A Procedure for Optimal Pulse Selection Strategy in Radar Imaging Systems

Miloš Daković, *Member, IEEE*, Ljubiša Stanković, *Fellow, IEEE*, Srdjan Stanković, *Senior Member, IEEE*
Faculty of Electrical Engineering, University of Montenegro
Dzordza Vasingtona bb, 81000 Podgorica, Montenegro
milos@ac.me, ljubisa@ac.me, srdjan@ac.me

***Abstract*—Commonly ISAR/SAR images can be considered as sparse. Therefore they can be obtained from a reduced set of measurements (pulses) by using Compressive Sensing reconstruction techniques. In this paper we analyze influence of the pulse selection strategy to the uniqueness of the obtained radar image from a reduced set of pulses. A simple algorithm for optimal pulse selection strategy is proposed. It is shown that the proposed method can significantly increase sparsity limit (maximal number of target points in a radar image) when the reconstruction uniqueness can be guaranteed.**

***Index Terms*—Radar, ISAR, SAR, Compressive sensing, Reconstruction uniqueness**

I. Introduction

Compressive sensing (CS) theory is introduced in [1]. Its possible applications, including signal processing [2], [3], are intensively studied in the past decade. It is shown that the CS can be used in many signal processing applications including radar signal processing.

Inverse Synthetic Radar Imaging (ISAR) and Synthetic Radar Imaging (SAR) are techniques for obtaining target image based on analysis of radar return signal [4], [5]. Target is illuminated with series of N radar pulses (lineary frequency modulated) and target image is obtained as 2D Fourier transform of the radar output. Especially in the ISAR case there is relatively small number of target scatterers and target image is sparse signal. Only a small part of image has non-zero values. These facts are intensively used for the CS based ISAR imaging [6]–[16]. From the CS point of view we can consider each pulse as a measurement and the obtained radar image as an output sparse signal. Reconstruction of the sparse signal based on a reduced set of measurements is important topic within CS framework. There exists many reconstruction methods [17]–[20]. In each of them the uniqueness of the obtained reconstruction is very important.

In the considered scenario radar omits some pulses during the coherent integration time (CIT) as illustrated in Fig. 1. The positions of omitted pulses are known and controllable by the radar. The problem addressed within this paper is in optimal strategy of selecting pulses that can be omitted, having in mind that the reconstruction should remain unique. Solution uniqueness is checked by recently proposed theorem [21]. It is

This research is supported by the Montenegro Ministry of Science projects CS-ICT (Grant No. 01-1002)

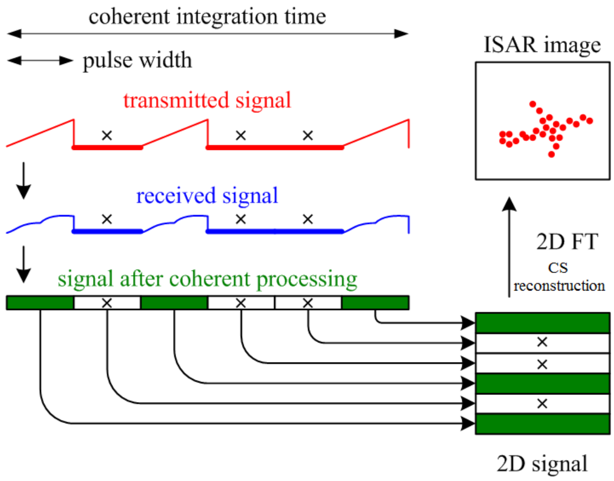

Fig. 1. Illustration of the CS based ISAR image formation. Three out of 6 pulses are omitted (marked with ×). In order to obtain clear ISAR image reconstruction of missing measurements should be performed.

shown that a random pulse selection is not optimal. A simple algorithm for optimal selection is presented.

Omitting some pulses reduces the total transmitted energy, making the radar system more energy efficient. This can also reduce the probability of radar detection.

Uniqueness theorem guaranties reconstruction uniqueness for sparsities lower than certain sparsity limit. In the radar imaging scenario this sparsity limit can be considered as the number of scatterers.

