University of Montenegro
Faculty of Electrical Engineering

Compressive sensing: Theory, Algorithms and Applications

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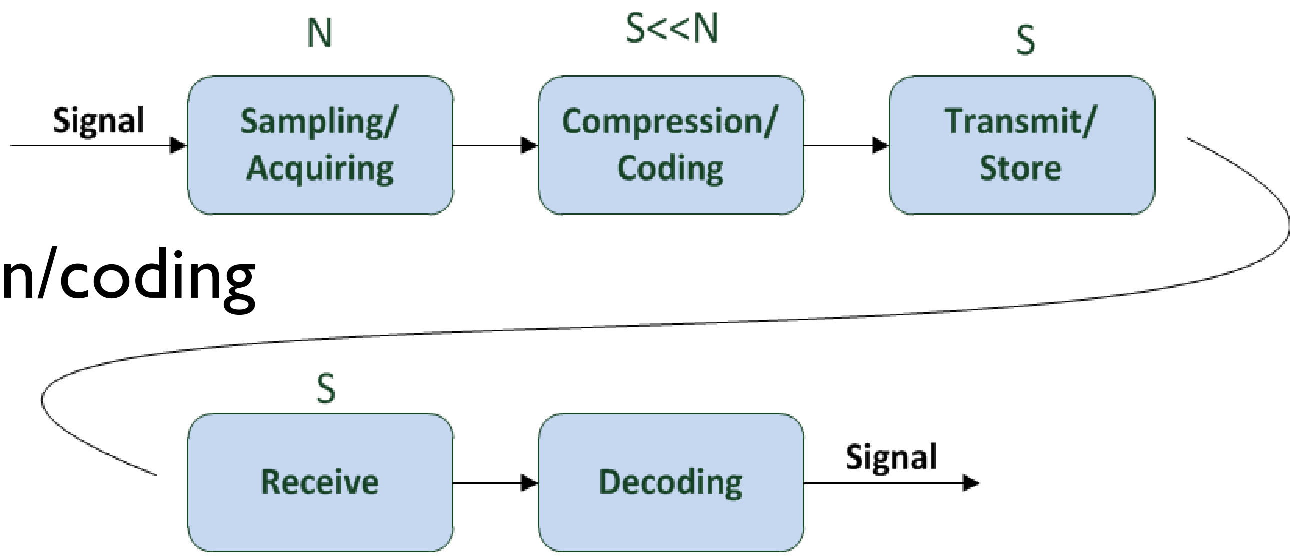
About CS group

- Project CS-ICT supported by the Ministry of Science of Montenegro
- 12 Researchers
  - 2 Full Professors
  - 3 Associate Professors
  - 1 Assistant Professor
  - 6 PhD students
- Partners:
  - INP Grenoble, FEE Ljubljana, University of Pittsburgh, Nanyang University, Technical faculty Split

Shannon-Nyquist sampling
Standard acquisition approach

- **Sampling frequency - at least twice higher than the maximal signal frequency ($2f_{max}$)**

- Standard digital data acquisition approach
  - sampling
  - compression/coding
  - decoding



Audio, Image, Video Examples

• **Audio signal:** - sampling frequency 44,1 KHz - 16 bits/sample	***Uncompressed:*** 86.133 KB/s	***MPEG 1*** – compression ratio 4: 21.53 KB/s
• **Color image:** -256x256 dimension – 24 bits/pixel - 3 color channels	***Uncompressed:*** 576 KB	***JPEG*** – quality 30% : 7.72 KB
• **Video:** - CIF format (352x288) -NTSC standard (25 frame/s) -4:4:4 sampling scheme (24 bits/pixel)	***Uncompressed:*** 60.8 Mb/s	***MPEG 1***- common bitrate 1.5 Kb/s **MPEG 4** 28-1024 Kb/s

Compressive Sensing / Sampling

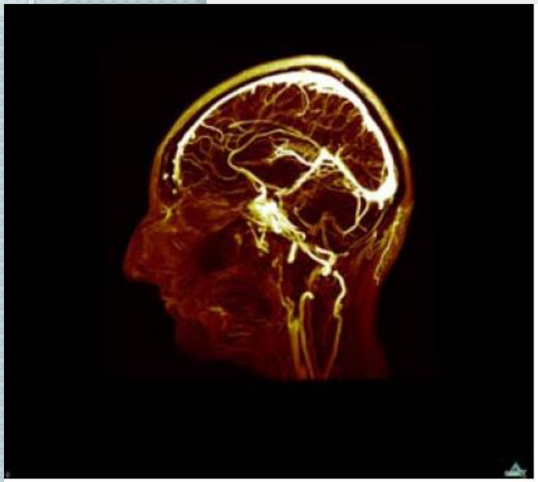
- Is it always necessary to sample the signals **according to the Shannon-Nyquist criterion**?
- Is it possible to apply the **compression during** the acquisition process?

⬇

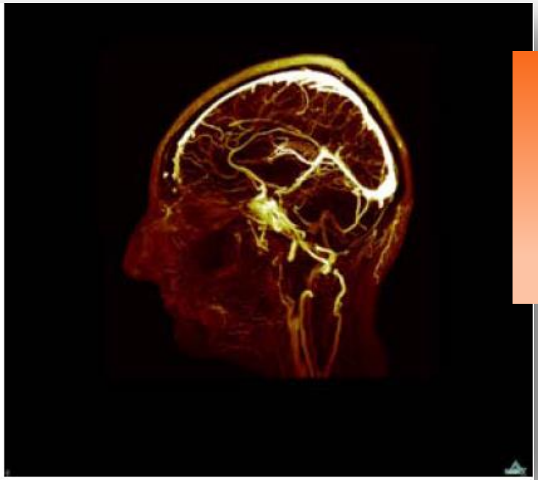
Compressive Sensing:

- **overcomes constraints** of the traditional sampling theory
- applies a concept of **compression during the sensing** procedure

CS Applications



Standard sampling



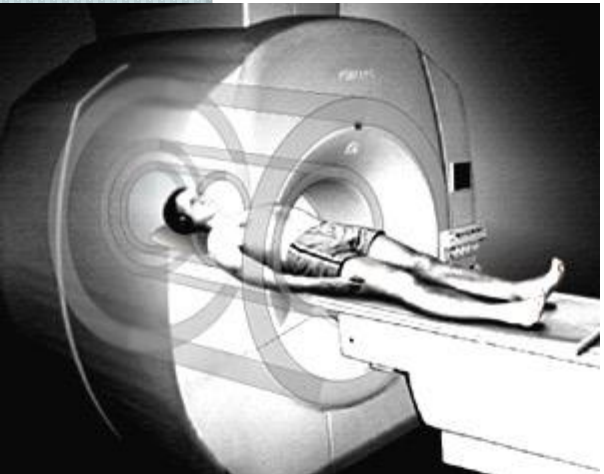
CS reconstruction using 1/6 samples

Biomedical Appl. MRI

Reconstruction from undersampled data
Standard (left)
CS (right)

Make entire "puzzle" having just a few pieces:

Reconstruct entire information from just few measurements/pixels/data

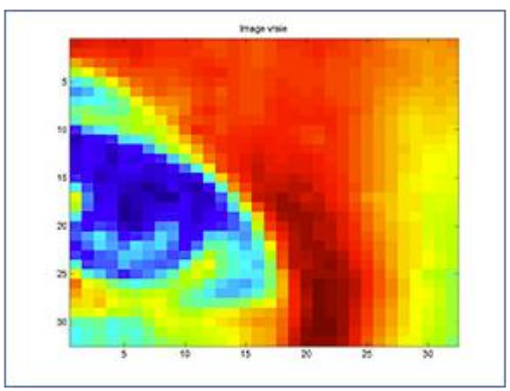


CS promises SMART acquisition and processing and SMART ENERGY consumption

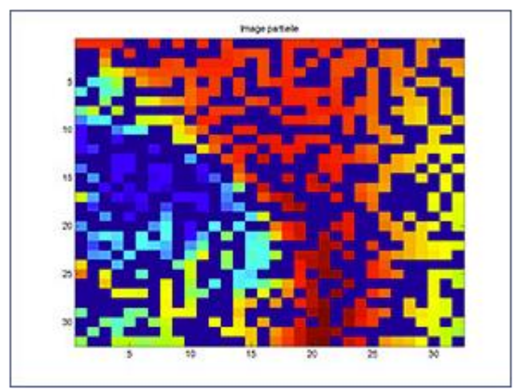
Compressive sensing is useful in the applications where people used to make a large number of measurements

CS Applications

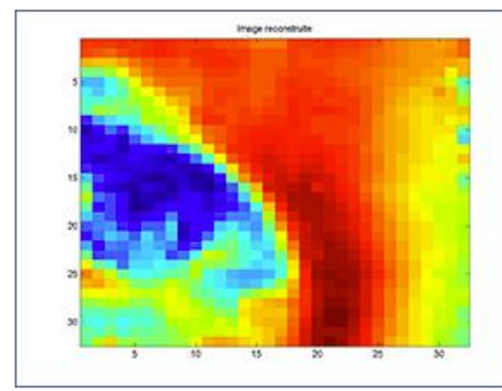
Reconstruction of Lenna's eye



Starting Image



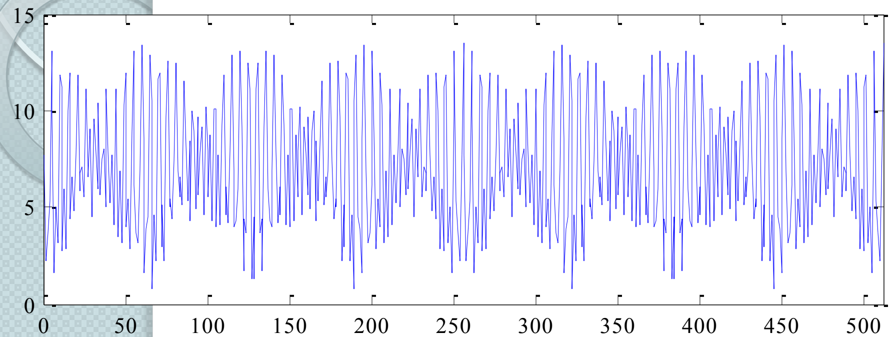
Partial Image

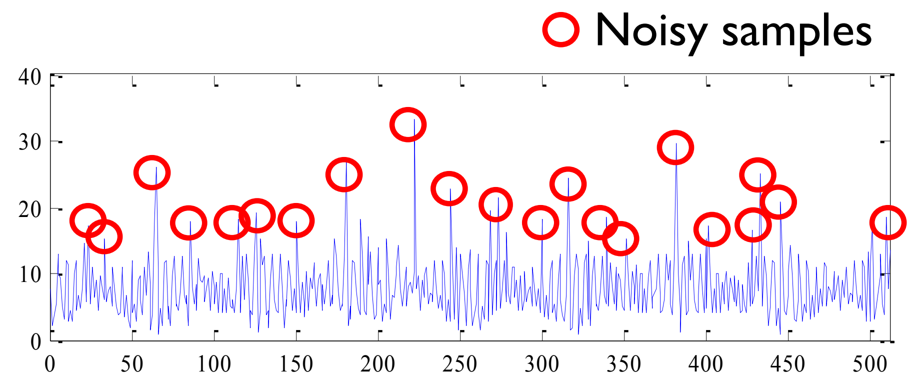


Reconstructed Image

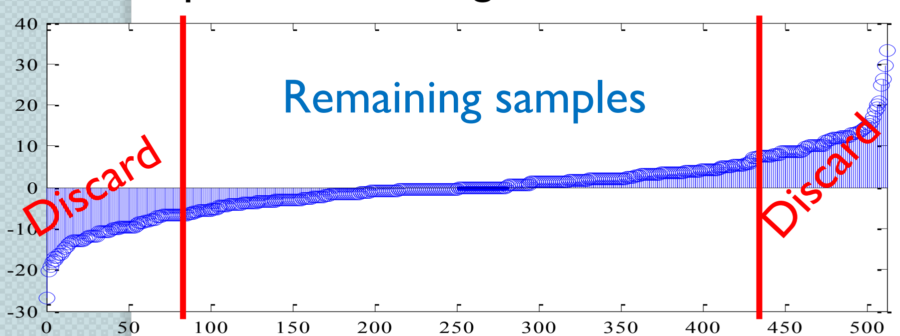
L-statistics based Signal Denoising

Non-noisy signal





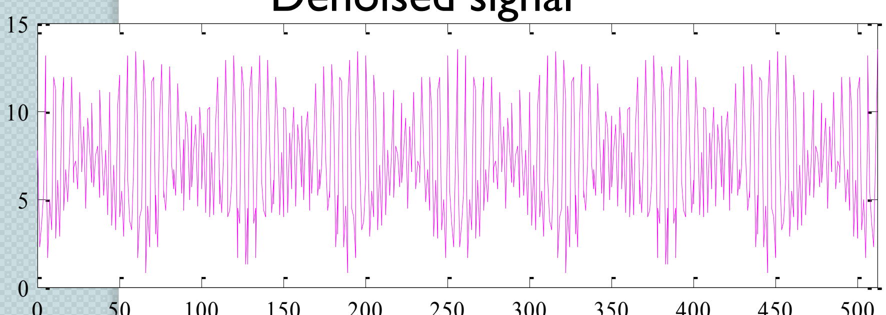
Sorted samples - Removing the extreme values



Discarded samples are declared as "missing samples" on the corresponding original positions in non-sorted sequence

This corresponds to CS formulation

Denoised signal



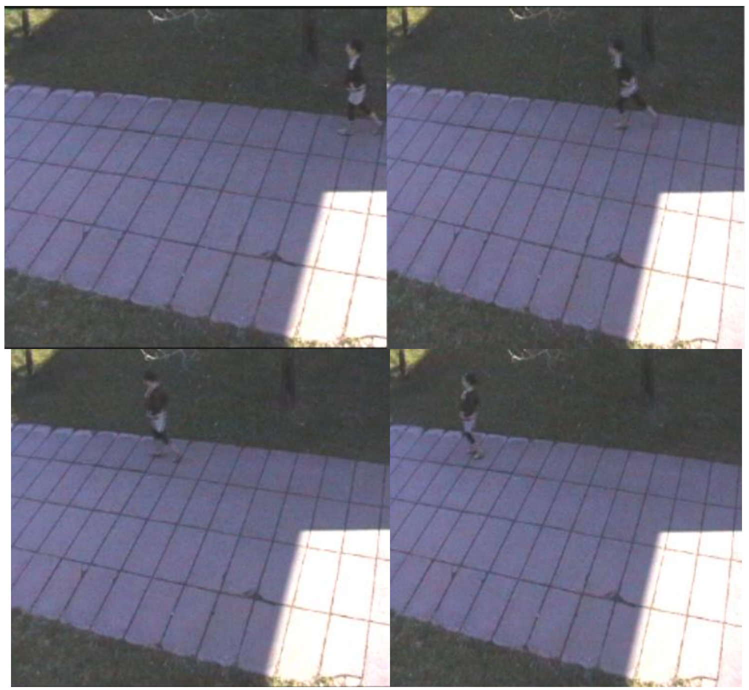
After reconstructing "missing samples" the denoised version of signal is obtained

