

Analysis of Noise and Nonsparsity in the ISAR Image Recovery from a Reduced Set of Data

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Abstract—Sparse inverse synthetic aperture radar (ISAR) images can be reconstructed using a reduced set of data and compressive sensing based theory. In real cases the ISAR images are noisy and only approximately sparse or not sparse. The influence of the additive input noise and nonsparsity of the ISAR data to the reconstructed images is analyzed in this paper. Simple and exact formula for the mean square error (MSE) in the reconstructed ISAR image is derived. Results are tested on examples and compared with statistical data in the cases of: 1) input additive noise and sparse ISAR images and 2) nonsparse ISAR images reconstructed assuming that they were sparse. Statistical data confirm the theoretical results.

Index Terms—Radar imaging, ISAR, noisy signal, sparse signal, compressive sensing

I. INTRODUCTION

Inverse Synthetic Aperture Radar (ISAR) is a technique in radar processing for obtaining a high resolution two-dimensional signal of the target of interest. The ISAR image is obtained as a two-dimensional Fourier transform of the received signal, after its appropriate pre-processing. Since the target commonly consists of a small number of reflectors, the signal can be considered as sparse in the transformation (ISAR image) domain. It means that the compressive sensing (CS) theory can be used for ISAR signal processing and reconstruction [1]–[7].

The aim of CS is to achieve full information of a signal from a reduced set of data/measurements/observations [8]–[15]. A reduced set of data in the ISAR imaging can occur for different reasons. Some parts of the signal may not be available due to the physical constraints. The radar signal can also be highly corrupted in some parts that it is better to omit them from the reconstruction [16]–[22]. In this case, the corrupted data will be declared as unavailable and the set is reduced to the ones which are not corrupted. The method of reducing the set of available measurements can be beneficial for the effectiveness of processing of radar signals by transmitting only a few random pulses of the signal as well.

In real cases the ISAR data are noisy and only approximately sparse or not sparse [23]–[26]. The influence of the additive input noise and nonsparsity of the ISAR data to the reconstructed images is analyzed in this paper. Simple and exact formula for the mean square error (MSE) in the

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reconstructed image is derived. Results are tested on examples and compared with statistical data in the cases of: 1) input additive noise and sparse ISAR images and 2) nonsparse ISAR images reconstructed assuming that they were sparse. Statistical data confirm theoretical results for the MSE.

The paper is organized as follows. The radar signal model for ISAR is presented in Section II. Model for missing samples in the initial signal is given in this section as well. Additive noise influence is reviewed in Section III. Relations for influence of nonsparse ISAR signal analysis, using the sparsity assumption, are given in Section IV. All theoretical relations are illustrated on numerical and statistical examples.

II. RADAR SIGNAL MODEL

Consider a radar that transmits continuous-wave linear frequency-modulated signals in a series of M chirps. The received signal is a delayed version of the transmitted signal. Assuming that a target consists of K scattering points, after common post-processing of the received signal is done, the signal component corresponding to the i th scattering point can be written as

$$q_i(m, t) = A_i e^{j\Omega_0 \frac{2d_i(t)}{c}} e^{-j2\pi B f_r (t - mT_r) \frac{2d_i(t)}{c}} \quad (1)$$

where Ω_0 is the radar operating angular frequency, c is the speed of light, A_i is the reflection coefficient of i th scattering point, while $2d_i(t)/c$ is the corresponding delay of the received signal component to the transmitted one. The distance of this point is denoted by $d_i(t)$. The single chirp repetition time is $T_r = 1/f_r$. The number of samples within a chirp is N . One revisit time is $T_c = MT_r$ (coherent integration time - CIT). The index m will be used for the slow time (chirp index). For a system of point scatterers, the signal is equal to the sum of the individual point scatterer responses, [1]. The Doppler part of the received signal will be denoted by

$$s_i(t) = A_i e^{j2d_i(t)\Omega_0/c} \quad (2)$$

The range part of the received signal can be written as

$$r_i(t) = e^{j2\pi\gamma n/N}$$

since $\exp(-j2\pi B f_r (t - mT_r) 2d(t)/c)$ reduces to with $\gamma = -B f_r T_s N (2d(t)/c)$ and $t - mT_r = nT_s$. The index of a signal sample within one chirp (fast time) is denoted by n and the sampling interval within that chirp is defined as $T_s = T_r/N$.

