ISAR Reconstruction from Incomplete Data using Total Variation Optimization

Andjela Draganić, Irena Orović, Srdjan Stanković

University of Montenegro,
Faculty of Electrical Engineering
Podgorica, Montenegro
e-mail address: andjelad@ac.me

Xiumei Li

Hangzhou Normal University
Hangzhou, China
e-mail address: lixiumei@pmail.ntu.edu.sg

Abstract—**Sparsity of the ISAR images is exploited with the aim to use the possibility of applying an under-sampling strategy as assumed by the compressive sensing approach. The signal sparsity is a desirable property that needs to be satisfied in order to reconstruct the signals and images from the compressive sensed data. It is assumed that certain amount of radar data is not available and the idea is to reconstruct the radar image from the rest of the data. The signal samples are observed in the spatial domain, and the reconstruction is based on the total variation minimization. The procedure is tested on both, synthetic and real ISAR image, showing satisfactory reconstruction quality with a small set of acquired samples.**

Keywords- Compressive Sensing, ISAR, incomplete set of samples, TwIST

I. INTRODUCTION

Radars have been used in large number of applications, such as imaging, remote sensing and global positioning. Remote-sensing radars, i.e. Synthetic Aperture Radar (SAR) and Inverse Synthetic Aperture Radar (ISAR), are techniques used for obtaining high resolution image of the target. They form an image based on the changes in viewing angle of the target with respect to the radar [1]-[6]. The SAR technique is used in the situations where the radar platform is moving while the target is stationary. On the other hand, the ISAR technique refers to the situations where the radar is stationary and the targets are moving (e.g. airplanes, ships, etc.). Useful information from both, SAR and ISAR images, can be derived by using proper post-processing techniques. Our focus in this paper will be on the ISAR images.

Inverse Synthetic Aperture Radar-ISAR provides imaging of the moving targets, in range-Doppler (or range and cross-range) domains. Therefore, it is an important tool in automatic target recognition applications. In ISAR, radar collects the scattering data from the target. This is done for different look angles, which results in resolving different points along the cross-range axis. The ISAR images are formed by using small number of target reflectors and they are post-processed in the 2D Fourier transform domain. Also, they show property of being sparse in this domain. Therefore, we consider the possibility of applying the algorithms for sparse reconstruction of ISAR images when some of the data are missing (not available).

The reconstruction of sparse signals, in the situations when a small set of samples is available, or when the signal is intentionally under-sampled, is done by using the Compressive Sensing (CS) approach [7]-[10]. Unlike the traditional Shannon-Nyquist theorem, which requires signal to be sampled with frequency twice higher than the maximal signal frequency for satisfactory signal reconstruction, the CS concept allows to recover the important information from very small set of signal samples. This leads to the lower memory requirements in various application, as well as faster signal acquisition.

In this paper, we assume that some of the positioned radar data are not available, in order to test the quality of ISAR image reconstruction using CS algorithms and considering different number of available target positions.

The paper is structured as follows: Section II provides theoretical background on ISAR image modeling, as well as the basic concepts of the CS theory. Section III describes the procedure for reconstruction of ISAR images from under-sampled radar data by using the CS techniques for 2D signal reconstruction. Section IV provides experimental results on synthetic and real ISAR signals. Conclusions are given in Section V.

II. THEORETICAL BACKGROUND

A. Model of the ISAR signal

The starting point, when defining an ISAR image, is to choose range and cross-range size. Let R denote the range size and C the cross-range size. Quality of the ISAR image is defined by range and cross-range resolution Δx and Δy, respectively. Frequency and angular resolution can therefore be defined as [4]:

$$\Delta f = \frac{c}{2R}, \quad \Delta \Theta = \frac{\lambda_c}{2C},\qquad(1)$$

where Δf is a frequency resolution, $\Delta \Theta$ is an angular resolution, c is the speed of light and λ_c is the wavelength. The frequency and angular bandwidth are defined as:

$$B = \Delta f \frac{R}{\Delta x}, \quad \Omega = \Delta \Theta \frac{C}{\Delta y}. \qquad (2)$$