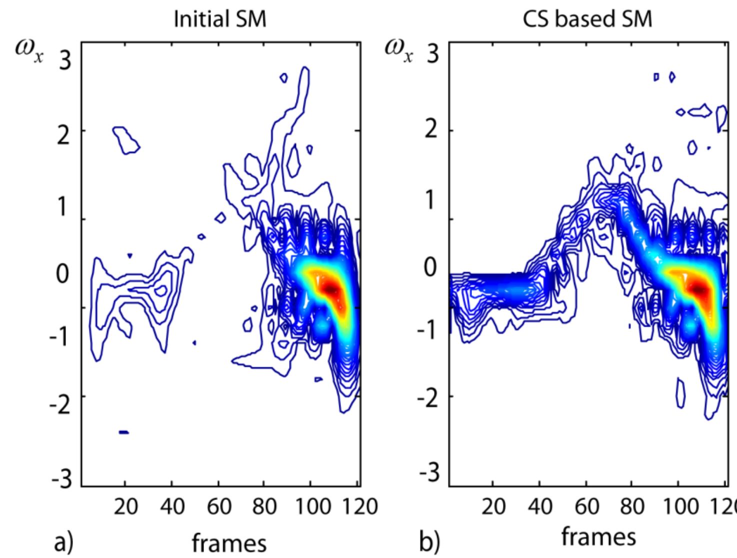
Video sequences

*Video Object Tracking

*Velocity Estimation

*Video Surveillance



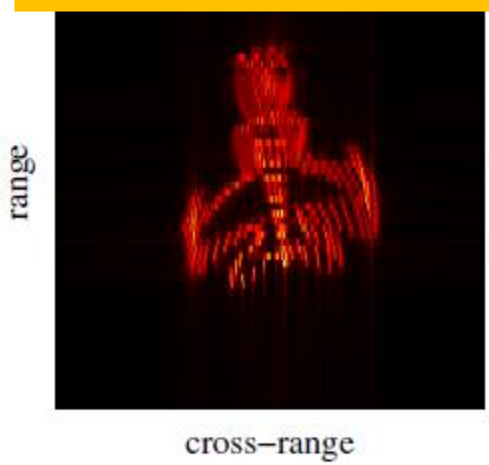


CS Applications

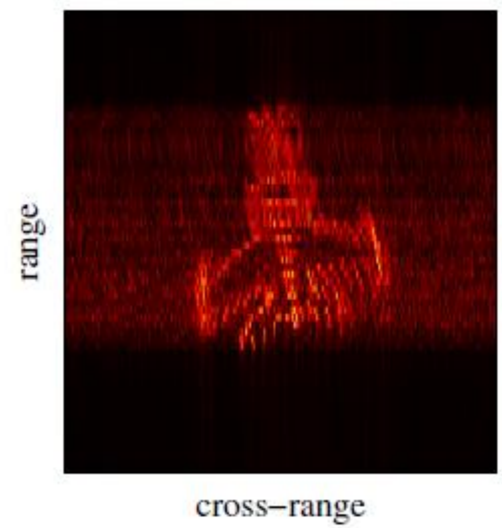
Reconstruction of the radar images

Mig 25 example

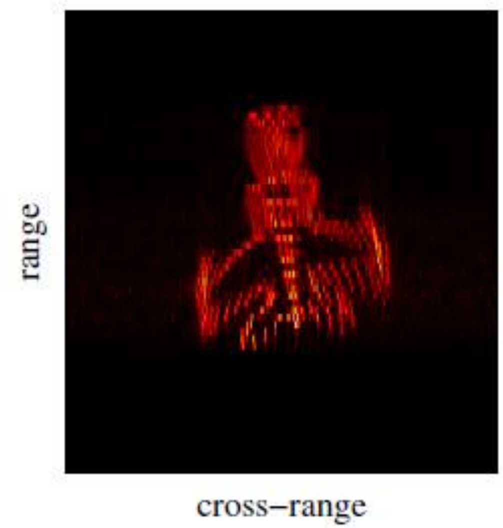
ISAR image with full data set



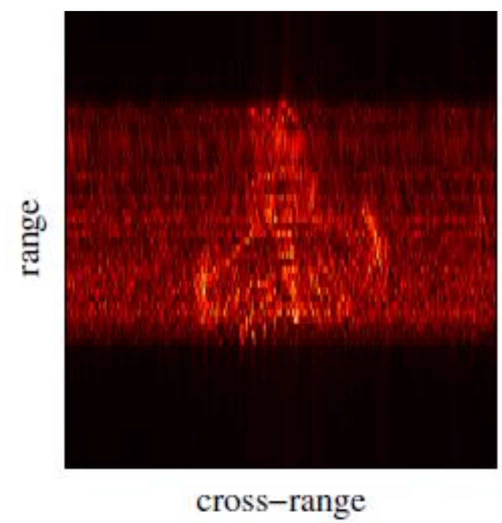
50% available pulses



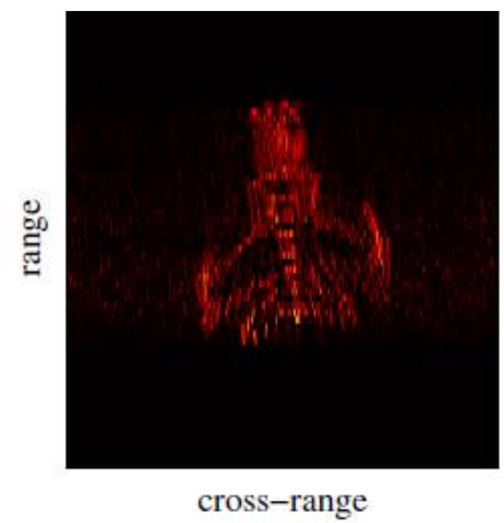
reconstructed image



30% available pulses

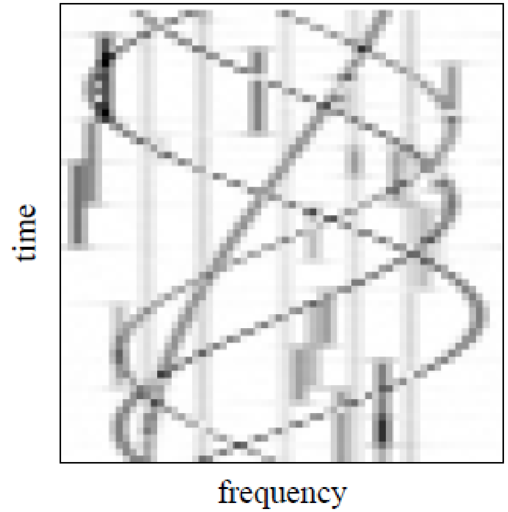


reconstructed image



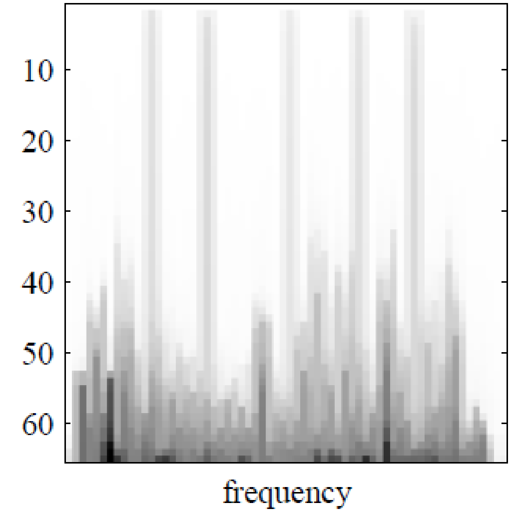
Compressive Sensing Based Separation of Non-stationary and Stationary Signals

Absolute STFT values



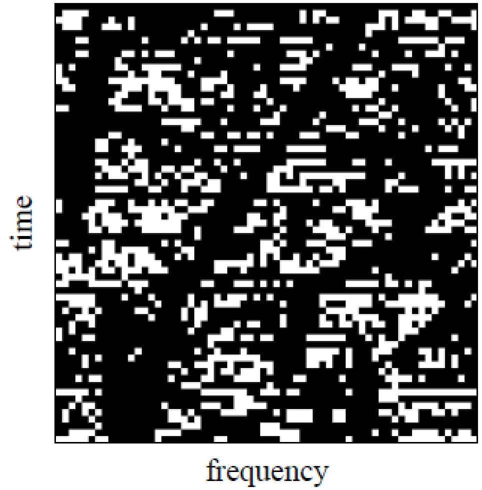
(a)

Sorted values



(b)

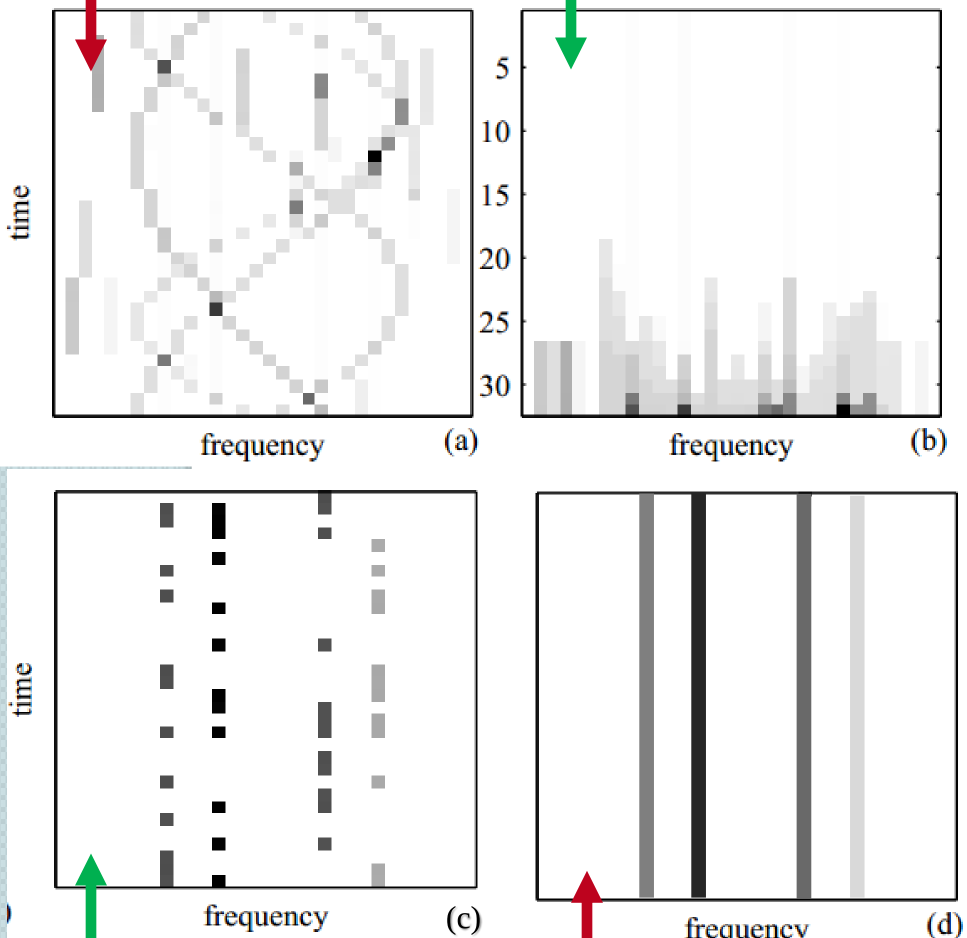
CS mask in TF



(c)

STFT of the composite signal

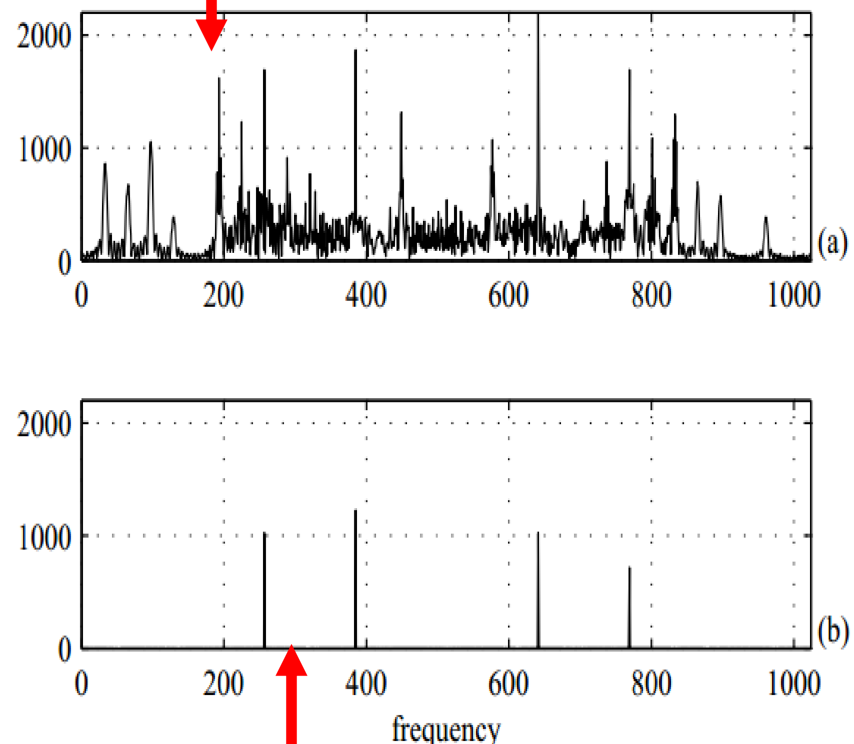
STFT sorted values



STFT values that remain after the L-statistics

Reconstructed STFT values

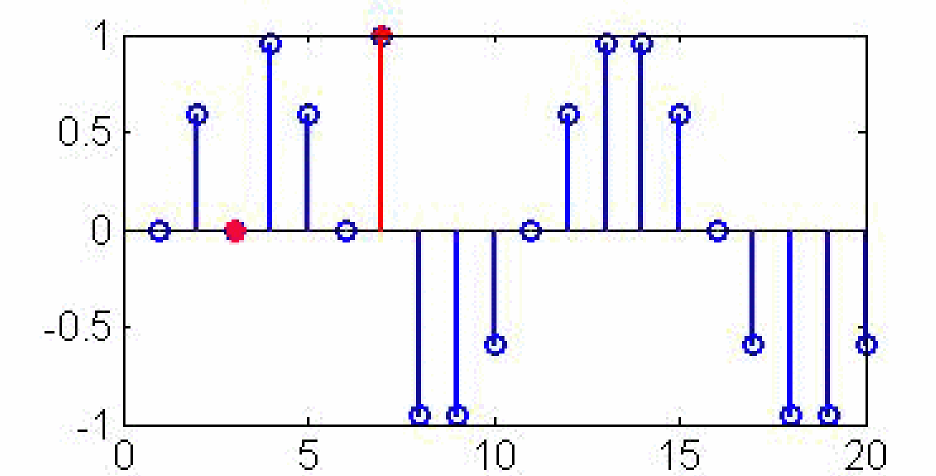
Fourier transform of the original composite signal