Reconstruction uniqueness is reviewed within Section II and the analysis of the worst and the best cases is performed. Based on this analysis optimal sampling strategy is proposed in Section III. Improvements in the sense of maximal allowed sparsity is analyzed and presented as well.

II. Reconstruction uniqueness check

Uniqueness of the signal reconstructed from a reduced set of samples is recently proposed [21]. It is assumed that the signal is sparse in the DFT domain calculated with $N = 2^r$. For an arbitrary signal uniqueness is checked with Theorem 1 and Corollary 2. Sparsity limit obtained by Theorem 1 is strict. It is shown that many cases covered with Theorem 1 are zero probability events leading to a relaxed uniqueness check, formulated as Corollary 2. Zero probability events that reconstructed signal components and their amplitudes are fully correlated to the positions of missing samples are neglected.

Theorem 1 *Consider a signal $x(n)$ that is sparse in the DFT domain with unknown sparsity. Assume that the signal length is $N = 2^r$ samples and that Q samples are missing at the instants $q_m \in \mathbb{N}_Q$. Assume that the reconstruction is performed and that the DFT of reconstructed signal is of sparsity s. The reconstruction result is unique if the inequality*

$$s < N - \max_{h=0,1,\dots,r-1} \left\{2^h \left(Q_{2^h} - 1\right)\right\} - s$$

holds. Integers Q_{2^h} are calculated as

$$Q_{2^h} = \max_{b=0,1,\dots,2^h-1} \left\{\operatorname{card}\{q : q \in \mathbb{N}_Q,\ \bmod(q, 2^h) = b\}\right\}$$

Corollary 2 *Consider the signal $x(n)$ that is sparse in the DFT domain. Assume that signal length is $N = 2^r$ samples and that Q samples are missing at the instants $q_m \in \mathbb{N}_Q$. Also assume that the reconstruction is performed and that the DFT of reconstructed signal is of sparsity s. Assume that the amplitudes of signal components are arbitrary with arbitrary phases so that the case when all of them can be related to the values defined by using the missing sample positions is a zero-probability event. The reconstruction result is not unique if the inequality*

$$s \geq N - \max_{h=0,1,\dots,r-1} \left\{2^h \left(Q_{2^h} - 1\right)\right\} - 1$$

holds. Integers Q_{2^h} are calculated in the same way as in the Theorem 1.

Consider now the best and the worst case for the sparsity limits obtained by Theorem 1 and Corollary 2. In both cases the value of

$$Q_m = \max_{h=0,1,\dots,r-1} \left\{2^h \left(Q_{2^h} - 1\right)\right\}$$

determine sparsity limit. Higher values of Q_m produce lower sparsity limits and vice versa.

Let us analyze values Q_{2^h}. They are obtained by partitioning set of missing samples $\mathbb{N}_Q$ with respect to the reminders obtained by division its elements with 2^h.

Let us start with $h = 0$. In this case the number of partitions is 1 and all elements of $\mathbb{N}_Q$ belong to single partition producing $Q_{2^0} = \operatorname{card} \mathbb{N}_Q = Q$.

For $h = 1$ we have two partitions: odd and even samples from $\mathbb{N}_Q$. In the best case missing samples are equally distributed producing $Q_{2^1} = \lceil Q/2 \rceil$ where $\lceil \cdot \rceil$ stands for ceiling operation (rounding to the greater integer). In the worst case one partition is largest possible. Consider that all $q \in \mathbb{N}_Q$ are $0 \leq q < N$ meaning that there is no more than $N/2$ even and $N/2$ odd elements in $\mathbb{N}_Q$. This limits maximal cardinality of each partition to $\min\{N/2, Q\}$. Now we can conclude that

$$\lceil Q/2 \rceil \leq Q_{2^1} \leq \min\{N/2, Q\}.$$

For $h = 2$ we have four partitions and limits

$$\lceil Q/4 \rceil \leq Q_{2^2} \leq \min\{N/4, Q\}.$$

In general, for any $h = 0, 1, ..., r - 1$ we have

$$\lceil Q/2^h \rceil \leq Q_{2^h} \leq \min\{N/2^h, Q\}$$

or

$$2^h(\lceil Q/2^h \rceil - 1) \leq 2^h(Q_{2^h} - 1) \leq 2^h(\min\{N/2^h, Q\} - 1).$$