Then, the E_S signal is formed as [4]:

$$E_S(k, \Theta) = \sum_{i=1}^{P} A_i e^{-j2k(\cos\Theta \cdot x_i + \sin\Theta \cdot y_i)}, \qquad (3)$$

where A_i is the backscattered field amplitude for the each point scatterer for the P points scatterer, $k=2\pi f/c$ is the wave number for the frequency f and Θ is the look-angle. ISAR image is obtained as 2D inverse DFT of the E_S:

$$ISAR(x, y) = F_{2D}^{-1}\{E_S(k, \Theta)\} =$$
$$\int_{\Theta_1}^{\Theta_2} \int_{k_1}^{k_2} E_S(k, \Theta) e^{j2k(\cos\Theta \cdot x + \sin\Theta \cdot y)} dk \cdot d\Theta, \qquad (4)$$

where backscattered electric field is collected for the spatial frequencies from k_1 to k_2 and for the angles from Θ_1 to Θ_2. From the relations (3) and (4) follows:

$$ISAR(x, y) = \sum_{i=1}^{P} A_i \delta(x - x_i, y - y_i), \qquad (5)$$

where $\delta(x,y)$ is the 2D delta function in the x-y plane. Here, the assumption that look angle Θ is small is used, and therefore, $cos(\Theta) \sim 1$ and $sin(\Theta) \sim \Theta$ [4].

B. Compressive Sensing

Nowadays, there is an increasing demand to reduce the amount of data in real applications such as to provide lower sensing time, lower memory requirements, and generally less resources, but still to be able to reconstruct the entire information afterwards. Consequently, some new approaches and algorithms for signal processing are developed. The alternative strategy for signal acquisition based on decreased number of measurements has been known as the CS approach. It deals with sparse signals, sampled according to the a priori defined procedure. Sampling procedure should provide successful reconstruction from the small set of acquired data, which is usually assured by random signal samples selection.

If the discrete signal $\mathbf{x}$ is sparse in the transform domain $\Im$, then it can be reconstructed from the vector of acquired samples $\mathbf{y}$ (measurements) by using the powerful mathematical algorithms for optimization, i.e. [11]-[14], [22], [23]. Mathematically, the measurement vector $\mathbf{y}$ can be defined as [10]:

$$\mathbf{y} = \Omega \Im^{-1} \mathbf{X}, \qquad (6)$$

where matrix Ω is used to randomly under-sample the signal, $\mathbf{X}$ is the vector of the transform domain coefficients and $\Im^{-1}$ is the inverse transform matrix, i.e.:

$$\mathbf{X} = \Im \mathbf{x}. \qquad (7)$$

The system of equations (6) is under-determined and various optimization techniques are used for obtaining the optimal solution in terms of sparsity. Optimization techniques may differ for 1D and 2D signals [11]-[21]. In this paper, we will focus on the commonly used techniques for the reconstruction of the under-sampled images, based on the total variation minimization (TV) [16].

III. PROCEDURE OF THE ISAR IMAGES RECONSTRUCTION BASED ON THE TOTAL VARIATION OPTIMIZATION

Image reconstruction from the reduced set of samples is based on the gradient minimization approach. This approach is more appropriate for the reconstruction of 2D signals than commonly used CS approaches, because of the fact that images are not strictly sparse neither in spatial nor in the frequency domain. Image gradient expresses sparsity property and therefore can be suitable in minimization problems. Popular regularization approach is the TV regularization, due to its ability to preserve image edges. Let us observe the signal $\mathbf{y}$ [16]:

$$\mathbf{y} = \mathbf{Af} + \mathbf{n}, \qquad (8)$$

where $\mathbf{A}$ is a matrix that models samples selection process, $\mathbf{y}$ is measurement vector, $\mathbf{f}$ is signal to be estimated and $\mathbf{n}$ is noise. The goal is to estimate $\mathbf{f}$ in a way that $\mathbf{y}$ correspond to the product $\mathbf{Af}$. In that sense, let us form an objective function:

$$O(\mathbf{f}) = \partial(\mathbf{y}, \mathbf{Af}) + \varepsilon R(\mathbf{f}), \qquad (9)$$

where $\partial(\mathbf{y}, \mathbf{AF})$ is the function which models the difference between $\mathbf{y}$ and $\mathbf{f}$, $R(\mathbf{f})$ is a regularization function and commonly is chosen to correspond to the ℓ_1-norm, and parameter ε is >0. In most applications, the objective function is defined as:

$$O(\mathbf{f}) = \|\mathbf{y} - \mathbf{Af}\|_{\ell_2}^2 + \varepsilon \|\mathbf{f}\|_{\ell_1}. \qquad (10)$$