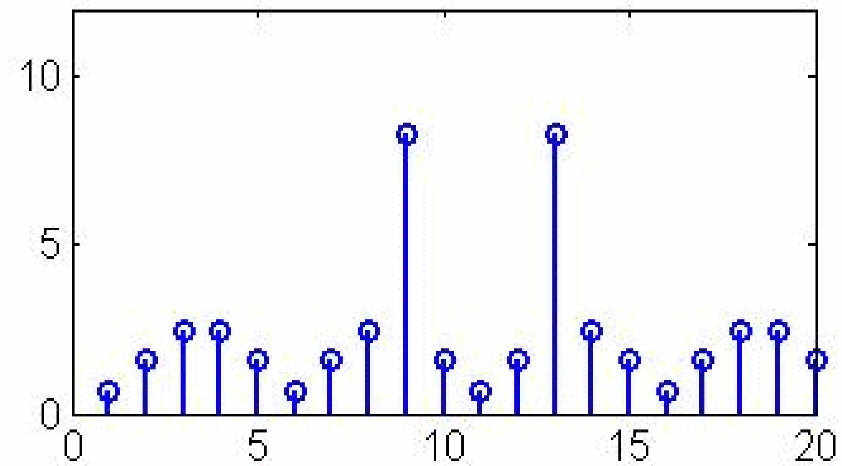


The reconstructed Fourier transform by using the CS values of the STFT

CS Applications

- **Simplified case: Direct search reconstruction of two missing samples (marked with red)**





Time domain

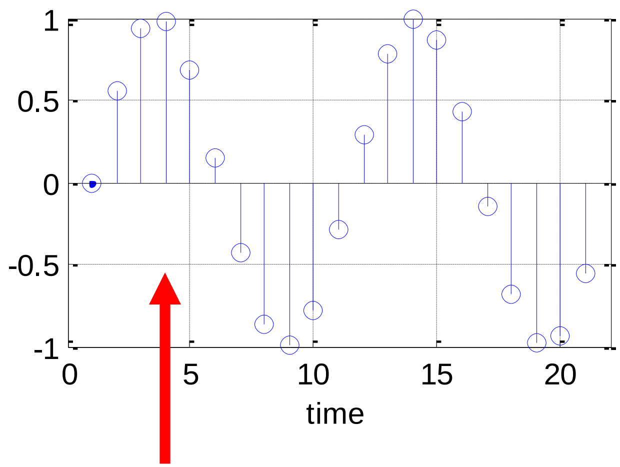
Frequency domain

If we have more missing samples, the direct search would be practically useless

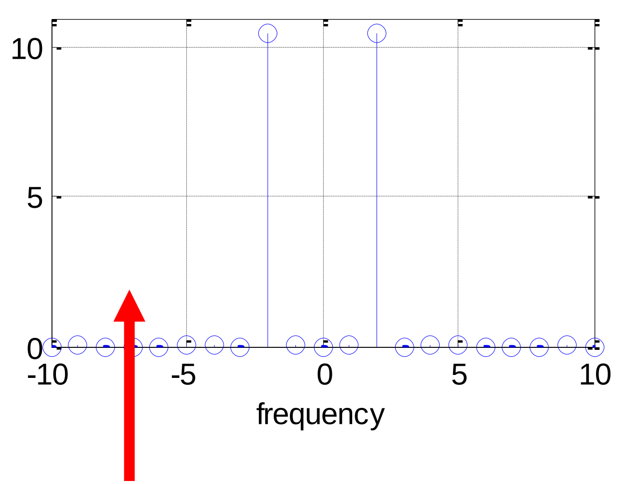
CS Applications-Example

- Let us consider a signal: $f_x(n) = \sin\left(2 \cdot \pi \cdot \left(2/N\right) \cdot n\right) \quad for \;\; n = 0,..,20$
- The signal is **sparse in DFT**, and vector of DFT values is:

$$\mathbf{F}_x = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 10.5i \;\; 0 \;\; 0 \;\; 0 \;\; -10.5i \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0];$$



Signal *fx*



DFT values of the signal *fx*

- **CS reconstruction using small set of samples:**

1. Consider the elements of inverse and direct Fourier transform matrices, denoted by Ψ and Ψ^{-1} respectively (relation $\mathbf{f}_x = \Psi \mathbf{F}_x$ holds)

$$\Psi = \frac{1}{21}\begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & e^{j\frac{2\pi}{21}} & e^{2j\frac{2\pi}{21}} & e^{3j\frac{2\pi}{21}} & \dots & e^{20j\frac{2\pi}{21}} \\ 1 & e^{2j\frac{2\pi}{21}} & e^{4j\frac{2\pi}{21}} & e^{6j\frac{2\pi}{21}} & \dots & e^{40j\frac{2\pi}{21}} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & e^{19j\frac{2\pi}{21}} & e^{38j\frac{2\pi}{21}} & e^{57j\frac{2\pi}{21}} & \dots & e^{380j\frac{2\pi}{21}} \\ 1 & e^{20j\frac{2\pi}{21}} & e^{40j\frac{2\pi}{21}} & e^{60j\frac{2\pi}{21}} & \dots & e^{400j\frac{2\pi}{21}} \end{bmatrix}$$

$$\Psi^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & e^{-j\frac{2\pi}{21}} & e^{-2j\frac{2\pi}{21}} & e^{-3j\frac{2\pi}{21}} & \dots & e^{-20j\frac{2\pi}{21}} \\ 1 & e^{-2j\frac{2\pi}{21}} & e^{-4j\frac{2\pi}{21}} & e^{-6j\frac{2\pi}{21}} & \dots & e^{-40j\frac{2\pi}{21}} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & e^{-19j\frac{2\pi}{21}} & e^{-38j\frac{2\pi}{21}} & e^{-57j\frac{2\pi}{21}} & \dots & e^{-380j\frac{2\pi}{21}} \\ 1 & e^{-20j\frac{2\pi}{21}} & e^{-40j\frac{2\pi}{21}} & e^{-60j\frac{2\pi}{21}} & \dots & e^{-400j\frac{2\pi}{21}} \end{bmatrix}$$

2. Take *M* random samples/measurements in the time domain
It can be modeled by using **matrix Φ:**

$$\mathbf{y} = \Phi \mathbf{f}_x$$

- Φ is defined as a **random permutation matrix**
- $\mathbf{y}$ is obtained by **taking *M* random** elements of $\mathbf{f}_x$

- Taking 8 random samples (out of 21) on the positions:

$$\begin{bmatrix} 5 & 9 & 10 & 12 & 13 & 15 & 18 & 20 \end{bmatrix}$$

$$\mathbf{y} = \mathbf{\Phi\Psi F}_x = \mathbf{AF}_x$$

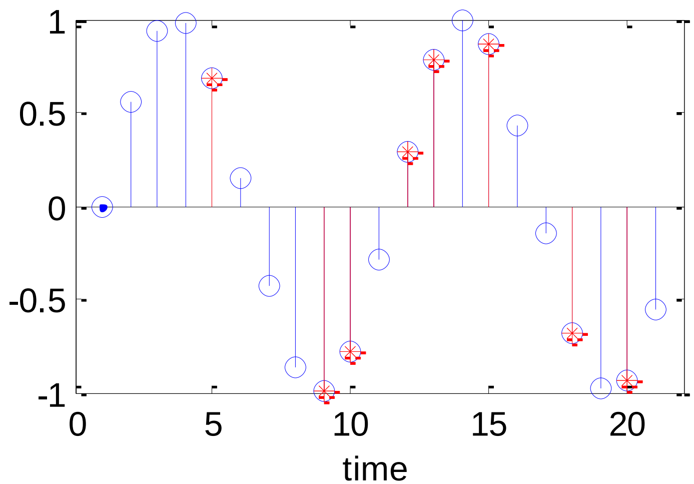
$$\mathbf{A} = \mathbf{\Phi\Psi}$$

obtained by using the **8** randomly chosen rows in $\mathbf{\Psi}$

$$\mathbf{A}_{MxN} = \mathbf{\Phi\Psi} = \frac{1}{21}\begin{bmatrix}
1 & e^{4j\frac{2\pi}{21}} & e^{8j\frac{2\pi}{21}} & \dots & e^{80j\frac{2\pi}{21}} \\
1 & e^{8j\frac{2\pi}{21}} & e^{16j\frac{2\pi}{21}} & \dots & e^{160j\frac{2\pi}{21}} \\
1 & e^{9j\frac{2\pi}{21}} & e^{18j\frac{2\pi}{21}} & \dots & e^{180j\frac{2\pi}{21}} \\
1 & e^{11j\frac{2\pi}{21}} & e^{22j\frac{2\pi}{21}} & \dots & e^{220j\frac{2\pi}{21}} \\
1 & e^{12j\frac{2\pi}{21}} & e^{24j\frac{2\pi}{21}} & \dots & e^{240j\frac{2\pi}{21}} \\
1 & e^{14j\frac{2\pi}{21}} & e^{28j\frac{2\pi}{21}} & \dots & e^{280j\frac{2\pi}{21}} \\
1 & e^{17j\frac{2\pi}{21}} & e^{34j\frac{2\pi}{21}} & \dots & e^{340j\frac{2\pi}{21}} \\
1 & e^{19j\frac{2\pi}{21}} & e^{38j\frac{2\pi}{21}} & \dots & e^{380j\frac{2\pi}{21}}
\end{bmatrix}$$

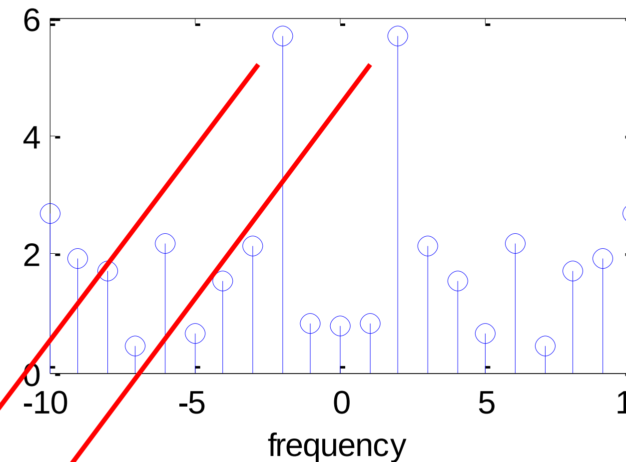
$\mathbf{A}_\Omega$

- The system with 8 equations and 21 unknowns is obtained



Blue dots – missing samples

Red dots – available samples



The initial Fourier transform

Components are on the positions -2 and 2 (center-shifted spectrum), which corresponds to 19 and 2 in nonshifted spectrum

$$\mathbf{A}_\Omega = \frac{1}{21}\begin{bmatrix} e^{4j\frac{2\pi}{21}} & e^{72j\frac{2\pi}{21}} \\ e^{8j\frac{2\pi}{21}} & e^{144j\frac{2\pi}{21}} \\ e^{9j\frac{2\pi}{21}} & e^{162j\frac{2\pi}{21}} \\ e^{11j\frac{2\pi}{21}} & e^{198j\frac{2\pi}{21}} \\ e^{12j\frac{2\pi}{21}} & e^{216j\frac{2\pi}{21}} \\ e^{14j\frac{2\pi}{21}} & e^{253j\frac{2\pi}{21}} \\ e^{17j\frac{2\pi}{21}} & e^{306j\frac{2\pi}{21}} \\ e^{19j\frac{2\pi}{21}} & e^{342j\frac{2\pi}{21}} \end{bmatrix}$$

$\mathbf{A}_\Omega$ is obtained by taking the 2nd and the 19th column of $\mathbf{A}$

$$\Omega=\{2,19\}$$

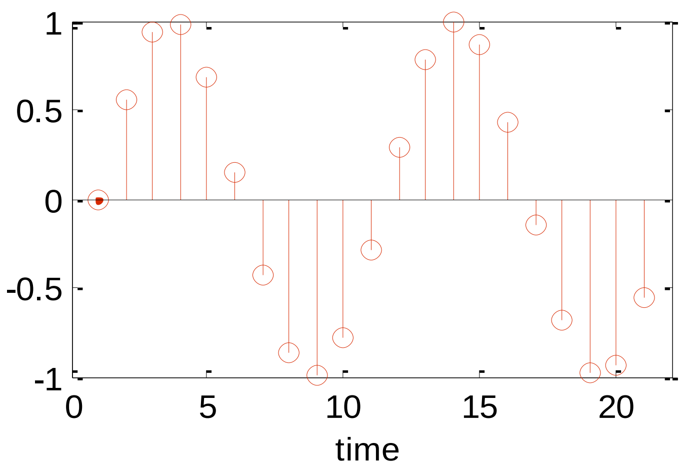
Least square solution

Problem formulation:

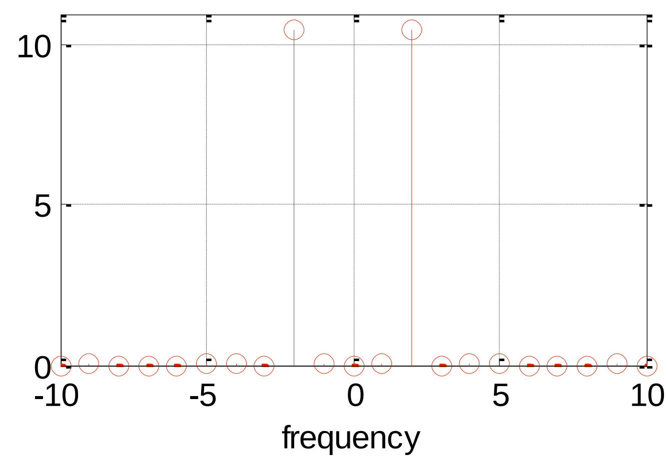
$$\mathbf{A}_\Omega \tilde{\mathbf{F}}_x = \mathbf{y} \quad \Rightarrow \quad \mathbf{A}_\Omega^T \mathbf{A}_\Omega \tilde{\mathbf{F}}_x = \mathbf{A}_\Omega^T \mathbf{y}$$

$$\tilde{\mathbf{F}}_x = (\mathbf{A}_\Omega^T \mathbf{A}_\Omega)^{-1} \mathbf{A}_\Omega^T \mathbf{y}$$

$$\mathbf{A}_\Omega = \frac{1}{21}\begin{bmatrix} e^{12j\frac{2\pi}{21}} & e^{216j\frac{2\pi}{21}} \\ e^{9j\frac{2\pi}{21}} & e^{162j\frac{2\pi}{21}} \\ e^{14j\frac{2\pi}{21}} & e^{252j\frac{2\pi}{21}} \\ e^{11j\frac{2\pi}{21}} & e^{198j\frac{2\pi}{21}} \\ e^{8j\frac{2\pi}{21}} & e^{144j\frac{2\pi}{21}} \\ e^{17j\frac{2\pi}{21}} & e^{306j\frac{2\pi}{21}} \\ e^{4j\frac{2\pi}{21}} & e^{72j\frac{2\pi}{21}} \\ e^{19j\frac{2\pi}{21}} & e^{342j\frac{2\pi}{21}} \end{bmatrix}$$