The upper limit of Q_m can be obtained by analyzing

$$\max_{h=0,1,\dots,r-1} \left\{2^h(\min\{N/2^h, Q\} - 1)\right\}.$$

It is equal to

$$Q_m^{(\max)} = N - 2^{\log_2 N - \lfloor \log_2 Q \rfloor}$$

where $\lfloor \cdot \rfloor$ stands for the floor rounding operation.

Note that this limit is equal to $N - 2$ for $Q \geq N/2$. It is equal to $N - 4$ for $N/4 \leq Q < N/2$, and so on.

The lower limit of Q_m can be obtained by analyzing

$$\max_{h=0,1,\dots,r-1} \left\{2^h(\lceil Q/2^h \rceil - 1)\right\}.$$

It is equal to

$$Q_m^{(\min)} = Q - 1$$

Let us now recall sparsity limits obtained by Theorem 1 in the best and in the worst case. Lowest sparsity limit will be obtained for the maximal Q_m as

$$2s < 2^{\log_2 N - \lfloor \log_2 Q \rfloor}$$

and the highest sparsity limit when the uniqueness could be guarantied is

$$2s < N - Q + 1.$$

For Example if $N = 128$ and $Q = 60$ we get $2s < 4$ in the worst case meaning that only signals with sparsity $s = 1$ can be uniquely reconstructed with $128 - 60 = 68$ available samples out of total 128 samples. If we consider best possible case then $2s < 69$ meaning that signals with up to 34 components can be uniquely reconstructed with 68 available and 60 missing (omitted) samples.

The same analysis can be used in the Corollary 2 limits resulting in obtaining non-unique reconstruction in the worst case for

$$s \geq 2^{\log_2 N - \lfloor \log_2 Q \rfloor} - 1$$

and for the best case

$$s \geq N - Q.$$

Note that the reconstruction will be unique with a high probability if conditions from Corollary 2 are not satisfied.

Presented analysis lead us to the conclusion that the decision which sample/pulse will be omitted and which one will be used for reconstruction is very important if we want to uniquely reconstruct signals with higher number of components (sparsity).

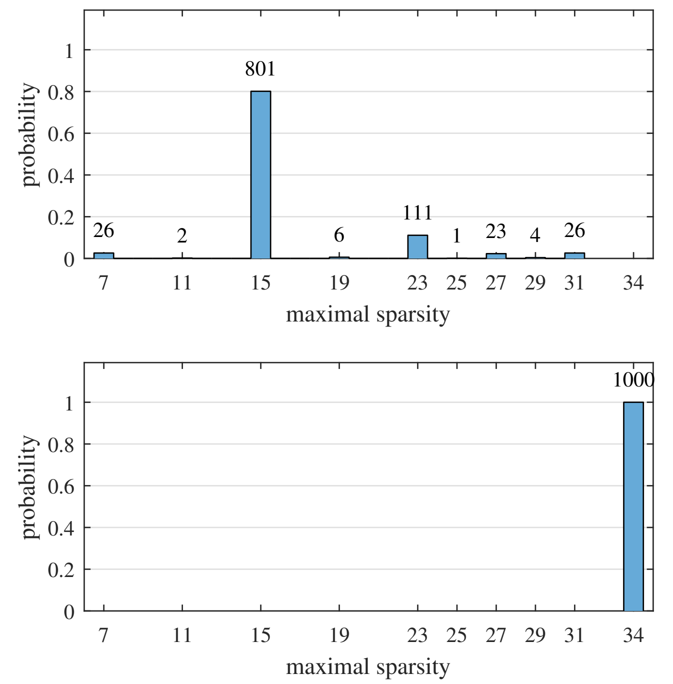

Fig. 2. Histogram of sparsity limits obtained with Theorem 1 for 1000 realizations of randomly positioned $Q = 60$ samples out of total $N = 128$ samples (upper subplot). Sparsity limit for samples positioned according to the proposed method (lower subplot). Number of realizations is given at top of each histogram bin.

