient algorithm is considered in this paper. Two approaches are used in the reconstruction. In the first approach, the image is pre-processed using 8x8 blocks, such that the smallest DCT coefficients are set to zero in order to make the image sparse. The second approach reconstructs the image without the pre-processing step. It has been assumed that the sparsity is an intrinsic property of the analyzed image. An adaptive gradient based algorithm is used to recover a large number of missing pixels in the image. In order to improve the calculation complexity, in this paper we propose an improved version of recently proposed adaptive gradient algorithm, which is now reduced to a single, automatically determined parameter. The previous reconstruction of black and white and colour images is repeated with a significant calculation efficiency improvement**.

Keywords - **compressive sensing, gradient-based algorithm, image processing, image reconstruction, sparse signals**

I. INTRODUCTION

Compressive Sensing (CS) is a new, growing field in signal processing with an intensive development in the last ten years. Sparse signals are in the focus of compressive sensing theory and applications. Most of the signals are processed and analysed in some of the transform domains, and then reconstructed via the inverse transforms. Signals can be observed as sparse in a transformation domain if the number of non-zero coefficients in that domain is much fewer than the number of signal samples. Such signals can be reconstructed using a small number of non-zero coefficients, recovered from a small set of signal samples. This formulation is in the core of CS. Namely, it has been shown that just a small number of randomly positioned samples is required to reconstruct all values of a sparse signal [1]-[8]. Various techniques are developed to deal with the reconstruction of sparse signals from randomly chosen measurements. All these techniques belong to two large groups. The first one is based on the analysis of signal in a transformation domain. The reconstruction is performed in the transformation domain where the signal is sparse. The other group of reconstruction techniques is the gradient based. Image-related applications, such as medical imaging, image compression, denoising, photography, holography, facial recognition, radar and array signal processing are some of the applications of CS techniques.

Common images usually have just a few significant DCT coefficients within each transformation block that should be considered as non-zero. Thus, they comply with the CS algorithm requirement that the image is a sparse signal in the considered transformation domain [9]. Since the images have a huge number of pixels, the reconstruction algorithm should be applicable to the cases when a large number of the missing pixels are randomly positioned over the whole image. To this aim the adaptive gradient based recovery algorithm is used [10]. In this paper, its variant that uses only one parameter is applied. In this way the number of parameters is reduced. The only parameter is automatically determined. Computation efficiency of the algorithm is improved compared to the more complex form given in [11], [12], while the same quality of results is obtained.

The paper is organized as follows. The problem formulation is given in the Section 2. A reconstruction algorithm, with image recovery examples, is presented in Section 3. The results and conclusions are given in Section 4.

II. BASIC THEORY

Consider an image $x(n,m)$ whose random set of pixels is available at

$$(n,m) \in \{(n_1,m_1),(n_2,m_2),...,(n_M,m_M)\}.$$

The goal is to reconstruct the remaining pixels that are not available. In order to apply the CS reconstruction algorithms, the image sparsity is assumed in the transform domain (in this case DCT). The DCT of an image is usually calculated by using 8x8 blocks. Most of the common images could be considered as sparse in the DCT domain without any additional processing. If we want to be sure that the original image, which will be processed in our examples, is sparse we can pre-process it by calculating the DCT of its 8x8 blocks and set the lowest amplitude coefficients to zero. By making the image

I. Stankovic is an MSc student at the Imperial College London, (email: isidora.stankovic14@imperial.ac.uk).
Prof. S. Stankovic, and dr I. Orovic are with University of Montenegro (email: srdjan@ac.me, irenao@ac.me).

sparse in the DCT domain we will not make a notable visual difference with respect to the original image. Nevertheless, even if we do not use this step, most of the images can be considered as sparse in the DCT domain, and the presented method can be used without pre-processing of the original image. For the analysis we use the same image as in [11], [12]. The original image "Isi" and its pre-processed version with "sparsified" DCT representation are shown in Fig. 1. These two images will be used in the analysis and reconstruction procedure.

We have assumed that the analyzed image is sparse and that its values are available only for the set of pixels:

$$x(n,m), \text{ for } (n,m) \in \{(n_1,m_1),(n_2,m_2),...,(n_M,m_M)\} \quad (1)$$

Using the available pixels (measurements), an incomplete image $y(n,m)$ is formed. It assumes the original image values at the positions of available pixels, while the missing pixels are set to zero value. This new image is defined as:

$$y(n,m) = \begin{cases} x(n,m), \text{ if } (n,m) \in \{(n_1,m_1),...,(n_M,m_M)\} \\ 0, \text{ elsewhere} \end{cases} \quad (2)$$

Note that for the missing pixels any value within the possible image values range can be assumed in the initial step. The algorithm will reconstruct the true image values at these positions. For graphical representation of missing pixels the value 255 corresponding to a white pixel will be used instead of 0. The missing pixels are then represented as blank (white) pixels, Fig. 2.