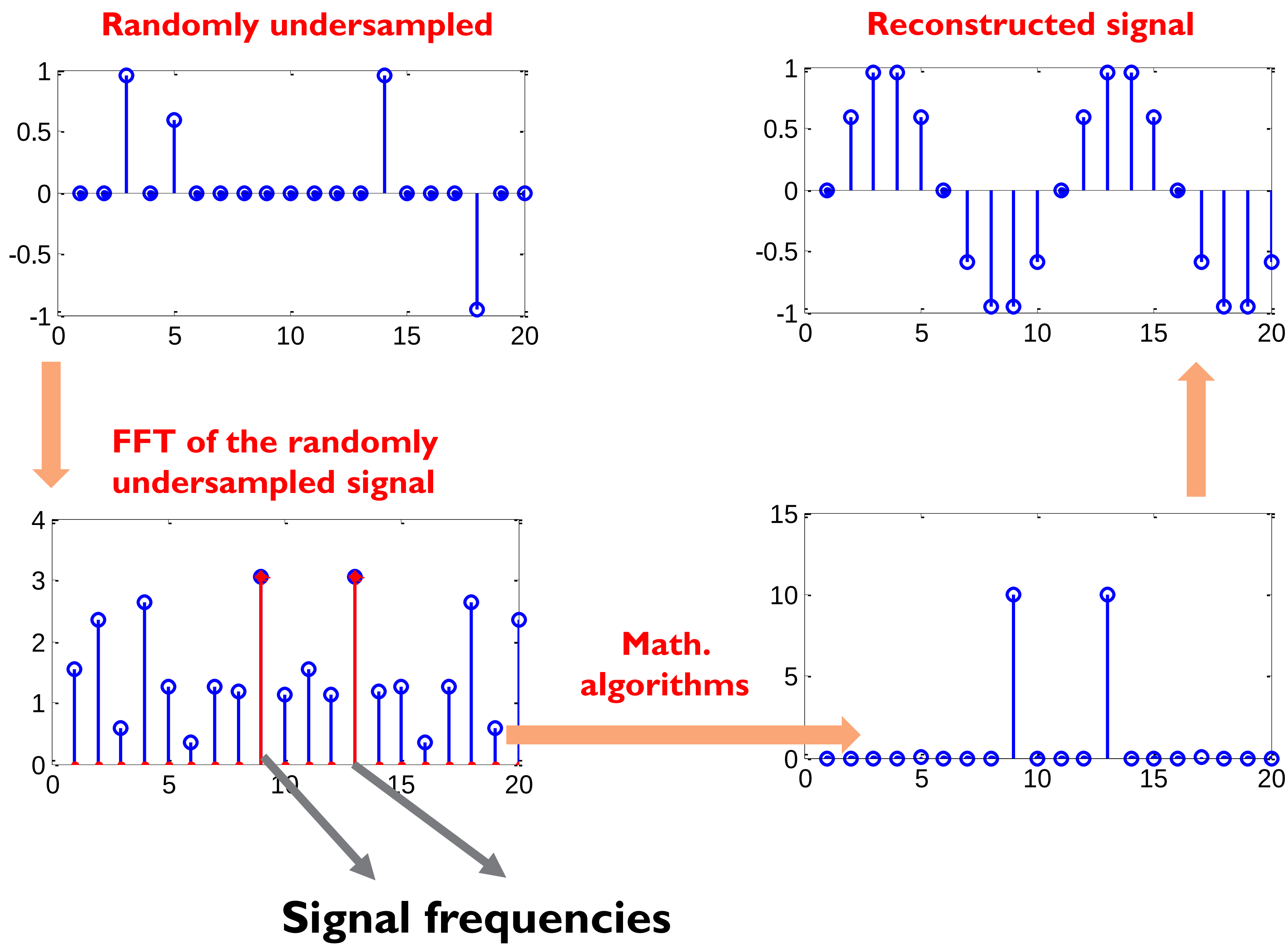


Reconstructed



Reconstructed

CS Applications



CS problem formulation

- The method of solving the undetermined system of equations , by searching for the sparsest solution can be described as:

$$\min \|\mathbf{x}\|_0 \ \ subject\ to\ \mathbf{y} = \mathbf{A}\mathbf{x}$$

$\|\mathbf{x}\|_0$ l_0 - norm

- We need to search over all possible sparse vectors **x** with *K* entries, where the subset of *K*-positions of entries are from the set {1,…,*N*}. The total number of possible *K*-position subsets is

$$\binom{N}{K}$$

CS problem formulation

- A more efficient approach uses the near optimal solution based on the l1-norm, defined as:

$$\min \|\mathbf{x}\|_1 \ \ subject\ to\ \mathbf{y} = \mathbf{A}\mathbf{x}$$

- In real applications, we deal with noisy signals.
- Thus, the previous relation should be modified to include the influence of noise:

$$\min \|\mathbf{x}\|_1 \ \ subject\ to\ \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \varepsilon$$

$$\|\mathbf{e}\|_2 = \varepsilon$$

L2-norm cannot be used because the minimization problem solution in this case is reduced to minimum energy solution, which means that all missing samples are zeros

CS conditions

- CS relies on the following conditions:

Sparsity – related to the signal nature;

- Signal needs to have concise representation when expressed in a proper basis ($K<<N$)

Incoherence – related to the sensing modality; It should provide a linearly independent measurements (matrix rows)
Random undersampling is crucial

Restriced **I**sometry **P**roperty – is important for preserving signal isometry by selecting an appropriate transformation

Summary of CS problem formulation

Signal $\mathbf{f}$ linear combination of the orthonormal basis vectors

$$f(t) = \sum_{i=1}^{N} x_i \psi_i(t), \quad or: \quad \mathbf{f} = \mathbf{\Psi x}.$$

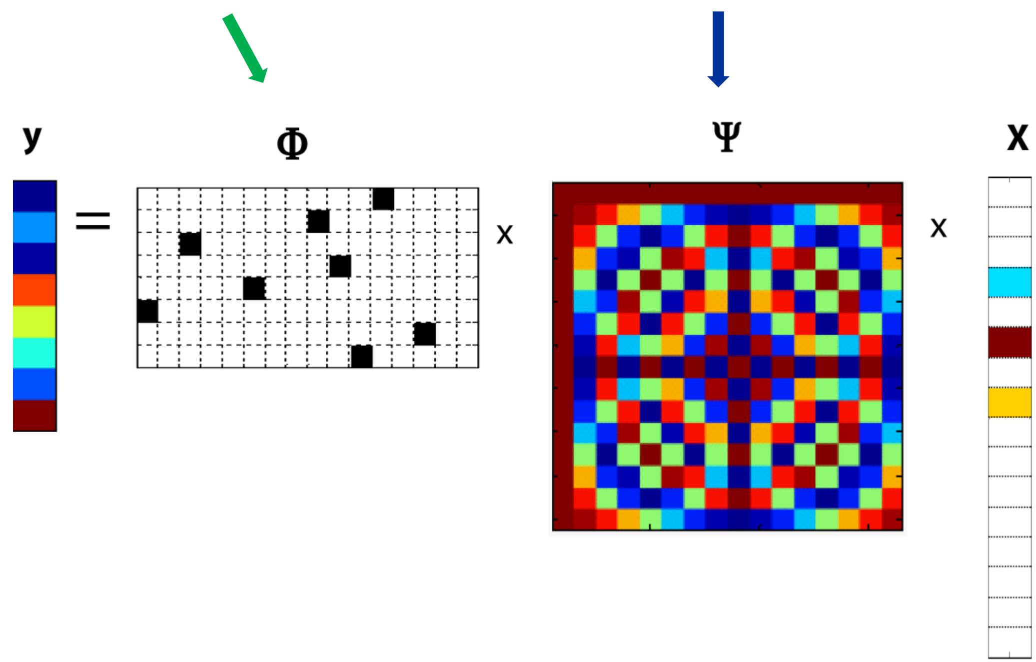
Set of random measurements:

$$\mathbf{y} = \mathbf{\Phi f}$$

random measurement matrix

transform matrix

transform domain vector



CS conditions

- Restricted isometry property
  - Successful reconstruction for a **wider range of sparsity** level
  - Matrix **A** satisfies **Isometry Property** if it preserves the vector intensity in the *N*-dimensional space:

$$\|\mathbf{Ax}\|_2^2 = \|\mathbf{x}\|_2^2$$

  - If **A** is a full Fourier transform matrix, i.e. :

$$\mathbf{A} = N\mathbf{\Psi}$$

$$\frac{N\|\mathbf{\Psi x}\|_2^2 - \|\mathbf{x}\|_2^2}{\|\mathbf{x}\|_2^2} = 1$$

CS conditions

- RIP

- For each integer number *K* the isometry constant δ_K of the matrix **A** is the smallest number for which the relation holds:

$$(1-\delta_K)\|\mathbf{x}\|_2^2 \le \|\mathbf{A}\mathbf{x}\|_2^2 \le (1+\delta_K)\|\mathbf{x}\|_2^2$$

$$\left|\frac{\|\mathbf{A}\mathbf{x}\|_2^2 - \|\mathbf{x}\|_2^2}{\|\mathbf{x}\|_2^2}\right| \le \delta_K$$

$0 < \delta_K < 1$ - **restricted isometry constant**

CS conditions

- Matrix **A** satisfies RIP → the **Euclidian length** of sparse vectors is preserved

- For the RIP matrix **A** with $(2K, \delta_K)$ and $\delta_K < 1$, all subsets of $2K$ columns are **linearly independent**

↓

$$spark(\mathbf{A}) > 2K$$

spark - the smallest number of dependent columns

CS conditions

A (MXN) →

$$2 \leq spark(\mathbf{A}) \leq M+1$$

$spark(\mathbf{A})=1$ - one of the columns has all zero values

$spark(\mathbf{A})=M+1$ - no dependent columns

↓

$$K < \frac{1}{2} spark(\mathbf{A}) \leq \frac{1}{2}(M+1)$$

the number of measurements should be at least twice the number of components *K*:

$$M \geq 2K$$

Incoherence

- Signals sparse in the transform domain Ψ, should be dense in the domain where the acquisition is performed

- Number of nonzero samples in the transform domain Ψ and the number of measurements (required to reconstruct the signal) depends on the **coherence** between the matrices Ψ and $\mathbf{\Phi}$.

- Ψ and $\mathbf{\Phi}$ are **maximally coherent** - **all** coefficients would be required for signal reconstruction

Mutual coherence: the maximal absolute value of correlation between two elements from Ψ and $\mathbf{\Phi}$ →

$$\mu(\Phi,\Psi)=\max_{i\neq j}\left|\frac{\langle \phi_i,\psi_j\rangle}{\|\phi_i\|^2\,\|\psi_j\|^2}\right|$$

Incoherence

Mutual coherence:

$$\mu(\mathbf{A}) = \max_{i \neq j, 1 \leq i, j \leq M} \left| \frac{\langle A_i, A_j \rangle}{\|A_i\|^2 \|A_j\|^2} \right|, \quad \mathbf{A} = \mathbf{\Phi\Psi}$$

maximum absolute value of normalized inner product between all columns in A

$\boldsymbol{A_i}$ and $\boldsymbol{A_j}$ - columns of matrix **A**

- The maximal mutual coherence will have the value **1** in the case when certain pair of columns coincides
- If the number of measurements is: $M \geq C \cdot K \cdot \mu(\mathbf{\Phi}, \mathbf{\Psi}) \cdot \log N$

then the sparsest solution is exact with a high probability (*C* is a constant)

Reconstruction approaches

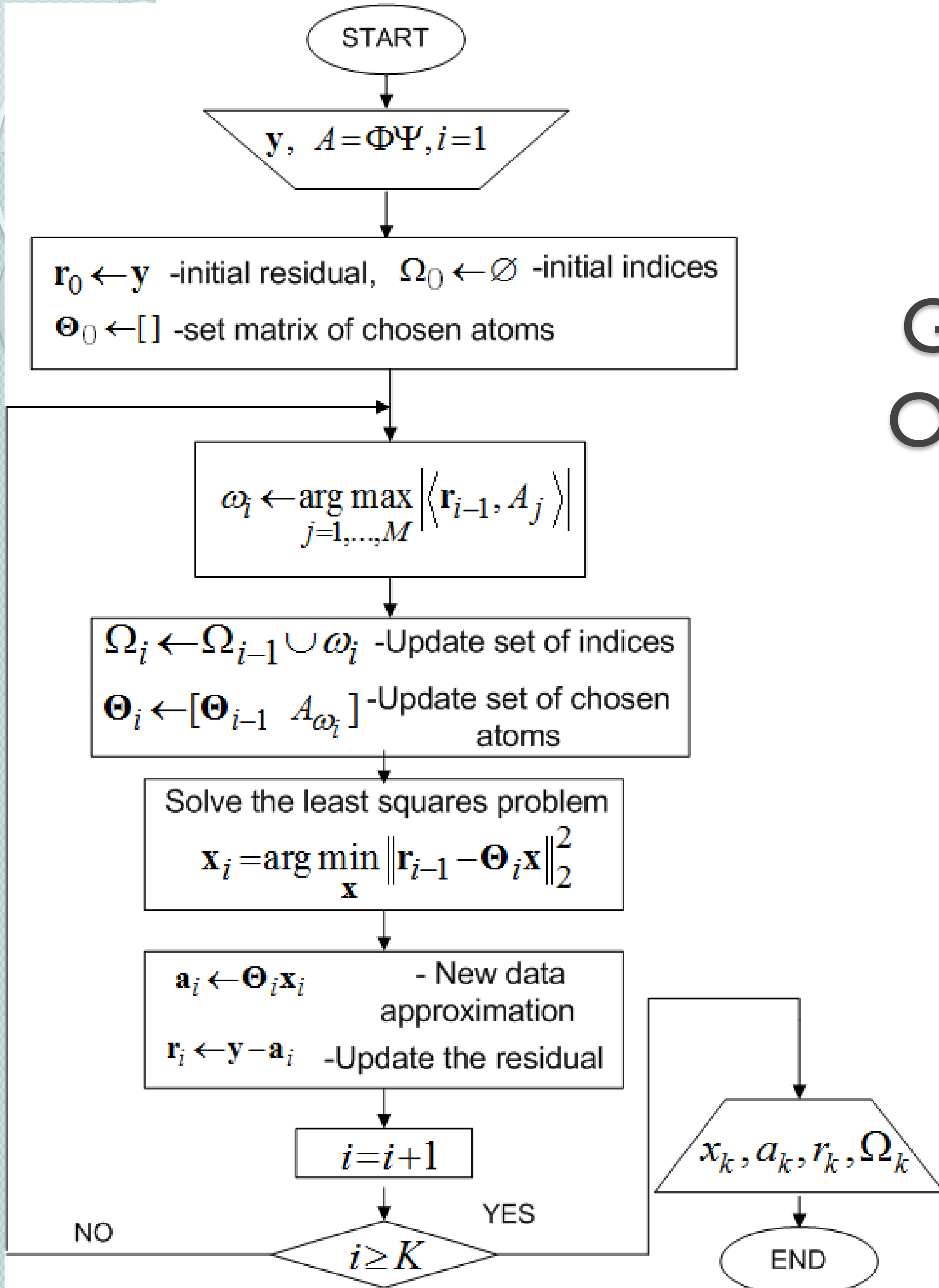
- The main challenge of the CS reconstruction: solving an **underdetermined system of linear equations** using sparsity assumption