Derived limits are analyzed as the worst and best case. In order to provide more details about sparsity limits we considered set of $N = 128$ samples in total and $Q = 60$ randomly positioned samples as missing ones. We analyze 1000 realizations and calculate sparsity limit according to Theorem 1 for each of them. Histogram of the obtained sparsity limits is presented in Fig. 2, upper subplot.

From Fig. 2 (upper subplot) we see that within the considered 1000 realizations, the best ($s = 34$) and the worst ($s = 1$) sparsity limits are not achieved. Instead we have a majority of realizations with sparsity limit of $s = 15$.

III. Procedure for optimal sampling strategy

According to the analysis presented in the previous section we can, for given N and Q select a set of Q missing (omitted) samples such that we achieve upper sparsity limit.

The procedure pseudocode is presented in Algorithm 1. Corresponding MATLAB code is given in Algorithm 2.

The main idea is to provide a maximal spread of elements from the set $\mathbb{N}_Q$ over each partition.

Note that selecting a half of the set (line 6 in Algorithm 1 and line 7 in Algorithm 2) in the case when the set $\mathbb{S}$ has odd cardinality $2m + 1$ should be done such that with equal probability we select m or $m + 1$ elements into set $\mathbb{S}_2$.

The selection procedure with $N = 128$ and $Q = 60$ is repeated 1000 times. For each subset of samples the Theorem 1 and Corollary 2 are checked for sparsity limits. Obtained sparsity limits for Theorem 1 are presented in Fig. 2, lower subplot. In all cases we get maximal sparsity limit, according to the analysis presented in Section II.

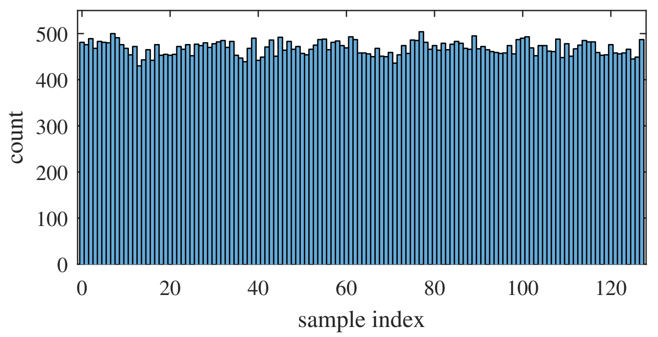

Fig. 3. Number of selections for each sample in 1000 realizations with $Q = 60$ and $N = 128$. Expected count, for uniform distribution, is $1000Q/N = 468.75$.

Next we checked uniformity of samples selection. For each realization we record selected set of samples and after 1000 realizations we counted how many times each sample was selected. The results are presented in Fig. 3. We can see that the selection procedure does not favor any sample, i.e., we obtain uniform distribution of selected samples.

In the next experiment we used $N = 128$ samples and varied number of missing samples Q from 1 to 127. For each pair (N, Q) we performed 100 iterations with randomly selected samples and samples selected according to the proposed procedure. Sparsity limits calculated for Theorem 1 are presented in Fig. 4. In the lower subplot we presented the increase in maximal allowed sparsity obtained with the proposed procedure.

Similar results (with approximately twice higher sparsity limits) are obtained for Corollary 2.

IV. Conclusion

A detailed analysis of sparsity limits obtained by the recently proposed uniqueness theorem is presented. The analysis lead to a simple procedure for the optimal samples selection. The presented procedure is checked statistically. A significant increase in the sparsity limits is achieved.

References

[1] D. Donoho, "Compressed sensing," *IEEE Trans. on Information Theory*, vol. 52, no. 4, 2006, pp. 1289–1306.