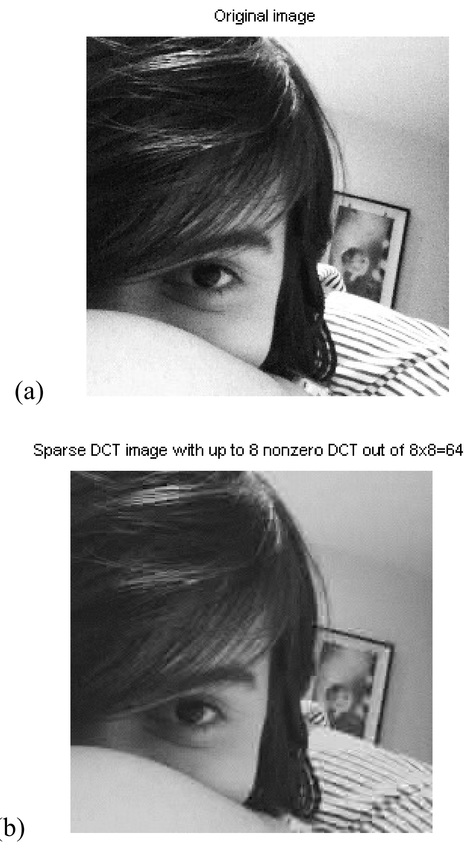


Figure 1: (a) Original image; (b) Image sparse in the DCT domain

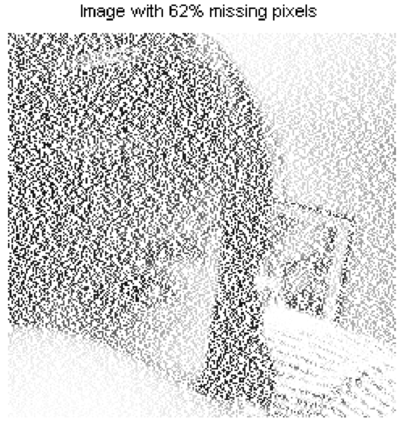


Figure 2: Image with a large number of missing pixels

III. RECONSTRUCTION ALGORITHM

In the analysis, the image is split into 8x8 blocks. The DCT is calculated for these blocks and the gradient algorithm is applied in the reconstruction of the missing samples [10], [11]. After reconstruction, the image blocks are recombined into a final image. The two-dimensional adaptive gradient algorithm is similar to the one-dimensional one [13]-[15] except that we use the position of the value in x (rows) and y (columns) directions rather than a value in a sequence. The L_1-norm is used over two-dimensional space of the image transform, i.e., as a sum the DCT values over all rows and columns.

Algorithm is implemented as follows:

Step 1:

Using the available image pixels $x(n,m)$, at the positions $n \in \{n_1, n_2, ..., n_M\}$ and $m \in \{m_1, m_2, ..., m_M\}$, a new image $y(n,m) = x(n,m)$ is formed. The new image $y(n,m)$ is formed by setting the unavailable values of $x(n,m)$, at $n \notin \{n_1, n_2, ..., n_M\}$ or $m \notin \{m_1, m_2, ..., m_M\}$, to zero:

$$y(n,m) = \begin{cases} x(n,m), \text{ if } (n,m) \in \{(n_1,m_1),...,(n_M,m_M)\} \\ 0, \text{ elsewhere} \end{cases}$$

Step 2:

For each position where the signal samples are not available, for $(k,l) \notin \{(n_1,m_1),(n_2,m_2),...,(n_M,m_M)\}$, within the same considered blocks:

$$\begin{aligned} y_1^{(k,l)}(n,m) &= y(n,m) + \Delta\delta(n-k, m-l) \\ y_2^{(k,l)}(n,m) &= y(n,m) - \Delta\delta(n-k, m-l) \end{aligned} \quad (4)$$

are formed.

Step 3:

The 2D-DCT is calculated as

$$Y_1^{(k,l)}(p,q) = DCT2\{y_1^{(k,l)}(n,m)\}$$
$$Y_2^{(k,l)}(p,q) = DCT2\{y_2^{(k,l)}(n,m)\} \quad (5)$$

The L_1 -norm of 2D-DCT is used:

$$\left\|Y_1^{(k,l)}\right\|_1 = \sum_{p=0}^{N-1}\sum_{q=0}^{M-1}\left|Y_1^{(k,l)}(p,q)\right|$$
$$\left\|Y_2^{(k,l)}\right\|_1 = \sum_{p=0}^{N-1}\sum_{q=0}^{M-1}\left|Y_2^{(k,l)}(p,q)\right| \quad (6)$$

The gradient corresponding to the change of the (k,l)-th pixel, $(k,l) \notin \{(n_1,m_1),(n_2,m_2),...,(n_M,m_M)\}$, is:

$$G(k,l) = \frac{\left\|Y_1^{(k,l)}\right\|_1 - \left\|Y_2^{(k,l)}\right\|_1}{2\Delta} \quad (7)$$

Step 4:

Each missing signal value, at the positions $(k,l) \notin \{(n_1,m_1),(n_2,m_2),...,(n_M,m_M)\}$, is then changed in the direction of the gradient for a step 2Δ :

$$y^{(i+1)}(k,l) = y^{(i)}(k,l) - 2\Delta G(k,l) \quad (8)$$

The available signal values are not changed.