ℓ_1 - optimization, based on linear programming methods, provide efficient signal reconstruction with high accuracy

- Linear programming techniques (e.g. convex optimization) may require a **vast number of iterations** in practical applications

- **Greedy and threshold based algorithms** are fast solutions, but in general less stable

Greedy algorithms – Orthogonal Matching Pursuit (OMP)



$\Psi - Transform\ \ matrix$

$\Phi - Measurement\ \ matrix$

$y = \Phi f - Measurement\ \ vector$

Influence of missing samples to the spectral representation

- **Missing samples produce noise in the spectral domain. The variance of noise in the DFT case depends on M, N and amplitudes *Ai*:**

$$\sigma^2_{MS} = \mathrm{var}\{F_{k \neq k_i}\} = M\,\frac{N-M}{N-1}\sum_{i=1}^{K} A_i^2$$

- **The probability that all (*N-K*) non-signal components are below a certain threshold value defined by T is (only K signal components are above T):**

$$P(T) = \left(1 - \exp\left(-\frac{T^2}{\sigma_{MS}^2}\right)\right)^{N-K}$$

Consequently, for a fixed value of *P*(*T*) (e.g. *P*(*T*)=0.99), threshold is calculated as:

$$T = \sqrt{-\sigma_{MS}^2 \log\left(1 - P(T)^{\frac{1}{N-K}}\right)} \approx \sqrt{-\sigma_{MS}^2 \log\left(1 - P(T)^{\frac{1}{N}}\right)}$$

When ALL signal components are above the noise level in DFT, the reconstruction is done using a Single-Iteration Reconstruction algorithm using threshold T

Optimal number of available samples M

- **How can we determine the number of available samples *M*, which will ensure detection of all signal components?**
- Assuming that the DFT of the *i*-th signal component (with the lowest amplitude) is equal to Ma_i, then the approximate expression for the probability of error is obtained as:

$$P_{err} = 1 - P_i \cong 1 - \left(1 - \exp\left(-\frac{M^2 a_i^2}{\sigma_{MS}^2}\right)\right)^{N-K}$$

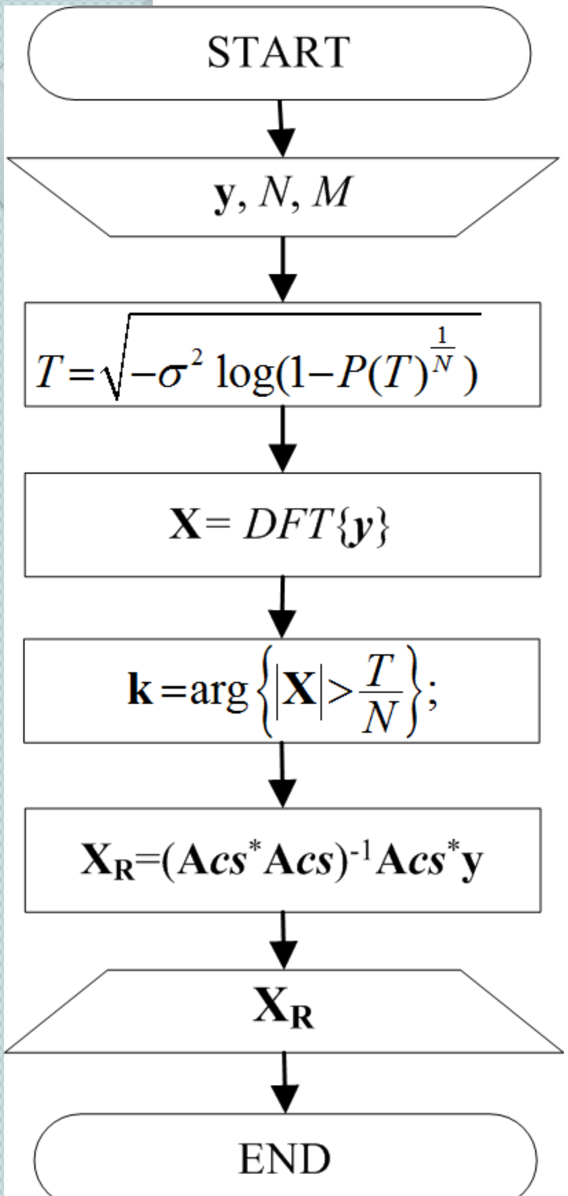
- For a fixed Perr, the optimal value of M (that allows to detect all signal components) can be obtained as a solution of the minimization problem:

$$M_{opt} \geq \arg\min_{M}\{P_{err}\}$$

For chosen value of P_{err} and expected value of minimal amplitude a_i, there is an optimal value of M that will assure components detection.

Algorithms for CS reconstruction of sparse signals

Single-Iteration Reconstruction Algorithm in DFT domain



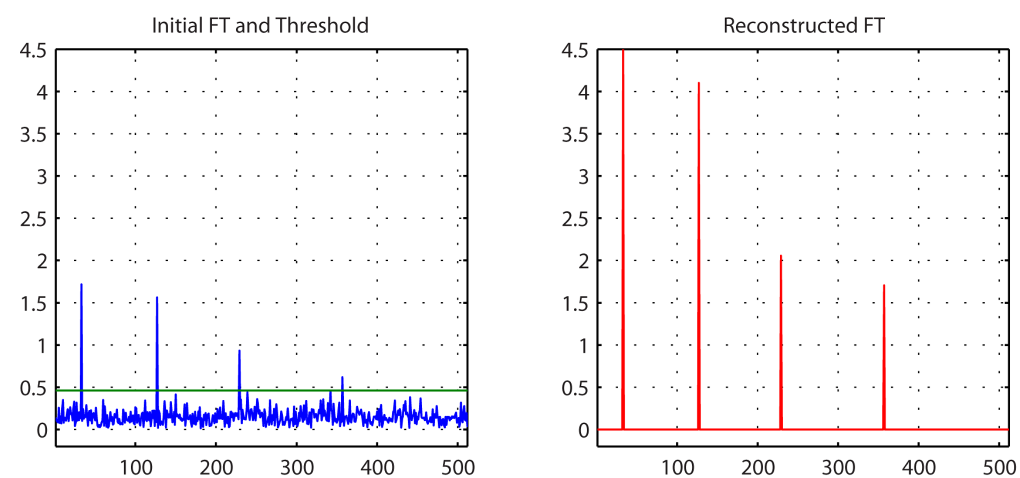
y – measurements

M - number of measurements

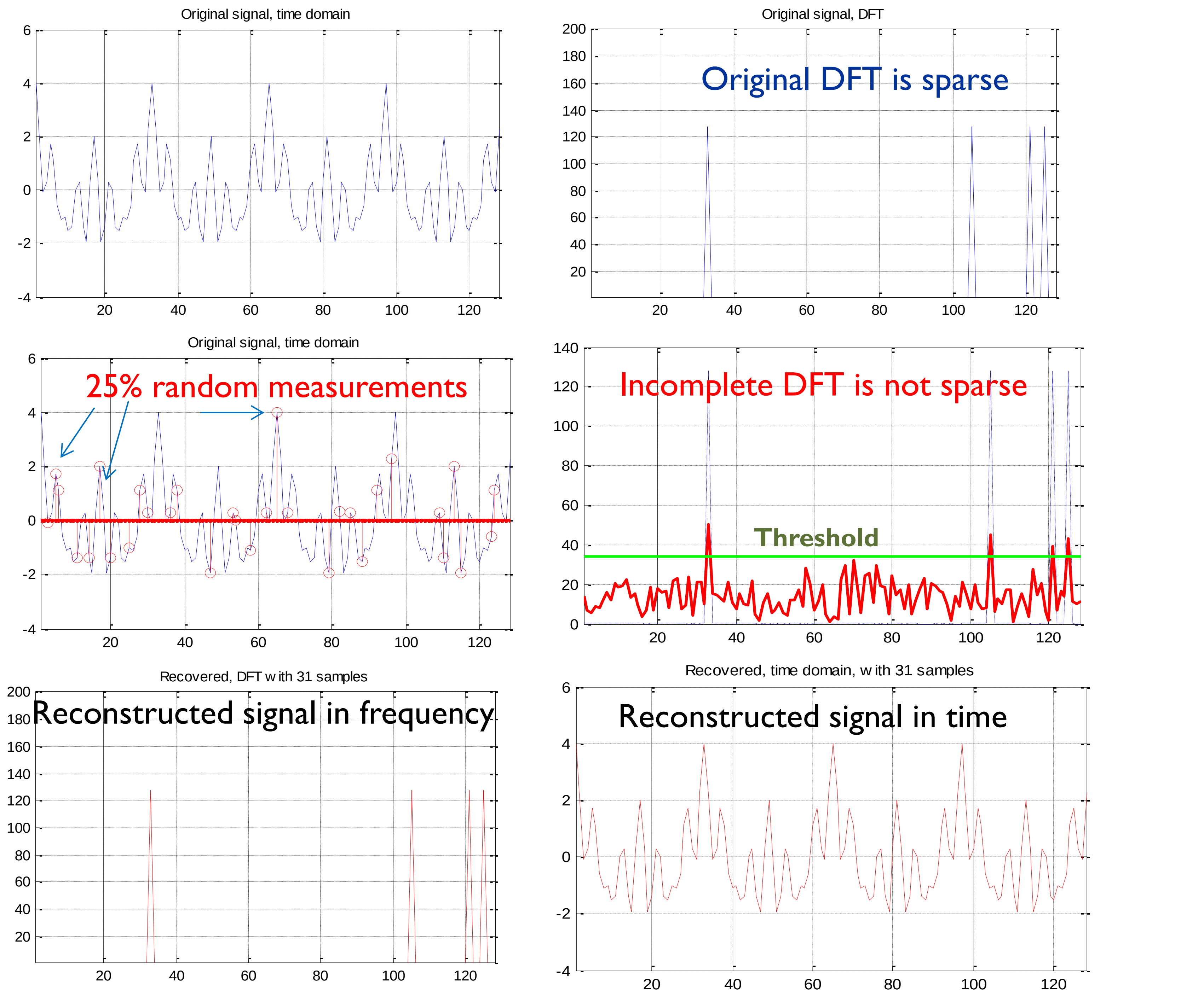
N – signal length

T - Threshold

- **DFT domain is assumed as sparsity domain**
- **Apply threshold to initial DFT components (determine the frequency support)**
- **Perform reconstruction using identified support**



Example 1: Single iteration

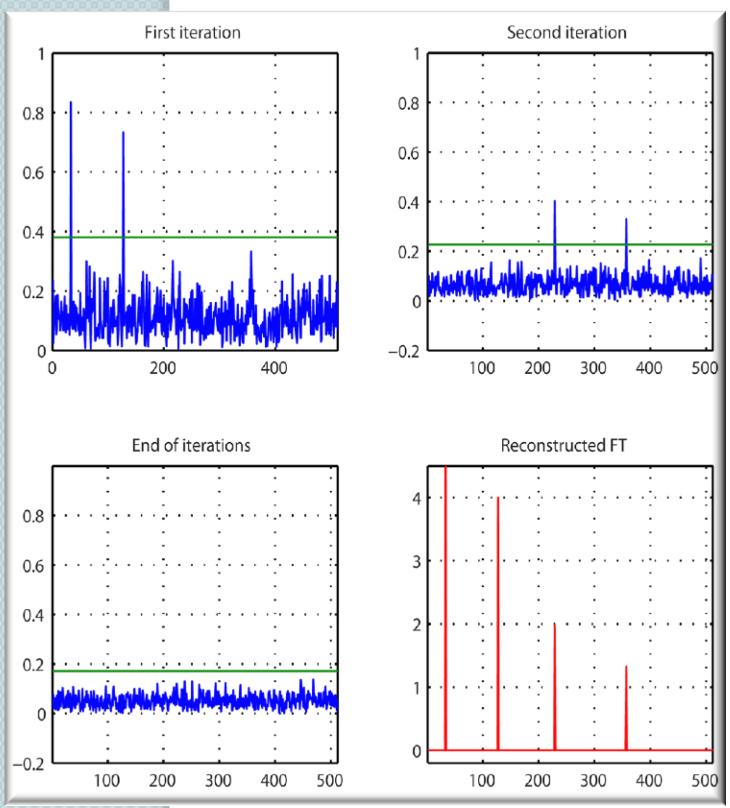


Case 3: External noise

- **External noise + noise caused by missing samples**

$$\sigma^2 = \sigma_{MS}^2 + M\sigma_N^2 = M\frac{N-M}{N-1}\sum_{i=1}^{K} A_i^2 + M\sigma_N^2$$

$$T = \sqrt{-\sigma^2 \log(1 - P(T)^{\frac{1}{N}})}$$



- **To ensure the same probability of error as in the noiseless case we need to increase the number of measurements M such that:**

$$\frac{\sigma_{MS}^2}{\sigma^2} = \frac{M\frac{N-M}{N-1}(A_1^2 + A_2^2 + \ldots + A_K^2)}{M_N\frac{N-M_N}{N-1}(A_1^2 + A_2^2 + \ldots + A_K^2) + M_N\sigma_N^2} = 1$$