[2] R. Baraniuk, "Compressive sensing," *IEEE Signal Processing Magazine*, vol. 24, no. 4, 2007, pp. 118–121.

[3] S. Stanković, I. Orović, E. Sejdić, *Multimedia Signals and Systems: Basic and Advanced Algorithms for Signal Processing*, Springer, 2016.

[4] V. C. Chen, H. Ling, *Time-frequency transforms for radar imaging and signal analysis*, Artech House, Boston, USA, 2002.

[5] L. Stanković, M. Daković, T. Thayaparan, *Time–frequency signal analysis with application*, Artech House, 2013

[6] L. Zhang, M. Xing, C. W. Qiu J. Li, Z. Bao, "Achieving higher resolution ISAR imaging with limited pulses via compressed sampling," *IEEE Geoscience and Remote Sensing Letters*, Vol.6, No.3, 2009, pp.567–571

[7] J. H. G. Ender, "On compressive sensing applied to radar," *Signal Processing*, Vol. 90, No. 5, 2010, pp.1402-1414

[8] Y. S. Yoon, M. G. Amin "Compressed sensing technique for high-resolution radar imaging," *Proc. SPIE 6968, Signal Processing, Sensor Fusion, and Target Recognition XVII*, 69681A (April 17, 2008); doi:10.1117/12.777175;

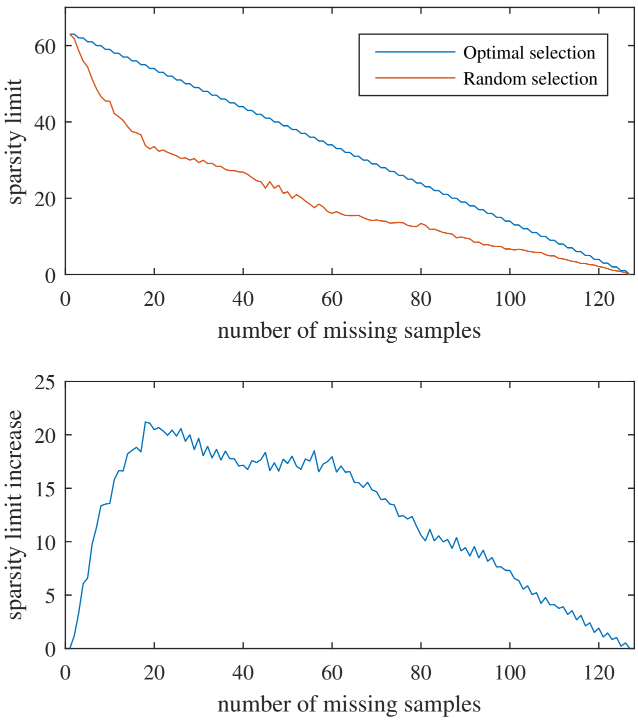

Fig. 4. Sparsity limit for Theorem 1 calculated for total number f samples $N = 128$ and number of selected (missing) samples $Q = 1, 2, \ldots, 127$ (upper subplot). Increase in sparsity limit caused by proposed selection procedure is presented (lower subplot).

Algorithm 1 Samples selection – pseudocode

Require:
- Total number of samples N
- Number of samples to be selected Q

Set $A(n) \leftarrow 0$ $\triangleright$ for $n = 1, 2, \ldots, Q$
Set $b \leftarrow \lceil \log_2(Q) \rceil$
for $k \leftarrow 0$ to $b-1$ **do**
  for $p \leftarrow 0$ to $2^k - 1$ **do**
    Find set $\mathbb{S} = \{n : A(n) = p\}$
    Select half of the set $\mathbb{S}$ into set $\mathbb{S}_2$
    for $n \in \mathbb{S}_2$ **do**
      Set $A(n) \leftarrow A(n) + 2^k$
    end for
  end for
end for
Generate random permutation $B(n)$ by selecting Q numbers form the set $\{0, 1, \ldots, N-1\}$
for $n \leftarrow 1$ to Q **do**
  Set $N_q(n) \leftarrow A(n) + 2^b \lfloor B(n)/2^b \rfloor$
end for
return $N_q(n)$