The parameter value is automatically determined as

$$\Delta = \max\{x(n,m)\}.$$

When the signal values do not cause the measure value change, the step size is reduced as $\Delta \to \Delta/\sqrt{10}$. The procedure is repeated until the desired reconstruction accuracy is achieved. The reconstruction accuracy is of the order of parameter Δ value.

Applying the algorithm on the sparse image, the reconstructed image is obtained, Figure 3.

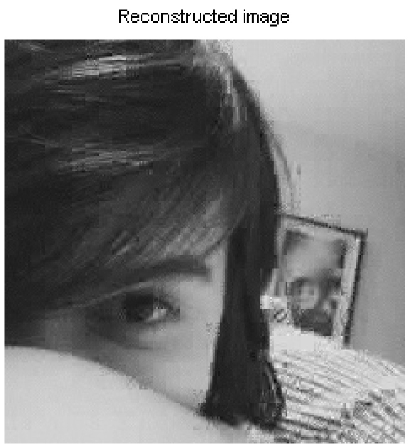


Figure 3: Reconstructed image from the image with 62% missing samples

The algorithm was tested on the same image but with 75% of missing samples. The image with 75% missing samples is presented in Figure 4.

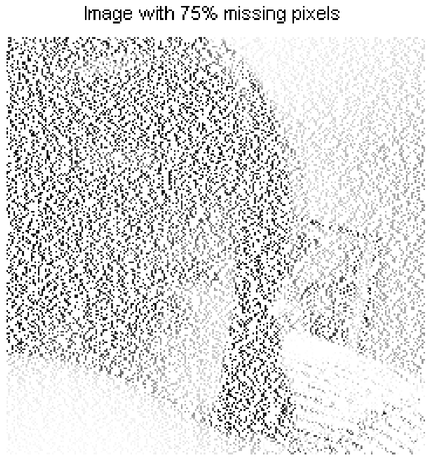


Figure 4: Image with 75% missing samples

The reconstructed image is shown in Figure 5. The picture already became blurrier. The 8x8 blocks started to be more visible. For this image we can conclude that the algorithm works properly with up to about 75% missing samples.

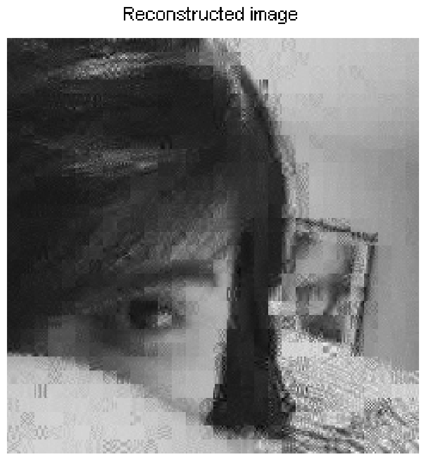
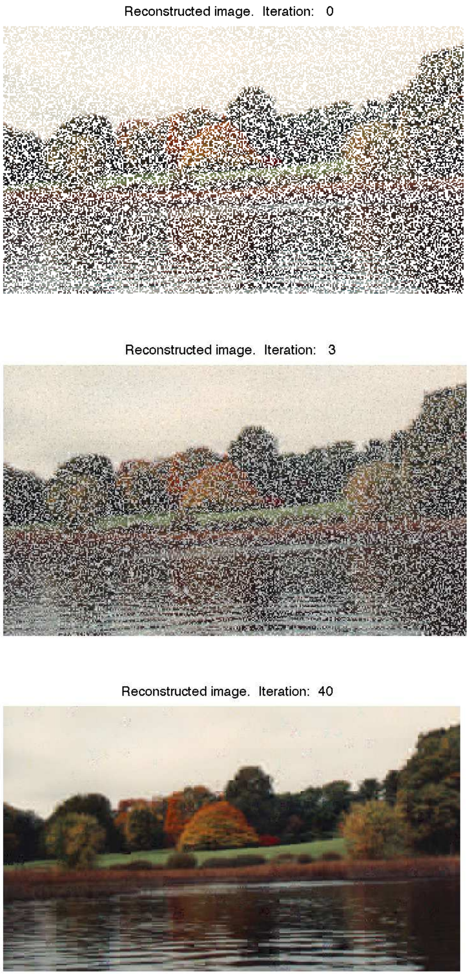


Figure 5: Reconstructed image from the image with 75% missing samples

The algorithm is also tested on colour images. For a colour images, each primary colour (red, green and blue) is reconstructed separately. The presented reconstruction method is used without pre-processing of the original image to make it sparse in an artificial manner by setting the DCT coefficients to zero. The number of corrupted pixels is 50% of the total number of pixels. The reconstruction of colour benchmark image "Autumn" from MATLAB is shown in Figure 6.