Consider a target consisting of K scattering points. Then the signal can be written as when the target motion may be considered as uniform within the CIT. The distance of the i th scattering point to radar can be written as $d_i(t) \cong d_0 + v_i t \cong d_0 + v_i m T_r$.

After the distance compensation, the received signal at the i th scattering point can be defined as

$$\begin{aligned} q_i(m, n) &= A_i e^{j\Omega_0 2v_i T_r m/c} e^{j2\pi\gamma_i n/N} \\ &= A_i e^{j2\pi\beta_i m/M} e^{j2\pi\gamma_i n/N}, \end{aligned}$$

where $v_i = y_{i0}\omega_R$, $\beta_i = 2\Omega_0 y_{i0}\omega_R T_r/c$ and $\gamma_i = -Bf_r T_s N(2d_i(t)/c)$ are the constants proportional to the velocity (cross-range y_{i0}) and range (after distance d_0 compensation $\gamma_i = -2Bx_{0i}/c$ since $T_s = T_r/N = 1/(f_r N)$).

The target signal with K scattering points is

$$q(m, n) = \sum_{i=1}^K q_i(m, n).$$

The two-dimensional FT of the signal $q(m, n)$ is

$$Q(k, l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} q(m, n) e^{-j(\frac{2\pi mk}{M} + \frac{2\pi nl}{N})}, \quad (3)$$

where $q(m, n)$ is the discrete received and processed signal and the indices k and l are the discrete two-dimensional FT frequencies, where k is proportional to the cross-range and l is proportional to the range.

Assume that some parts (few samples or blocks) of the signal are highly corrupted or not available at random positions. The samples are then omitted from the analysis [11]. The two-dimensional FT estimate of the new signal is

$$\hat{Q}(k, l) = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{N}_A(m)} q(m, n) e^{-j(\frac{2\pi mk}{M} + \frac{2\pi nl}{N})}. \quad (4)$$

where $\mathbb{N}_A(m)$ represents the set of available samples within the m -th chirp, and the total number of available samples is in the range $1 \ll N_A \leq MN$. Note that the unavailable/corrupted data can occur within one chirp only or spread over more chirps, including the possibility that a few chirps in a row are affected. Also note that it could happen that there are no available samples within m th chirp, i.e. $\mathbb{N}_A(m) = \emptyset$.

For a large number of randomly positioned unavailable samples $MN - N_A$ the value of $\hat{Q}(k, l)$ is a sum of vectors with quasi arbitrary phases (for k and l not corresponding to β_i and γ_i). It can be considered as a complex-valued variable (missing samples noise) with Gaussian distributed real and imaginary parts (as shown in [13]). Its variance is

$$\text{var}\{\hat{Q}(k, l)\} = N_A \frac{MN - N_A}{MN - 1} A_i^2. \quad (5)$$

For K scattering points and k and l we may write [13]

$$E\{\hat{Q}(k, l)\} = \sum_{i=1}^K A_i N_A \delta(k - \beta_i, l - \gamma_i) \quad (6)$$

$$\text{var}\{\hat{Q}(k, l)\} = N_A \frac{MN - N_A}{MN - 1} \sum_{i=1}^K A_i^2 (1 - \delta(k - \beta_i, l - \gamma_i)).$$

III. INFLUENCE OF ADDITIVE INPUT NOISE

Let us consider now that an additive noise $\varepsilon(m, n)$ exists in the available data. When the recovery is achieved, accuracy of the result is related to the input additive noise in signal samples since the missing samples noise influences the possibility to recover the signal only. Reconstruction of noisy signal is based on

$$\begin{aligned} q(m, n) + \varepsilon(m, n) &= \frac{1}{MN} \sum_{i=1}^K Q_K(k_i, l_i) e^{j(\frac{2\pi mk_i}{M} + \frac{2\pi nl_i}{N})}, \\ &\text{for } 0 \leq m \leq N - 1, \quad n \in \mathbb{N}_A(m). \end{aligned} \quad (7)$$

Its matrix form is

$$\mathbf{y} + \boldsymbol{\varepsilon} = \boldsymbol{\Psi} \mathbf{Q}_K.$$

This is a system of N_A linear equations with K unknowns $Q_K(k_i, l_i)$ in vector \mathbf{Q}_K . The transform indices can take a value from the set of detected values $(k, l) \in \{(k_1, l_1), (k_2, l_2), \dots, (k_K, l_K)\}$. The solution of this system of N_A equation for the unknown values of

$$\mathbf{Q}_K = (Q_K(k_1, l_1), Q_K(k_2, l_2), \dots, Q_K(k_K, l_K))$$

is found as

$$\mathbf{Q}_K = (\boldsymbol{\Psi}^H \boldsymbol{\Psi})^{-1} \boldsymbol{\Psi}^H (\mathbf{y} + \boldsymbol{\varepsilon}), \quad (8)$$

where $\mathbf{Q}_K = \mathbf{Q}_{KS} + \mathbf{Q}_{KN}$ contains the true transform coefficients and the noise values of the reconstructed transform coefficients, respectively [26].