In this paper we have used TwIST algorithm for solving problem (9). TwIST relies on the iterative shrinkage/thresholding (IST) and iterative re-weighted/shrinkage (IRS) algorithms. Regularization function $R(\mathbf{f})$ can be set to minimize ℓ_0-norm, ℓ_1-norm, ℓ_2-norm, TV semi-norm or similar. Solution is reached by an iterative procedure defined as [16]:

$$\mathbf{f}_1 = \Gamma_\varepsilon(\mathbf{f}_0),$$
$$\mathbf{f}_{t+1} = (1-\alpha)\mathbf{f}_{t-1} + (\alpha - \beta)\mathbf{f}_t + \beta\Gamma_\varepsilon(\mathbf{f}_t), \ t \geq 1. \qquad (11)$$

Function Γ is defined as:

$$\Gamma_\varepsilon(\mathbf{f}) = \Psi_\varepsilon(\mathbf{f} + \mathbf{A}^T(\mathbf{y} - \mathbf{Af})), \qquad (12)$$

where Ψ_ε is denoising operator:

$$\Psi_\varepsilon = \arg\min_{\mathbf{f}} \frac{1}{\mu}\sum_i \|D_i\mathbf{f}\| + \frac{1}{2}\|\mathbf{y} - \mathbf{Af}\|_{\ell_2}^2, \qquad (13)$$

and $\|D_i\mathbf{f}\|$ denotes gradient operator Each iteration of the algorithm has computational complexity similar to the

complexity of the Orthogonal Matching Pursuit algorithm, $2KMN+3K^2M$, where K is sparsity, M number of available samples, and N is the signal length [24].

Samples are acquired from the spatial domain, by using the radial line mask, shown in Figure 1.
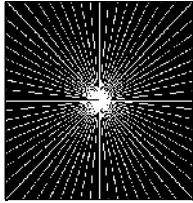

Figure 1.   Radial line mask used in the measurement procedure

The acquired samples correspond to the white regions in the masks. Note that in this case the vector of observations **y** denotes spatial coefficients, while vector **f** denotes coefficients from the transform domain.

IV.   EXPERIMENTAL RESULTS

The presented theory is demonstrated on two examples - simulated and real radar signals. In both examples, we assume that only small set of the coefficients is available, as defined by the radial-lines within the considered mask. The noisy free measurements are observed. In the noisy signal cases, the observations should be denoised prior to reconstruction procedure is applied.

Example 1: Synthetic radar signal

In the first example, the simulated radar signal is analyzed. The outline of the airplane is formed by using 110 point scatterers and backscattered electric field was collected for 64 frequencies and 64 angles. In this example, the radial line masks with different number of radial lines are used. Figure 2. Figure 2. b, 2c and 2d show the ISAR images obtained directly from the available pulses, as a 2D DFT of the available samples. It can be seen that the obtained images are noisy. Noise in the 2D DFT domain is a consequence of the missing information in the spatial domain. The available samples are used in optimization algorithm and results of the reconstruction are shown in Figure 2. In all considered cases, successful reconstruction is obtained by using the TV minimization algorithm. The reconstruction is done by using 33.4% (mask with 20 radial lines), 38.5% (mask with 24 radial lines) and 49.4% (mask with 32 radial lines) of the total number of samples.