Output:
- Selected set of samples $\{N_q(n) : n = 1, 2, \ldots, Q\}$

Algorithm 2 Samples selection – MATLAB code

```
function Nq = Select_Samples(N,Q)
A = zeros(1,Q);
b = nextpow2(Q);
for k = 0:(b−1)
    for p = 0:(2^k−1)
        S = find(A==p);
        m = round( length(S)/2 + 0.1∗(rand−0.5) );
        A(S(1:m)) = A(S(1:m)) + 2^k;
    end
end
B = randperm(N,Q) −1;
Nq = sort( A + 2^b∗floor(B/2^b) );
```


[9] V.M. Patel, G.R. Easley, D. M. Healy Jr., R. Chellappa, "Compressed synthetic aperture radar," *IEEE Journal of Selected Topics in Signal Processing*, Vol.4, No.2, 2010, pp.244-254

[10] G. Zhao, Z. Wang, Q. Wang, G. Shi, F. Shen, "Robust ISAR imaging based on compressive sensing from noisy measurements," *Signal Processing*, Vol. 92, No. 1, 2012, pp.120-129

[11] X. C. Xie, Y. H. Zhang. "High-resolution imaging of moving train by ground-based radar with compressive sensing." *Electronics letters* Vol. 46, No. 7, 2010, pp.529-531.

[12] L. Stanković, "On the ISAR Image Analysis and Recovery with Unavailable or Heavily Corrupted Data," *IEEE Trans. Aerospace and Electronic Systems*, Vol.51, July 2015.

[13] W. Qiu, E. Giusti, A. Bacci, M. Martorella, F. Berizzi, H. Z. Zhao, Q. Fu, "Compressive sensing for passive ISAR with DVB-T signal," *14th International Radar Symposium (IRS)*, June 2013, pp.113-118.

[14] W. Qiu, M. Martorella, F. Berizzi, "Interferometric ISAR imaging based on compressive sensing." *2nd International Workshop on Compressed Sensing Applied to Radar*, Sept. 2013, Bonn, Germany.

[15] Y. Wang, H. Ling, V. C. Chen, "ISAR motion compensation via adaptive joint time-frequency technique," *IEEE Trans. Aerospace and Electronic Systems*, Vol.34, No.2, 1998, pp.670–677

[16] M. Daković, LJ. Stanković, S. Stanković, "Gradient Algorithm Based ISAR Image Reconstruction From the Incomplete Dataset,"*3rd International Workshop on Compressed Sensing Theory and its Applications to Radar, Sonar and Remote Sensing (CoSeRa)* 2015, 17-19 June 2015, pp.6-10, doi: 10.1109/CoSeRa.2015.7330253

[17] E. Candes, J. Romberg and T. Tao. "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. on Information Theory*, vol. 52, pp. 489–509, 2006.

[18] L. Stanković, M. Daković, S. Vujović, "Adaptive variable step algorithm for missing samples recovery in sparse signals," *IET Signal Processing*, vol.8, no.3, 2014, pp.246–256, doi: 10.1049/iet-spr.2013.0385

[19] M. Daković, L. Stanković, I. Orović, "Adaptive gradient based algorithm for complex sparse signal reconstruction," *22nd Telecommunications Forum TELFOR 2014*, Nov. 2014, Belgrade, Serbia

[20] L. Stanković, M. Daković, S. Vujović, "Concentration measures with an adaptive algorithm for processing sparse signals," *8th International Symposium on Image and Signal Processing and Analysis (ISPA)*, Trieste 4-6 Sept. 2013, pp.425,430, doi: 10.1109/ISPA.2013.6703779

[21] L. Stanković, M. Daković, "On the Uniqueness of the Sparse Signals Reconstruction Based on the Missing Samples Variation Analysis," *Mathematical Problems in Engineering*, vol. 2015, Article ID 629759, 14 pages, doi:10.1155/2015/629759