If the full signal is available, i.e. there are no unavailable/corrupted samples, the input signal-to-noise ratio (SNR) would be

$$SNR_i = 10 \log \frac{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |q(m, n)|^2}{\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |\varepsilon(m, n)|^2} = 10 \log \frac{E_s}{E_\varepsilon}. \quad (9)$$

The scattering coefficient value (component amplitude) at (k_i, l_i) would be MNA_i if all signal samples are available. When N_A samples are available, the coefficient is scaled to $N_A NA_i$ and the reconstruction algorithm performs its rescaling by a factor of MN/N_A . It means that the noise in the transform coefficients is rescaled by the same factor MN/N_A . The noise energy is increased to $E_{\varepsilon A} (MN/N_A)^2$ during the reconstruction. The energy of noise in the available signal samples is denoted by $E_{\varepsilon A}$. The SNR in the reconstructed signal is

$$SNR_f = 10 \log \frac{E_s}{\frac{M^2 N^2}{N_A^2} E_{\varepsilon A}} \quad (10)$$

Having in mind that only K out of MN coefficients are used and considered as nonzero, the energy of the reconstruction error is reduced for the factor of $K/(MN)$. Therefore, the noise energy in the reconstructed signal is

$$E_{\varepsilon R} = \frac{K}{MN} \frac{M^2 N^2}{N_A^2} E_{\varepsilon A}.$$

For a white input noise, the energy of noise in the reconstructed signal is

$$E_{\varepsilon R} = \frac{K}{N_A} E_\varepsilon = \frac{K}{N_A} MN \sigma_\varepsilon^2,$$

TABLE I
RESULTS FOR THE RECONSTRUCTION ERROR IN [dB] FOR NOISE ONLY
CASE FOR $N_A = MN/3$ AND $N_A = MN/2$

$N_A = \frac{1}{3}MN$	$K = 10$	25	50	75	100
Statistics	-38.4	-34.5	-31.3	-29.5	-28.1
Theory	-38.3	-34.4	-31.4	-29.6	-28.3
$N_A = \frac{1}{2}MN$					
Statistics	-40.3	-36.2	-33.0	-31.3	-30.0
Theory	-40.1	-36.1	-33.1	-31.4	-30.1

since the variances of noise in all samples and in the available samples are the same, i.e.

$$\frac{1}{N_A}E_{\varepsilon_A} = \frac{1}{MN}E_{\varepsilon}.$$

In the case of additive input noise in the available signal samples, the output SNR will be increased if the estimated number of components K is as small as possible, for a given number of available samples N_A . In an ideal case, the value of K should be equal to the signal sparsity.

Example 1: Consider a sparse noisy signal

$$q(m, n) = \left(\sum_{i=1}^K A_i e^{j2\pi(k_{1i}m/M + k_{2i}n/N)} \right) / (MN) + \varepsilon(m, n)$$

where k_{1i}, k_{2i} are random frequency indices from 0 to $N-1$ and 0 to $M-1$. In this example we will use $M = N = 64$ and assume that the first K components of the signal are reconstructed. Signal amplitudes of randomly positioned components are $A_i = 1$, for $i = 1, \dots, K$. The additive noise is complex i.i.d. Gaussian with standard deviation of real and imaginary parts $\sigma_n = 0.1/(MN)$. Their variance is $\sigma_{\varepsilon}^2 = \sigma_n^2$. Factor $1/(MN)$ is used in the standard deviation to normalize its value to the signal amplitudes. Two cases are considered with $N_A = MN/2 = 2048$ and $N_A = MN/3 \approx 1365$. That is, we will consider the cases when 50% and 33% of the signal data is available. Additionally, K will be used from $K = \{10, 25, 50, 75, 100\}$. The reconstruction results for the squared error,

$$E_{stat} = 10 \log \left(\|\mathbf{Q}_K - \mathbf{Q}_R\|_2^2 \right) \quad (11)$$

$$E_{theory} = 10 \log \left(K \frac{(MN)^2}{N_A} \sigma_{\varepsilon}^2 \right), \quad (12)$$

averaged in [dB] over 100 realizations of randomly positioned available samples, are given in Table I. A complete MATLAB code (to fully reconstruct this calculation) is given at the end of this paper. Agreement with the theory is within the statistical confidence for the number of performed realizations.