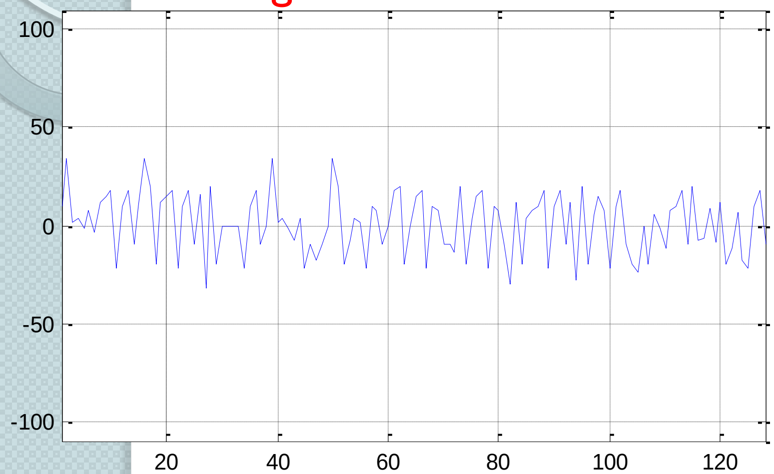
$$\frac{M(N-M)}{M_N}\frac{SNR}{SNR(N-M_N) + (N-1)} = 1$$

Solve the equation:

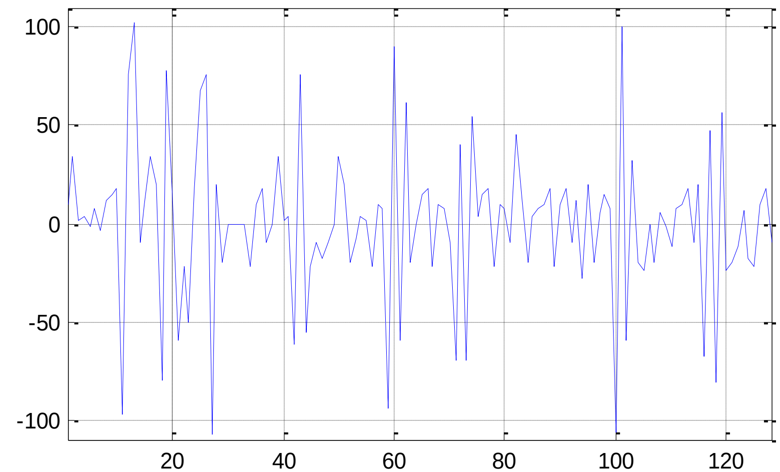
$$M_N^2 \cdot SNR - M_N(SNR \cdot N + N - 1) + SNR \cdot (MN - M^2) = 0$$

Dealing with a set of noisy data – L-estimation approach

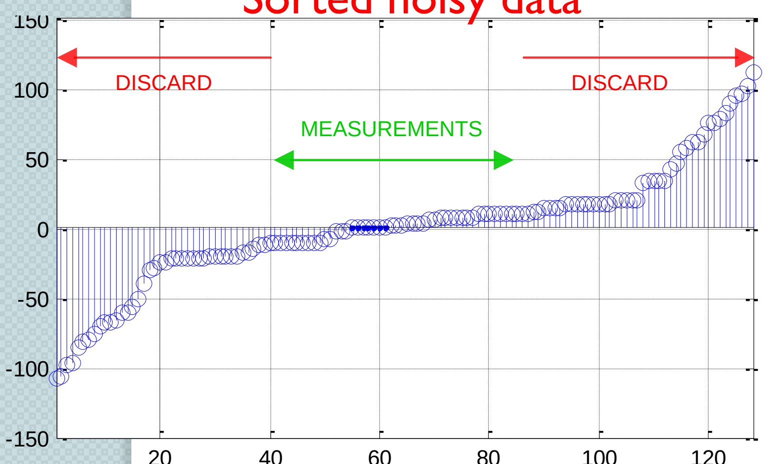
Original data- DESIRED



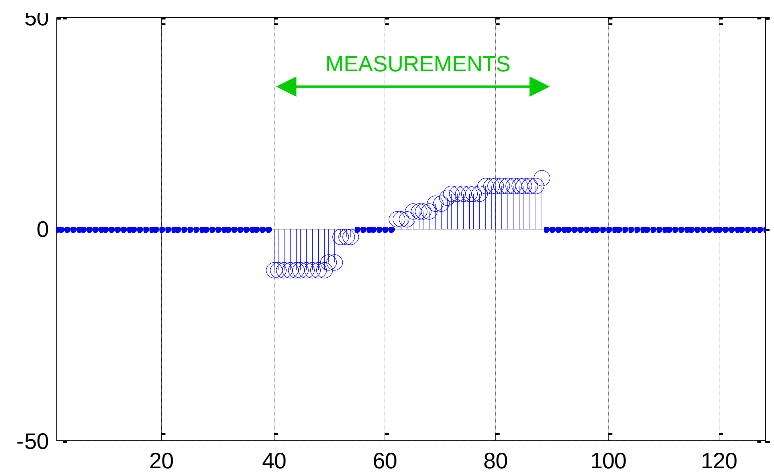
Noisy data - AVAILABLE



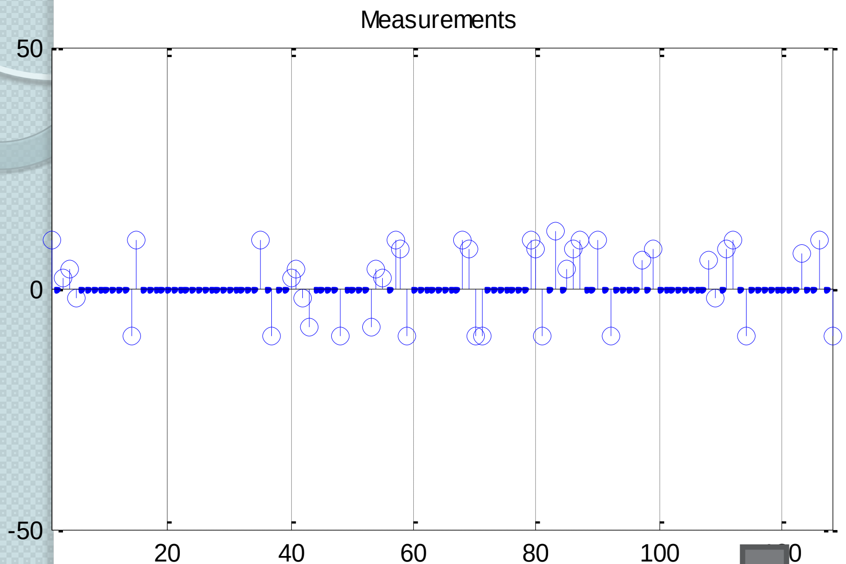
Sorted noisy data



Denoised data

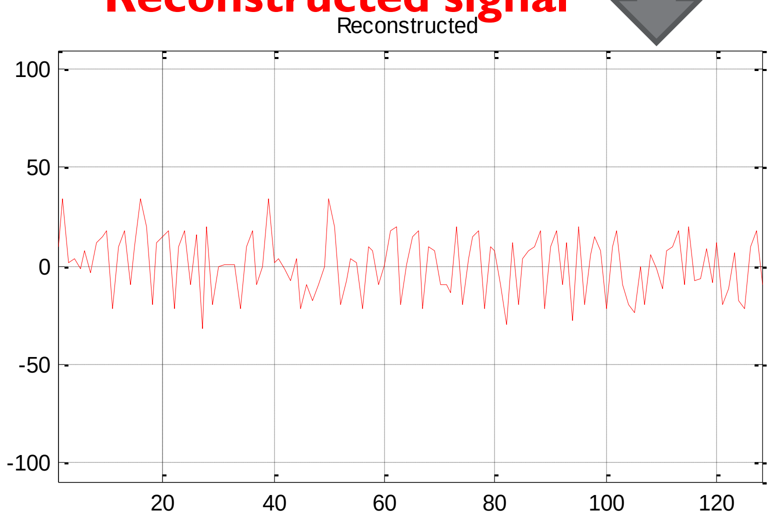


Dealing with a set of noisy data – L-estimation approach

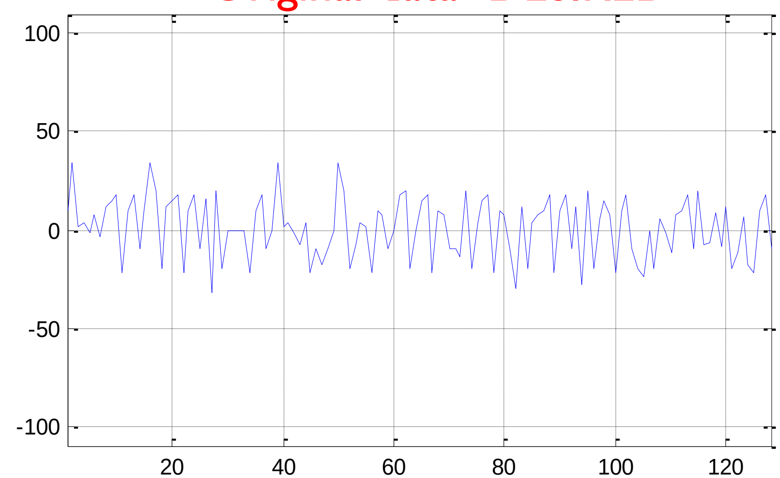


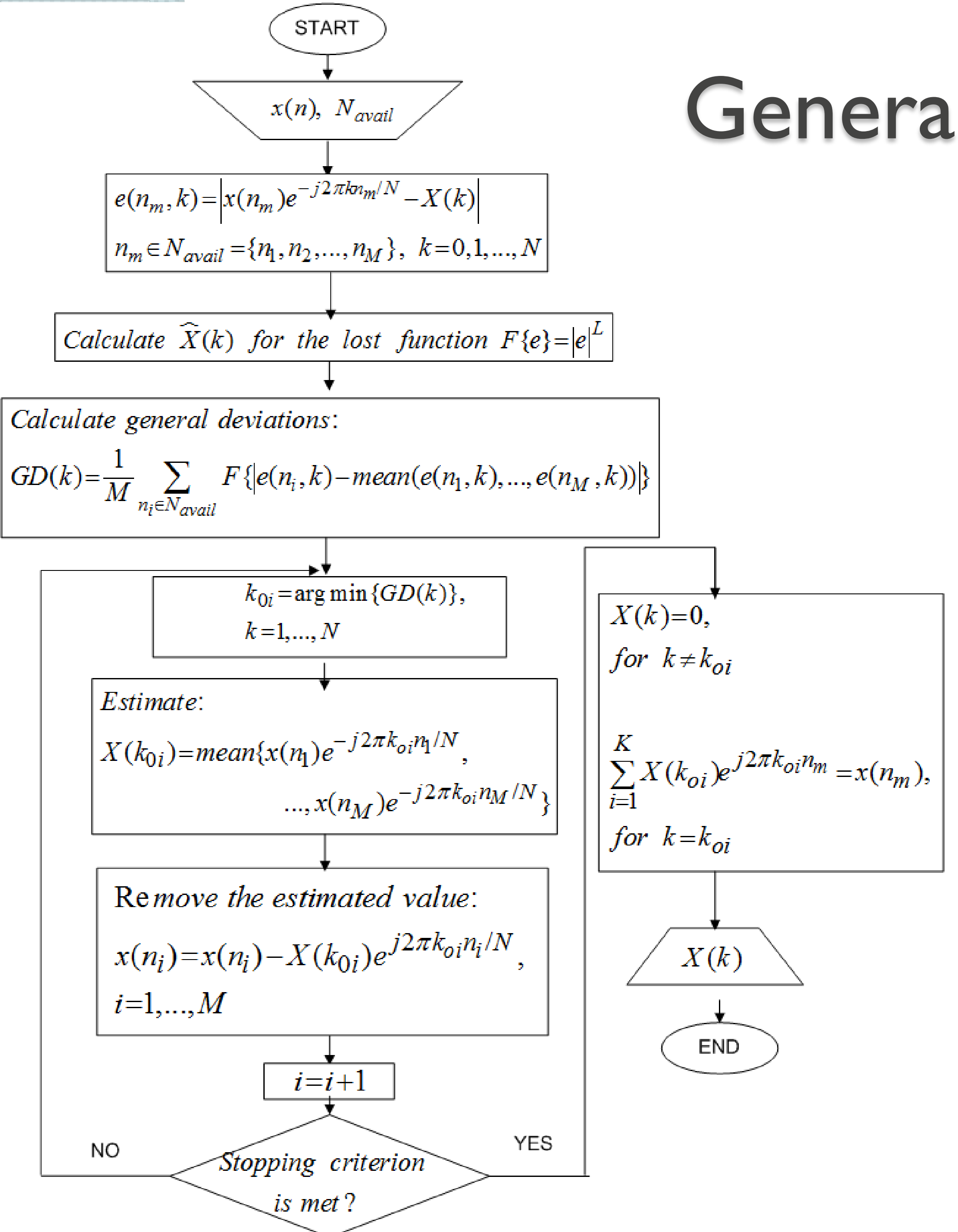
We end up with a random incomplete set of samples that need to be recovered

