

Decomposition of Signals in Dispersive Channels using Dual Polynomial Fourier Transform

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Abstract—The acoustic waves transmitted in a dispersive environments can be quite complex for decomposition and localization. A signal which is transmitted through a dispersive channel is usually non-stationary. Even if a simple signal is transmitted, it can change its characteristics (phase and frequency) during the transmission through an underwater acoustic dispersive communication channel. Commonly, several components with different paths are received. In this paper, we present a method of decomposition of multicomponent acoustic signals using the dual polynomial Fourier transform and time-frequency methods.

Keywords—dispersive channels; polynomial Fourier transform; time-frequency analysis; underwater acoustics

I. INTRODUCTION

The dispersivity in underwater channels has been a challenging topic in the recent years. Many channels with the dispersion phenomena have been studied. In this paper we will focus on the iso-velocity channels. The iso-velocity channels are channels with the same sound velocity over all volume [1].

A dispersive channel in underwater acoustics is a system which produces nonlinear signal transformations [1]–[5]. That is, it shifts the propagating signal in the phase. This causes that different frequencies are changed in time by different time instances. Other complex problem of a dispersive channel is that it is usually characterized by multipath propagation producing multicomponent signals. The multipath propagation can occur for various reasons. The main one is the scattering of acoustic signals on the sea bottom.

Signal analysis and processing tools can help in detection, extraction and localization of transmitted signals. The received signal in a dispersive channel is different from the transmitted signal. The received signal is a complex and non-stationary signal. Typically high frequencies are less disturbed than the lower frequencies (up to 200 Hz), [3], [4]. Because of the non-stationary nature of these signals, the time-frequency signal analysis is suitable tool for analysis.

In this paper, we present a method for decomposition of a signal which was transmitted through a dispersive environ-

ment. The decomposition was done using the dual polynomial Fourier transform.

The paper is organized as follows. In Section II the signal which is received from a dispersive channel in terms of signal processing will be modelled and explained. The traditional and the dual polynomial Fourier transform for analysis and localization of acoustic signals will be presented in Section III. Numerical results and conclusions are given in Sections IV and V, respectively.

II. MODELLING OF THE RECEIVED SIGNALS FROM DISPERSIVE CHANNELS

Let us assume that an underwater acoustic wave is transmitted. We will assume that the transmitter is located at the depth of z_s meters. The receiver is located at the depth of z_r meters. The distance between the transmitter and the receiver is denoted by r . We will consider the model as in [2]. The transfer function can then be written as

$$H(f) = \sum_{m=1}^{+\infty} g_m(z_s)g_m(z_r) \frac{\exp(jk_r(m, f)r)}{\sqrt{k_r(m, f)r}} \quad (1)$$

$$\approx \sum_{m=1}^{+\infty} \frac{A(m, f)}{\sqrt{r}} \exp\{jk_r(m, f)r\}, \quad (2)$$

where g_m are the modal functions of the m th mode and $k_r(m, f)$ are the horizontal wavenumbers. The modal functions are the solutions [2] of

$$\frac{\partial^2 g}{\partial z^2} + \left(\left(\frac{2\pi f}{c} \right)^2 - k_r^2(m, f) \right) g = 0. \quad (3)$$

The sound speed c in the case of underwater communications is $c = 1500$ m/s. It is obvious that the transfer function of a dispersive channel is of a multicomponent structure. The components depend on the wavenumber and their frequencies. The variable

$$A_t(m, f, r) = \frac{A(m, f)}{\sqrt{r}}$$

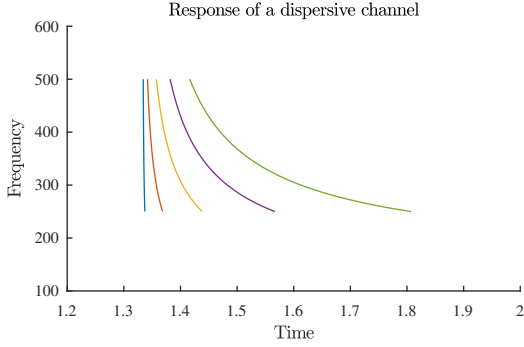


Fig. 1. The time-frequency representation of the impulse response of the five modes