IV. NONSPARSE SIGNAL RECONSTRUCTION

In Section II it has been shown that the missing samples in the initial signal can be represented by a noise. Assume a signal whose two-dimensional DFT is \mathbf{Q} with sparsity K . According to the presented analysis, if we apply a reconstruction algorithm on the signal whose DFT is not sparse (or not sufficiently sparse), then the nonreconstructed components will behave as additive noise. General bounds for

the reconstruction error for nonsparse signals, reconstructed with the sparsity assumption, are given in [12]. Here we will present an exact relation for the reconstruction error for the ISAR image. Denote by \mathbf{Q}_K the sparse signal with K nonzero coefficients equal to the largest K coefficients of \mathbf{Q} . The K -sparse original signal transform will be denoted as \mathbf{Q}_K where $Q_K(k) = Q(k)$ for $k \in \mathbf{K}$ and $Q_K(k) = 0$ for $k \notin \mathbf{K}$. We will assume that the sparsity K and the measurements matrix satisfy the reconstruction conditions of CS. Then the reconstruction algorithm can detect (one by one or at once) the largest K components (A_1, A_2, \dots, A_K) and perform signal reconstruction to get reconstructed signal \mathbf{Q}_R . The remaining $MN - K$ nonreconstructed signal components with amplitudes ($A_{K+1}, A_{K+2}, \dots, A_{MN}$) will be considered as noise in the K largest reconstructed components. Then we get the energy of error in the reconstructed components as a function of the energy of nonreconstructed components

$$\|\mathbf{Q}_R - \mathbf{Q}_K\|_2^2 = K \frac{MN - N_A}{N_A MN} \|\mathbf{Q} - \mathbf{Q}_K\|_2^2.$$

The proof, along with a generalization to the nonsparse noisy ISAR signal, is presented in [23]. This relation will be illustrated and confirmed on examples.

Example 2: Consider a nonsparse signal

$$q(m, n) = \left(\sum_{i=1}^S A_i e^{j2\pi(k_{1i}m/M + k_{2i}n/N)} + \sum_{i=S+1}^{MN} A_i e^{j2\pi(k_{1i}m/M + k_{2i}n/N)} \right) / (MN)$$

where k_{1i}, k_{2i} are random frequency indices. Signal amplitudes are $A_i = 1$, $i = 1, \dots, S$ at the randomly positioned components, while the remaining components are $A_i = 0.5e^{-i/(2S)}$, $i = S + 1, \dots, MN$. Using $M = N = 64$, the first $K = S$ components of the signal are reconstructed (cases when $K \neq S$ are analyzed in [23] as well). The remaining $MN - S$ signal components are considered as a disturbance. Reconstruction of the nonsparse signal $q(m, n)$ for several values of assumed sparsity K is done. The square error for 100 realizations with random frequency positions and positions of available samples is calculated. When the nonsparse signal is noise-free, the results of

$$E_{stat} = 10 \log \left(\|\mathbf{Q}_K - \mathbf{Q}_R\|_2^2 \right) \quad (13)$$

$$E_{theory} = 10 \log \left(K \frac{MN - N_A}{N_A MN} \|\mathbf{Q} - \mathbf{Q}_K\|_2^2 \right), \quad (14)$$

averaged in [dB] in 100 realizations, for $N_A = MN/3$ and $N_A = MN/2$, are given in Table II.

The analysis is done on simulated ISAR Boeing data [1]. The simulation uses X-band radar operating at a center frequency of 9 GHz. The bandwidth of the waveform is 150 MHz with a range resolution of 1 m. The original ISAR image is presented in Fig.1. Assuming different sparsities, the results are presented in Fig.2. The sparsities K correspond to 0.7812%, 3.125%, 6.25%, and 12.5% of the total number of points $MN = 16384$. The square error normalized to

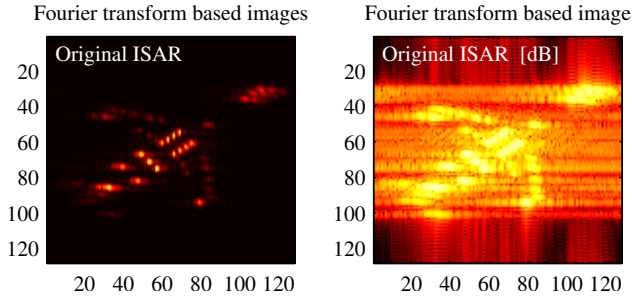


Fig. 1. Original ISAR using all signal samples. The logarithmic scale is on the right.