Reconstructed signal



Original data- DESIRED





General deviations-based approach

$x(n)$ -compressive sampled signal, K-sparse in DFT domain

-**Navail** – positions of the available samples

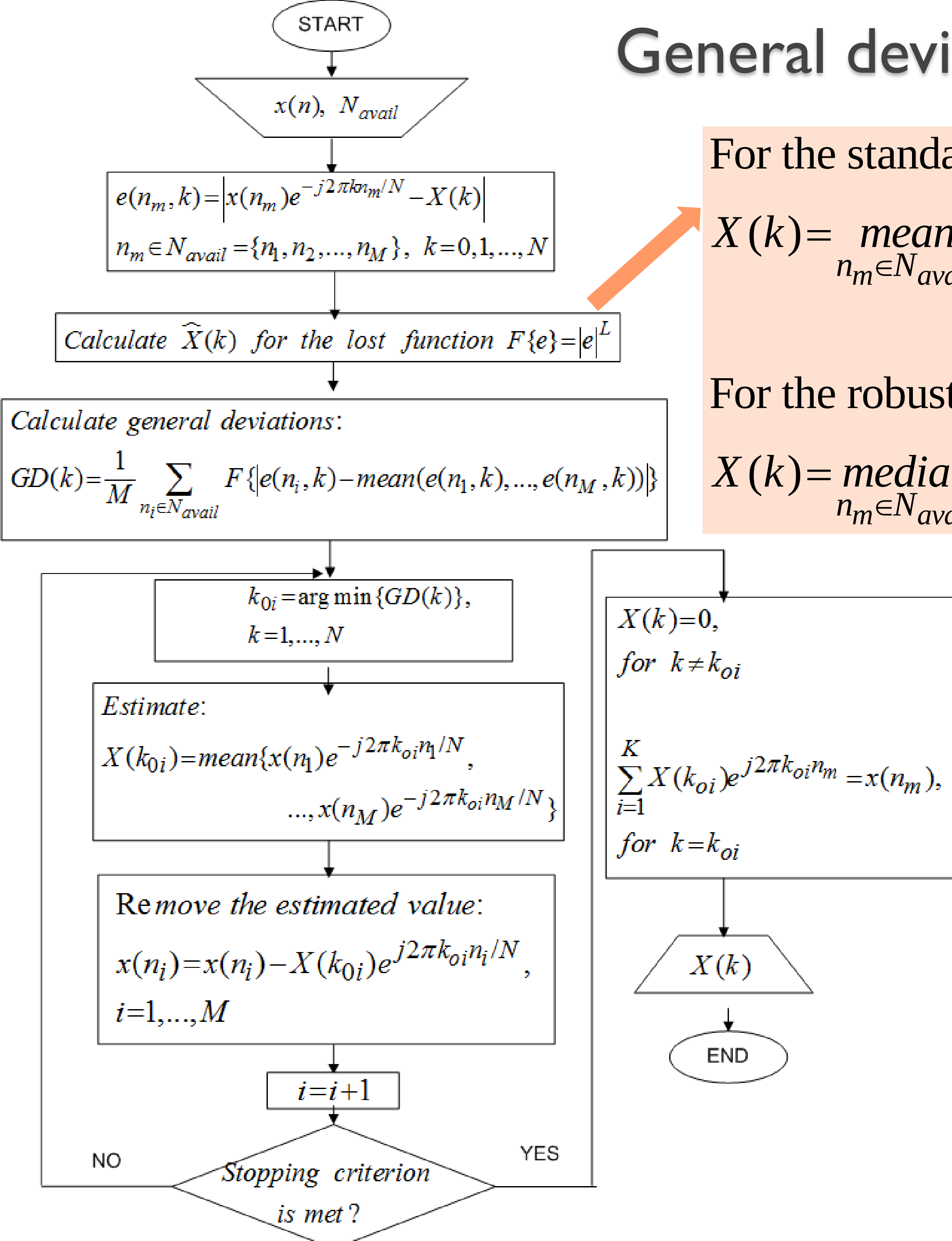
- **M**-number of available samples

$$F\{e(n)\}=F\{\left|x(n)e^{-j2\pi kn/N}-X(k)\right|\}$$

Loss function

$$F\{e(n)\}=\begin{cases}|e|^2 - \text{standard form}\\ |e| - \text{robust form}\end{cases}$$

General deviations-based approach



For the standard form, FT of the signal $x(n)$ is:

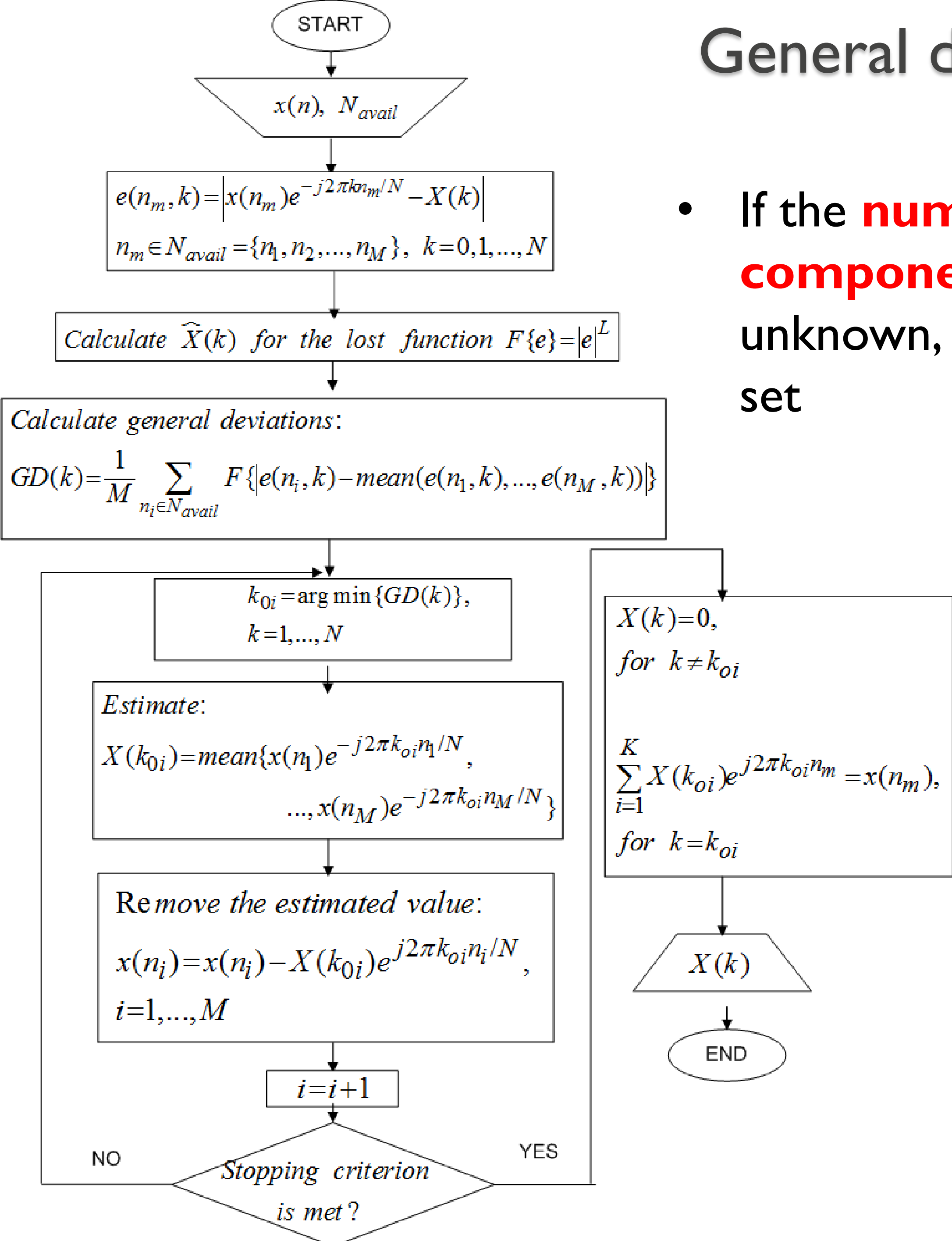
$$X(k)=\underset{n_m\in N_{avail}}{mean}\{x(n_1)e^{-j2\pi kn_1/N},...,x(n_M)e^{-j2\pi kn_M/N}\}$$

For the robust form, FT of the signal $x(n)$ is:

$$X(k)=\underset{n_m\in N_{avail}}{median}\{x(n_1)e^{-j2\pi kn_1/N},...,x(n_M)e^{-j2\pi kn_M/N}\}$$

- The **number of components** and the **number of iterations** might be known from the nature of physical processes appearing in real application

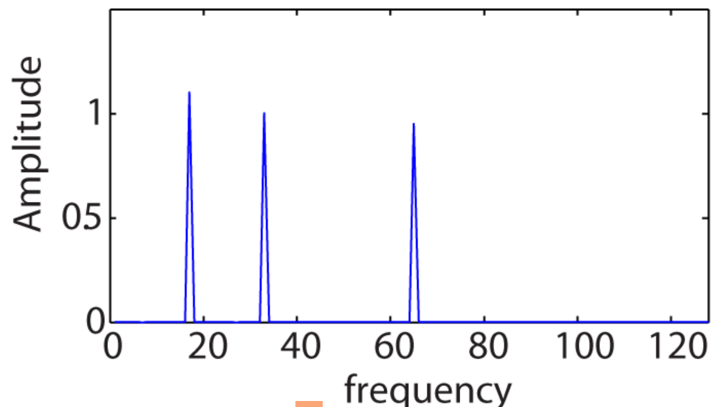
General deviations-based approach

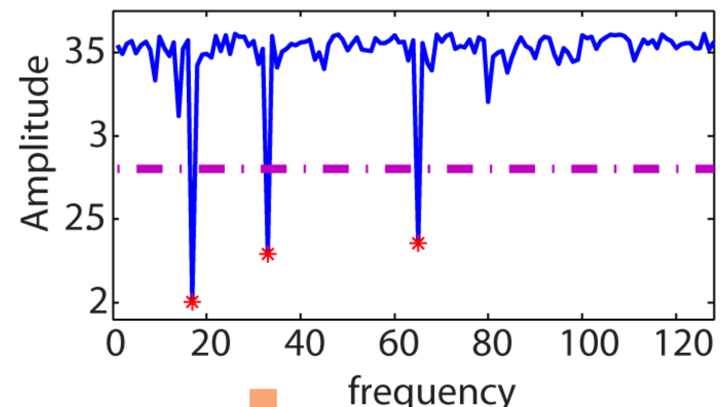


- If the **number of components/number of iterations** is unknown, the **stopping criterion** can be set

- **Stopping criterion:** Adjusted based on the l2-norm bounded residue that remains after removing previously detected components