Example 2: Real radar signal

Commonly used signal for ISAR image testing, MIG 25, is considered in this example. As we are dealing with real signal, it is only approximately sparse in each range bin. Signal has 256 pulses and 64 samples in each pulse (due to the algorithm requirements we have used image of size 64×64). Original image is shown in Figure 3. ISAR images obtained directly from the samples belonging to the radial lines are shown in the Figure 4. a, b and c, and these are quite disturbed due to incomplete data. Reconstructed images after TV optimization algorithm are shown in Figure 4. d, 4e and 4f. Masks with the same number of radial lines as in the Example 1 are used (20, 24 and 32 lines, respectively).
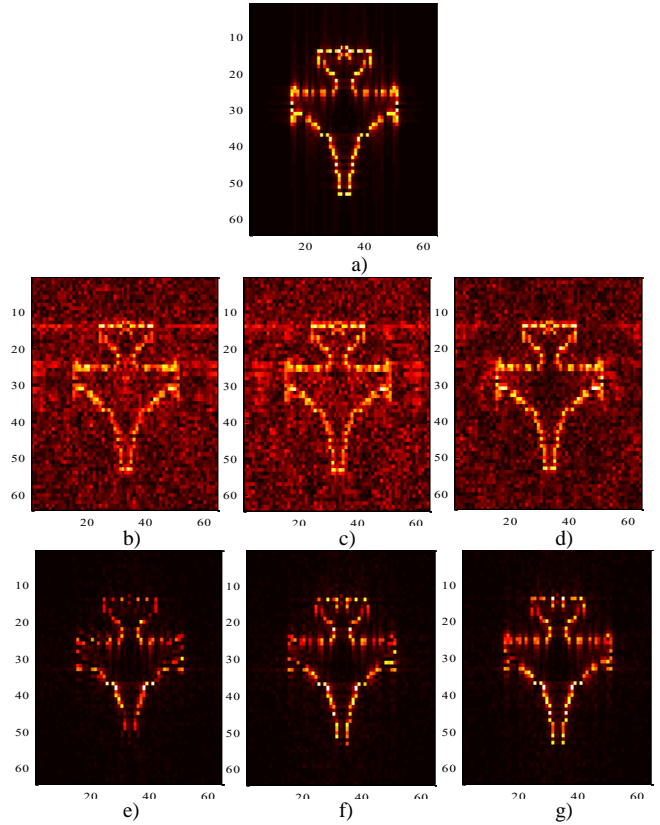

Figure 2.   a) ISAR image of the simulated radar signal, b), c) and d)) ISAR image obtained from the available samples, by using 20, 24 and 32 lines radial mask, respectively; e), f) and g) Reconstructed ISAR image by using TV optimization algorithm, from 20, 24 and 32 radial line mask, respectively
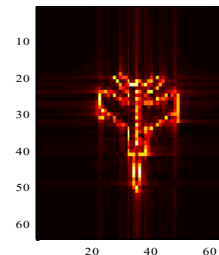

Figure 3.   ISAR image of the Mig25

We can observe that the ISAR image can be successfully reconstructed in all considered cases: by using 33.4%, 38.5%, and 49.4% of the total number of samples. Even the image reconstructed using only 33.4% of the total number of samples, is very similar to the original one.
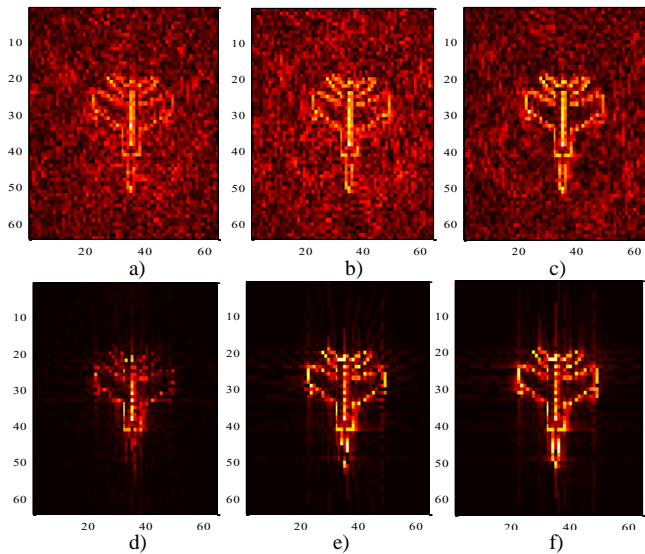
Figure 4.   a), b) and c) ISAR image obtained from the available samples, using 20, 24 and 32 lines radial mask, respectively; d), e) and f) Reconstructed ISAR image from the radial line mask, with 20, 24 and 32 lines using TV based optimization

V.   CONCLUSION

Images obtained by using the ISAR technology exhibit sparsity property in the 2D Fourier transform domain. This fact is used for testing the possibility to apply CS approach for the reconstruction of the ISAR images. The samples, called measurements, are acquired from the spatial domain. The measurement process is modeled by using the radial mask as it is done in biomedical imaging applications. It is shown that the ISAR images can be successfully recovered from small number of acquired samples – approximately with 33% of the total number of samples. The performance of the algorithm in the presence of external noise could be considered in the future work.

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