(a)

(b)

(c)

Figure 6: (a) Image with missing/corrupted samples; (b) Reconstruction after 3 iterations; (c) Reconstructed image

IV. CONCLUSIONS

A reconstruction of images with a large number of missing samples is considered. The case of missing samples can be considered due to reduced number of available samples or due to our desire to use a small number of signal samples in the process of information storage or transmission. In the image processing, the 8x8 block-based DCT is commonly used. Since, in general, images are not strictly sparse then we have considered two possibilities. One was to make the analysed image sparse in the 2D-DCT domain by appropriate pre-processing, by setting only a few of its largest values as non-zero. The processing and reconstruction of missing samples is then based on this “sparsified” image version. Common images contain just a few significant 2D-DCT coefficients in the 8x8 blocks (which means they are approximately sparse), and thus the pre-processing step can be avoided. This is considered in this paper as well. In all cases the image is reconstructed using the compressive sensing adaptive gradient algorithm. Both cases produced similar results for the analysed image. In this paper a simplified, one parameter form, of this algorithm is used. The parameter is automatically determined based on the image values. It has been shown that the recovery can be succesful with the number of noisy samples equal to the number of missing samples (up to 75% missing pixels). The algorithm has been applied on both black and white and colour images. High reconstruction accuarcy reconstructions are repeated with the simplified algorithm as in the case of the original two-parameter algorithm.

REFERENCES

[1] E.J. Candes and M. Wakin, “An Introduction to Compressive Sampling”, *IEEE Signal Processing Magazine,* vol. 25, no. 2, pp. 21-30, March 2008.

[2] M. Duarte, M. Davenport, M. Wakin, and R. Baraniuk, "Sparse signal detection from incoherent projections", *IEEE International Conference on Acoustic, Speech and Signal Processing,* vol. 3, May 2006.

[3] M. Duarte and Y. Eldar, “Structured Compressed Sensing: From Theory to Applications”, *IEEE Transactions on Signal Processing,* vol. 59, no. 9, pp. 4053-4085, September 2011.

[4] E.J. Candes, J. Romberg, and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information”, *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 489–509, February 2006.

[5] D.L. Donoho, “Compressed sensing,” *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, April 2006.

[6] R. Baraniuk, “Compressive Sensing”, *IEEE Signal Processing Magazine,* vol. 24, no.4, pp. 118-121, July 2007.

[7] S. Stankovic, I. Orovic, and E. Sejdic, "Multimedia Signals and Systems", Springer, 2012.

[8] L. Stankovic, S. Stankovic, and M. Amin, “Missing Samples Analysis in Signals for Applications to L-estimation and Compressive Sensing,” *Signal Processing*, vol. 94, Jan 2014, pp. 401-408.

[9] B. Zhang, X. Yin, W. Wang, and Q. Lei, "Research on image processing with compressed sensing algorithm: Base on the improved layered discrete cosine transform", *IEEE International Conference on Communications*, pp. 6357-6361, June 2012.

[10] L. Stankovic, M. Dakovic and S. Vujovic, “Adaptive Variable Step Algorithm for Missing Samples Recovery in Sparse Signals”, *IET Signal Processing*, vl.8, no.3, May 2014, pp. 246 -256, (arxiv.org/abs/ 1309.5749).

[11] I. Stankovic, "Recovery of images with missing samples using a gradient compressive sensing algorithm", arXiv:1407.3695, April 2014.

[12] I. Stankovic, "Sparse Signals: Analysis and Recoinstruction", Final project (project advisor D. Barjamovic), University of Westminster, June 2014.

[13] M.A. Figueiredo, R. D. Nowak, and S. J. Wright, “Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems”, *IEEE Journal of Selected Topics in Signal Processing,*, vol. 1, no. 4, pp. 586–597, 2007.

[14] R. Chartrand, "Exact Reconstruction of Sparse Signals via Nonconvex Minimisation", *IEEE Signal Processing Letters*, vol. 14, no. 10, pp. 707-710, October 2007.

[15] J. R. Deng, Guanghui, J. Yansheng, and N. Wenjing, "Iterative Weighted Gradient Projection for Sparse Reconstruction", *Information Technology Journal*, vol. 10, no. 7, pp. 1409-1414, June 2011.