General deviations-based approach
Example

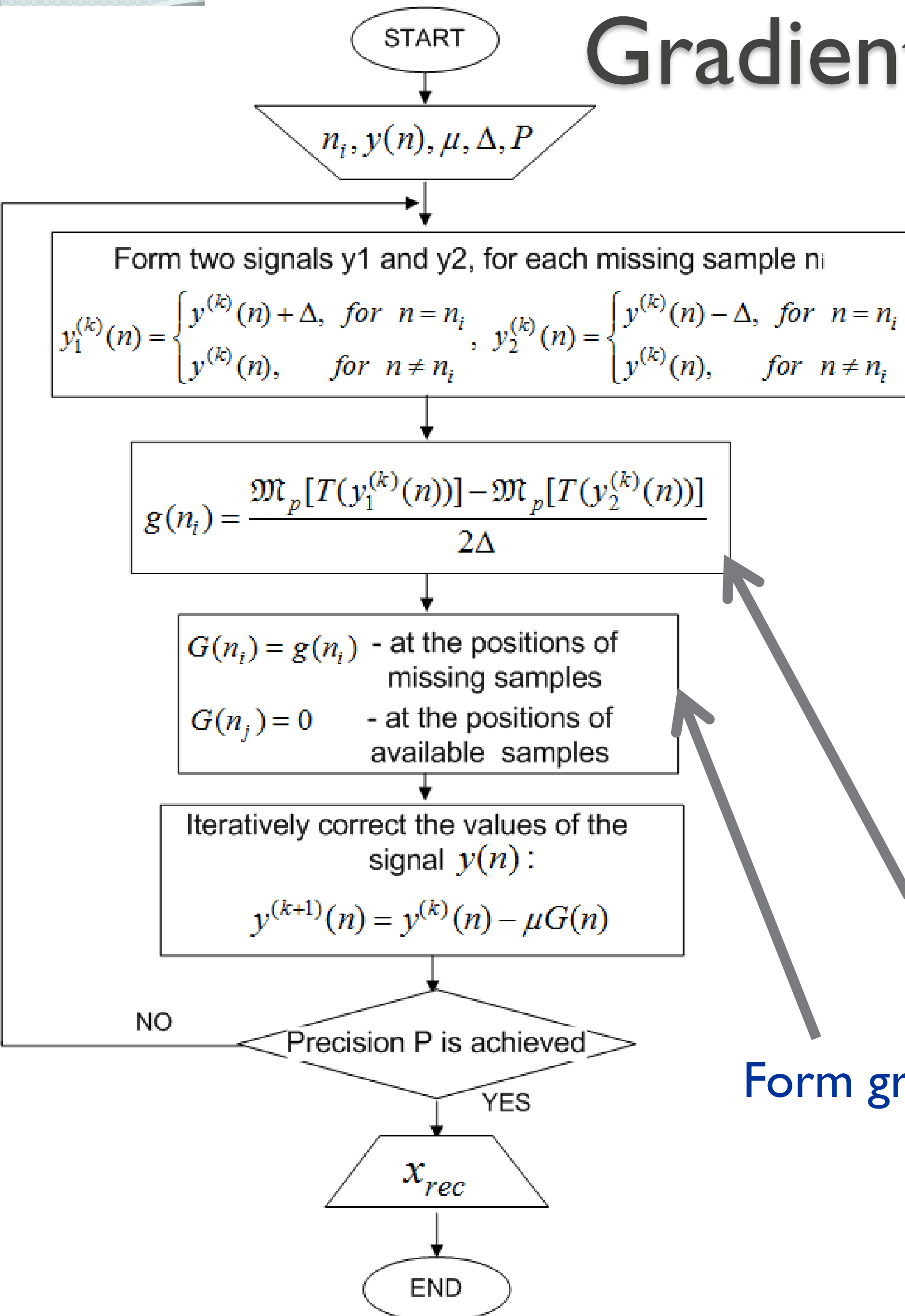




Fourier transform of original signal

Variances at signal (marked by red symbols) and non-signal positions

Gradient algorithm



n_i - missing samples positions

n_j - Available samples positions

$y(n)$ - Available signal samples

Δ - Constant; determines whether sample should be decreased or increased

μ - Constant that affect algorithm performance

P - Precision

Estimate the differential of the signal transform measure

Form gradient vector G

$$\mathfrak{M}_p[T(x(n))] = \frac{1}{N}\sum_k |X(k)|^{1/p},$$

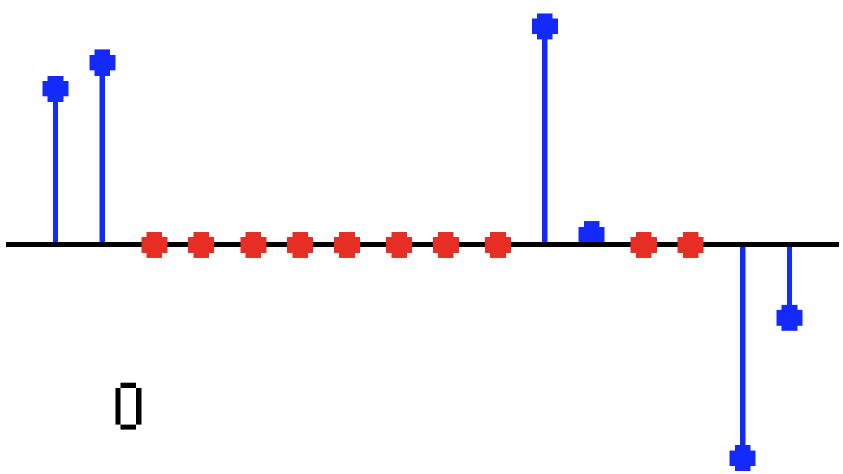
$$1 \leq p < \infty$$

Gradient algorithm - Example

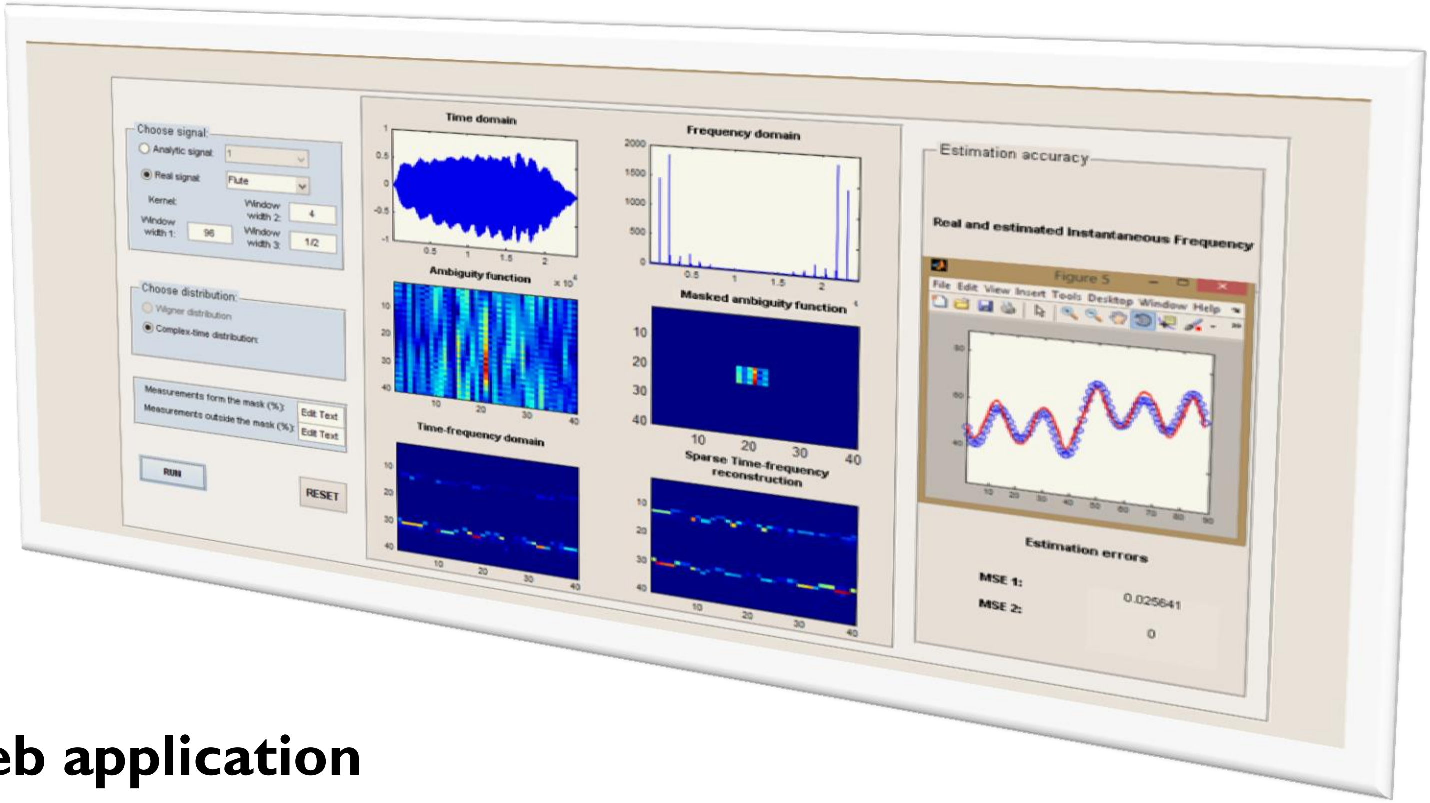
Signal contains 16 samples

Missing – 10 samples (marked with red)

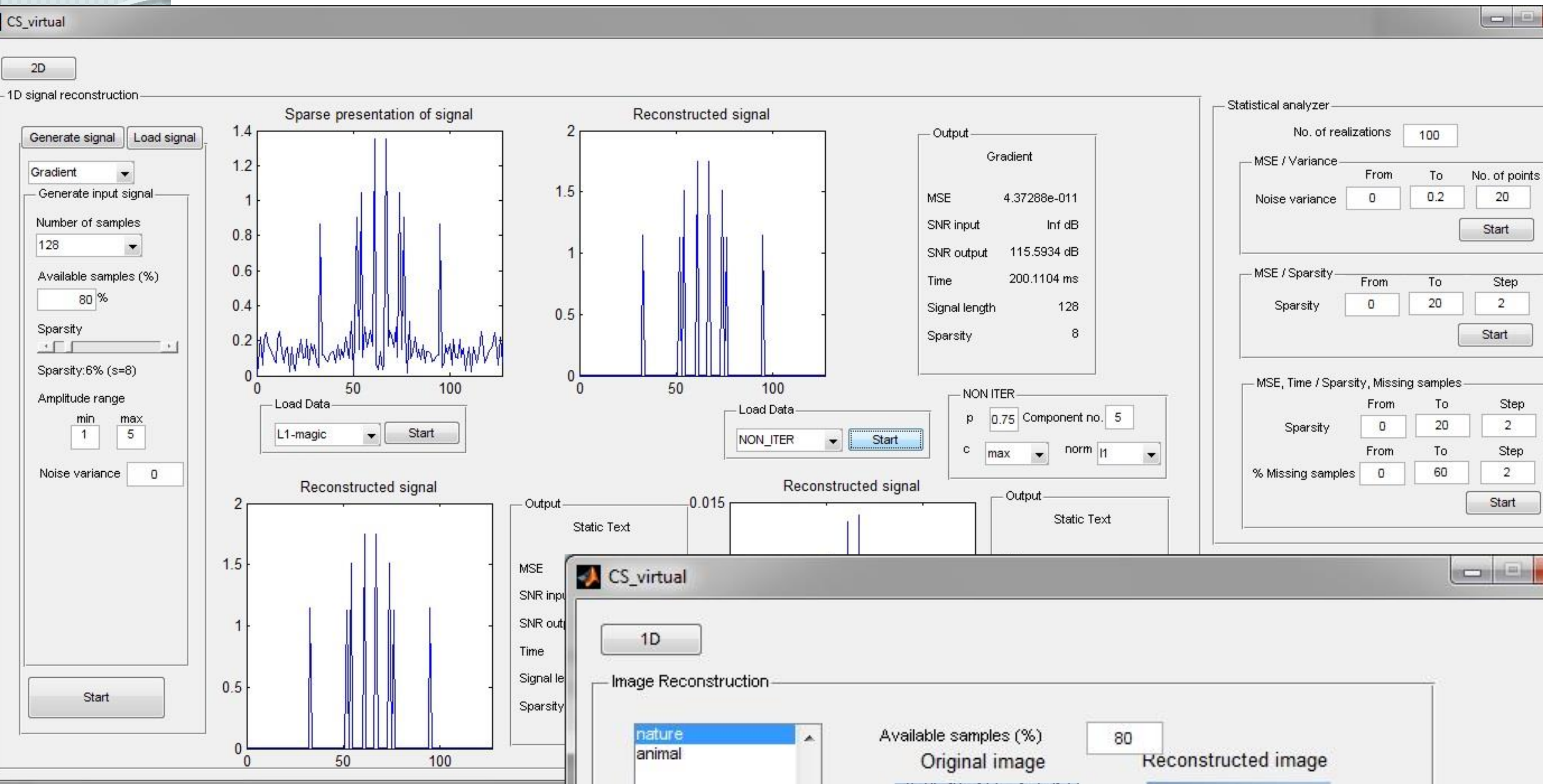
Signal is iteratively reconstructed using **Gradient algorithm**

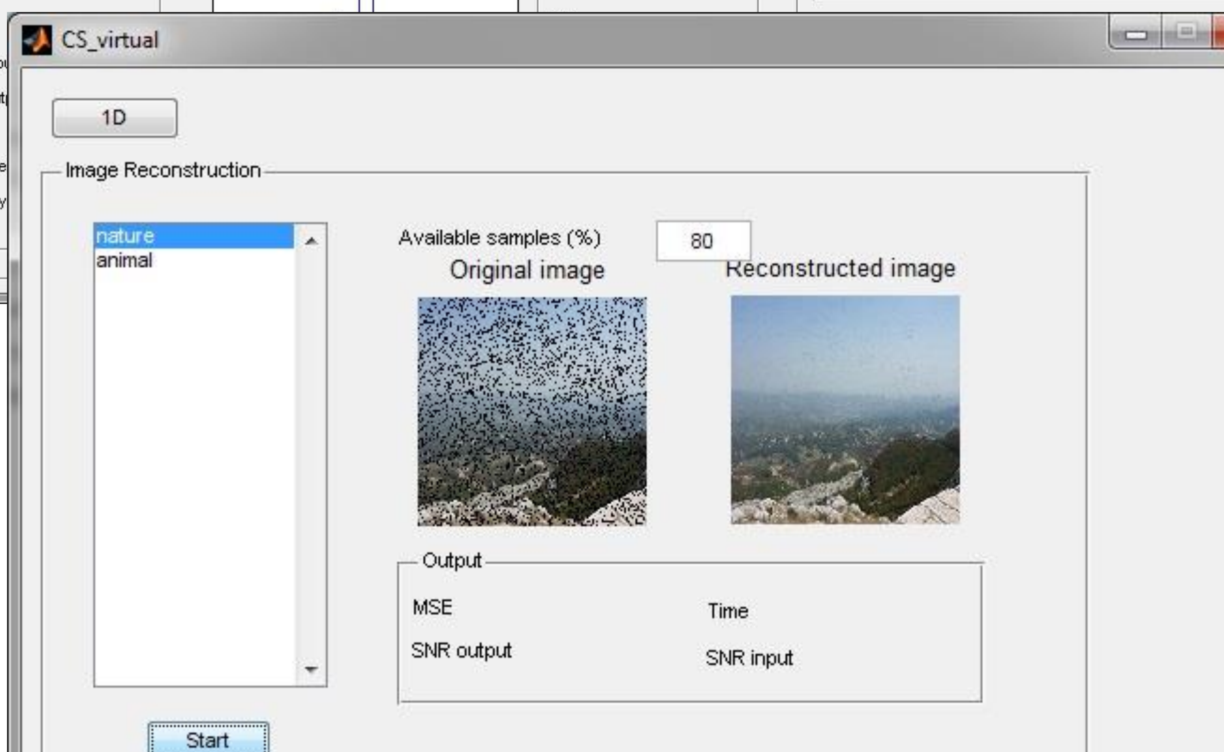


Some Developments



Web application

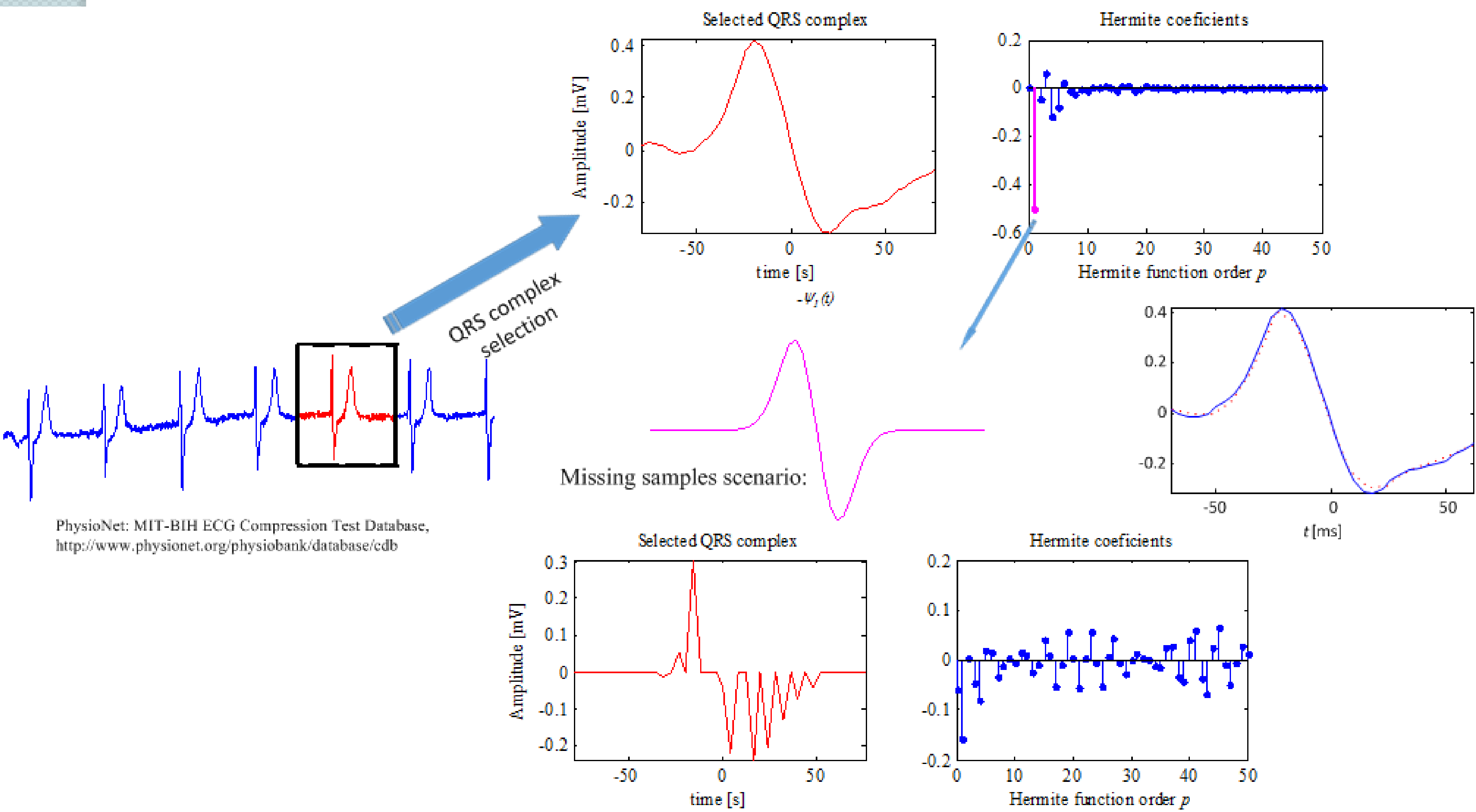




Virtual instrument for Compressive sensing

EEG signals: QRS complex is sparse in Hermite transform domain, meaning that it can be represented using just a few Hermite functions and corresponding coeffs.

CS of QRS complexes in the Hermite transform domain



PhysioNet: MIT-BIH ECG Compression Test Database,
http://www.physionet.org/physiobank/database/cdb

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