is the attenuation rate. It depends on g_m , $k_r(m, f)$ and r . The response to a monochromatic signal, $\exp(j2\pi f_0 n)$, can be written as

$$s_m(n) \approx D(m, f_0) \exp(j2\pi f_0 n - jk_r(m, f_0)r), \quad (4)$$

where $D(m, f_0)$ is the depth of the receiver. The phase and group velocities are

$$\nu_\varphi(m, f) = \frac{2\pi f}{k_r(m, f)} \quad (5)$$

$$\nu_g(m, f) = 2\pi \frac{\partial f}{\partial k_r(m, f)}. \quad (6)$$

An ideal time-frequency representation of the impulsive response of a dispersive channel environment is shown in Fig. 1.

III. DUAL POLYNOMIAL FOURIER TRANSFORM

Because of the non-stationary nature of the dispersive channels, the most suitable tool is the time-frequency analysis. Several techniques were developed for the localization in the underwater dispersive channels, like those using the phase continuity of the signals, [2]. Here, we will analyze the dispersive channel using the dual polynomial Fourier transform (PFT) of the third order.

The idea behind the traditional PFT is to find the parameters where the signal gives the maximum value. In this way we can extract all components and localize their positions [6], [7]. Let us assume a signal $x(n)$. Its PFT is calculated as

$$X_{\alpha_2, \alpha_3, \dots, \alpha_N}(k) = \sum_n x(n) e^{-j \frac{2\pi}{N} (kn + \alpha_2 n^2 + \alpha_3 n^3 + \dots + \alpha_N n^N)}, \quad (7)$$

where $\alpha_2, \alpha_3, \dots, \alpha_N$ are the parameters.

Assume that the analyzed signal is a polynomial phase signal (PPS) of the N -th order

$$x(n) = A \exp\left(j \sum_{l=1}^N a_l n^l\right).$$

The signal will be highly concentrated in the PFT space of parameters where the maximum of the transform is achieved (where the transform of this signal is the best concentrated), i.e.

$$(\hat{a}_2, \hat{a}_3, \dots, \hat{a}_N) = \arg \max_{(k, \alpha_2, \dots, \alpha_N)} |X_{\alpha_2, \dots, \alpha_N}(k)|. \quad (8)$$

It means that the PFT of a signal $x(n)$ will have the best concentration when $(\alpha_2, \dots, \alpha_N) = (a_2, \dots, a_N)$. Then the goal to estimate $a_2 \approx \hat{a}_2, \dots, a_N \approx \hat{a}_N$ is achieved.

For the signals whose spectral content is concentrated within short time interval, with changes in frequency the dual PFT is more appropriate tool. The discrete dual PFT is defined as:

$$x_{\beta_2, \beta_3, \dots, \beta_N}(n) = \sum_k X(k) e^{j \frac{2\pi}{N} (nk + \beta_2 k^2 + \dots + \beta_N k^N)}. \quad (9)$$

The maximum of dual PFT, i.e., the maximum of the Eq. (9) is achieved when

$$(\hat{b}_1, \hat{b}_2, \dots, \hat{b}_N) = \arg \max_{(n, \beta_2, \dots, \beta_N)} |x_{\beta_2, \dots, \beta_N}(n)|. \quad (10)$$

A local form of the dual PFT, corresponding to the local PFT (known as LPFT) would be obtained using a frequency domain window function $W(k)$. It reads

$$x_{\beta_2, \beta_3, \dots, \beta_N}(n, k) = \sum_l W(l) X(k+l) \times e^{j \frac{2\pi}{N} (nk + \beta_2 k^2 + \dots + \beta_N k^N)}.$$

This kind of transform can be used for analysis of quite complex non-stationary acoustic signals in the dispersive media.