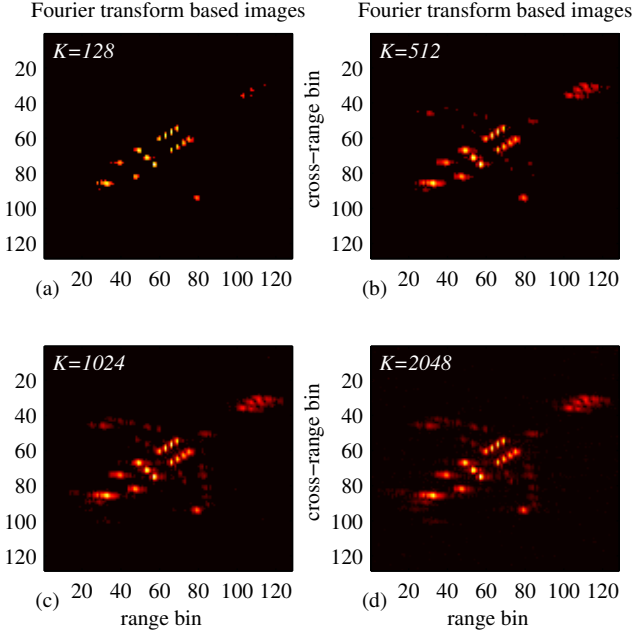


Fig. 2. Original ISAR and reconstructed assuming different sparsities, $K = 128$, $K = 512$, $K = 1024$, and $K = 2048$.

the sparsity K and the total maximal coefficient value are calculated as:

$$E_{stat} = 10 \log \left(\frac{\frac{1}{K} \|\mathbf{Q}_K - \mathbf{Q}_R\|_2^2}{\max_{k,l} \{|Q(k,l)|^2\}} \right) \quad (15)$$

$$E_{theory} = 10 \log \left(\frac{\frac{MN - N_A}{N_A MN} \|\mathbf{Q} - \mathbf{Q}_K\|_2^2}{\max_{k,l} \{|Q(k,l)|^2\}} \right).$$

TABLE II
RESULTS FOR THE RECONSTRUCTION ERROR IN [dB] FOR NONSPARSE NOISE FREE CASE IN FOR $N_A = MN/3$ AND $N_A = MN/2$

$N_A = \frac{1}{3}MN$	$K = 10$	25	50	75	100
Statistics	-24.0	-15.5	-9.4	-5.8	-3.1
Theory	-23.7	-15.6	-9.5	-6.0	-3.5
$N_A = \frac{1}{2}MN$					
Statistics	-26.9	-18.8	-12.4	-8.7	-6.2
Theory	-26.7	-18.6	-12.6	-9.0	-6.5

TABLE III
THE MEAN SQUARE ERROR IN THE ISAR RECONSTRUCTED COEFFICIENTS FOR BOEING DATA AND VARIOUS ASSUMED SPARSITY K .

$N_A = MN/3$	$K = 128$	512	1024	2048
Theory	-25.28	-30.51	-34.53	-38.38
Statistics	-25.66	-30.12	-34.04	-38.13

TABLE IV
THE MEAN SQUARE ERROR IN THE ISAR RECONSTRUCTED COEFFICIENTS FOR MIG DATA AND VARIOUS ASSUMED SPARSITY K .

$N_A = MN/2$	$K = 50$	150	250	350
Theory	-20.92	-24.72	-28.71	-31.60
Statistics	-20.19	-24.36	-28.17	-30.34

The errors are checked statistically by using 33% of the available samples at random positions in 100 realizations. The mean normalized error energy in K reconstructed components, obtained statistically and using the energy of remaining components, is given in Table III. The accuracy of the reconstruction result is proportional to the energy of remaining content.

The same analysis is performed on the ISAR MIG data [1]. The simulation parameters are: X-band radar operating at 9 GHz, with 512MHz bandwidth and a range resolution of 0.293 m. The ISAR image is presented in Fig.3.

For various assumed sparsities, the reconstructions with available 50% of the original data are presented in Fig.4. The sparsities $K = 50, 150, 250$, and 350 are considered. The results are checked statistically by using random realizations of the available samples in 100 trials. The mean normalized error energy in K reconstructed components is given in Table IV, calculated statistically and using the theory.

V. CONCLUSIONS

In this paper, we examined the influence of noise and non-sparsity on the ISAR image reconstruction. Simple relations are tested on a simulated example. The cases of noisy and non-sparse data are considered. The results are tested on Boeing and MIG ISAR data with various assumed sparsity levels.

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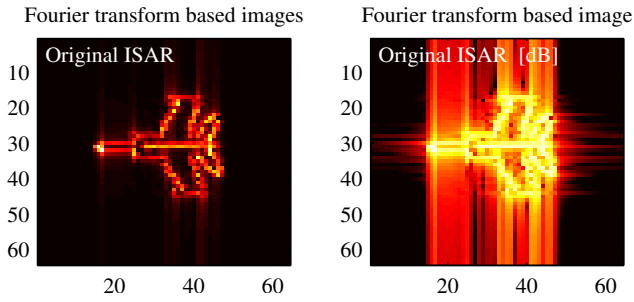


Fig. 3. Original ISAR using all signal samples. The logarithmic scale is on the right.

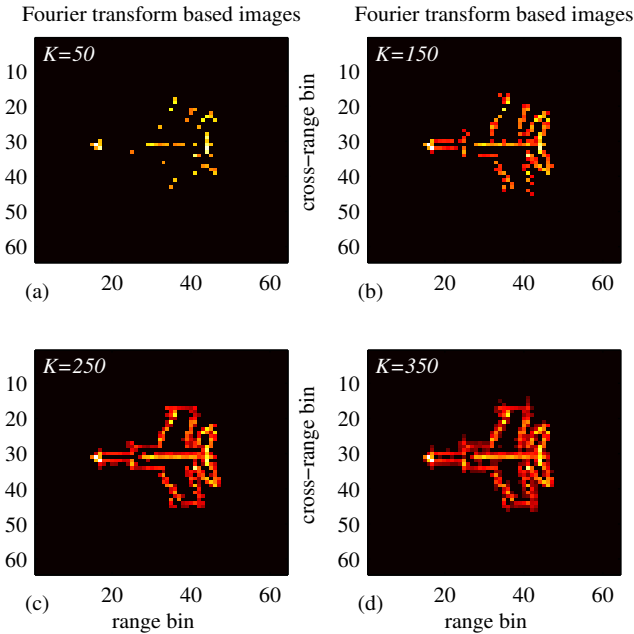


Fig. 4. Original ISAR and reconstructed assuming different sparsities, $K = 50$, $K = 150$, $K = 250$, and $K = 350$.

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% MATLAB code for Tables I and II
N=64; NA=2048; K=50; S=K;
ks=randperm(N*N); Q=ones(N^2,1);
% _____nonsparsity and noise_____
Q(ks(S+1:N*N))=1*0.5*exp(-(S+1:N*N)/S/2);
Sn=0*0.1/N^2;
% _____
q=ifft2(reshape(Q,N,N));% original signal
p=randperm(N*N); NA_v=p(1:NA);
DFT2_mtx=conj(kron(dftmtx(N)/N,dftmtx(N)/N));
A=DFT2_mtx(NA_v,:); % measuremet matrix
y=A*Q; % available samples y=q(NA_v')
y=y+Sn*randn(size(y))+j*Sn*randn(size(y));
% _____
% Reconstruction
KB=[]; y0=y; Kstep=1; %for MIG Kstep=50;
for iter=1:round(K/Kstep)
    if iter > 1;
        q_R_v=q_R(:); y=y0-q_R_v(NA_v);
    end
    Q0=(A*N^2)'*y;
    [Xv,Kv]=sort(abs(Q0));
    KB=[KB; Kv(end-Kstep+1:end)];
    A_K=A(:,KB);
    Q_R=zeros(N*N,1);
    Q_R(KB)=pinv(A_K)*y0;
    q_R=ifft2(reshape(Q_R,N,N));
end
% _____
% Display the results for one realization
EStat=10*log10(sum(abs(Q_R(KB)-Q(KB)).^2));
KT=1:N^2; KT(KB)=[]; C=K*(N^2-NA)/NA/N^2;
E=10*log10(C*sum(abs(Q(KT)).^2)+K/NA*2*Sn^2*N^4)

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