IV. NUMERICAL RESULTS

Let us consider a dispersive channel with $M = 5$ modes. The channel depth is $D = 20$ meters and the distance between the transmitter and receiver is $r = 2000$ meters. The frequency range f is between $f_{min} = 250$ Hz and $f_{max} = 500$ Hz.

We consider the system with impulse response shown in Fig. 1. Its analytic form in the frequency and time domain is

$$H(k) = \sum_{m=1}^M A_m \exp(jk_r(m, f)r) \quad (11)$$

$$h(n) = \sum_{m=1}^M A_m s_m(n), \quad (12)$$

where $s_m(n)$ is defined by Eq. (4). It contains $k_r(m, f)$ which is defined by

$$k_r(m, f) = (2\pi f/c)^2 - ((m-0.5)\pi/D)^2. \quad (13)$$

The impulse response of each mode independently is shown in Fig. 2 (top). The frequency response of each mode separately is shown in Fig. 2 (bottom). Since each mode is in

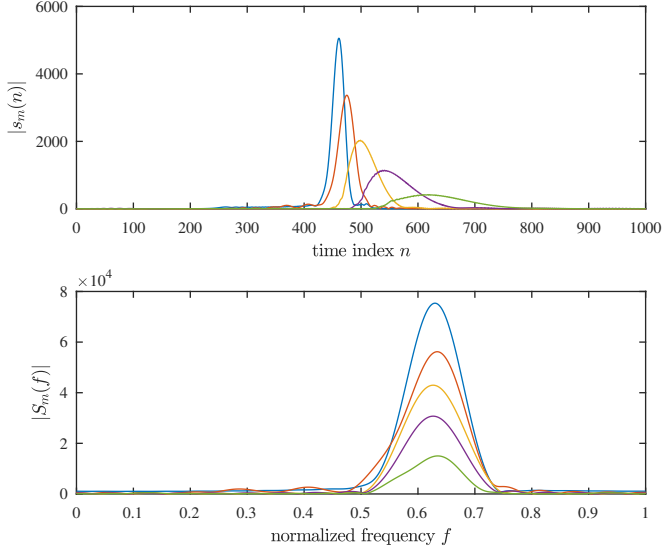


Fig. 2. Impulse and frequency response of the dispersive channel

the range between 250 and 500 Hz, their magnitudes are set on the top of each other. Amplitude is attenuated and it is $A_m = (6 - m)W(f)$ where $W(f)$ is the Hanning window in frequency domain, for the considered example.

We will use linearly frequency modulated signal as the transmitted signal, i.e., as input to the analyzed system

$$u(n) = e^{j\pi an^2}. \quad (14)$$

Received signal can be obtained as the convolution of the transmitted signal and the impulse response of the system

$$x(n) = u(n) * h(n). \quad (15)$$

For the analysis of this signal we have used the dual PFT of the third order as in Eq. (9),

$$x_{\beta_2, \beta_3}(n) = \sum_k X(k) e^{j \frac{2\pi}{N} (nk + \beta_2 \frac{2\pi}{N} k^2 + \beta_3 (\frac{2\pi}{N})^2 k^3)}. \quad (16)$$

Variables $\beta_{2,3}$ are arbitrarily varied in the range of -0.2 to 0.2 and -0.3 to 0.3 , respectively. The received signal (the signal which goes through the dispersive environment) is shown in Fig. 3 (top left).

The optimal parameter values for various modes are detected iteratively. When we find the first set of parameters β_2, β_3 the peak in the dual PFT correspond to single mode. We can remove the peak from the dual PFT and continue to estimate other components. The parameters corresponding to the maximal values for each component/mode are shown in Table I.

The decomposition results are non-stationary single component signals. They are shown in the time-frequency domain

 TABLE I
PARAMETERS CORRESPONDING TO THE MAXIMAL DUAL PFT VALUES FOR EACH MODE

Parameters/Mode	1	2	3	4	5
β_2	0.136	0.150	0.082	0.050	-0.022
β_3	-0.012	-0.003	0.063	0.141	0.282

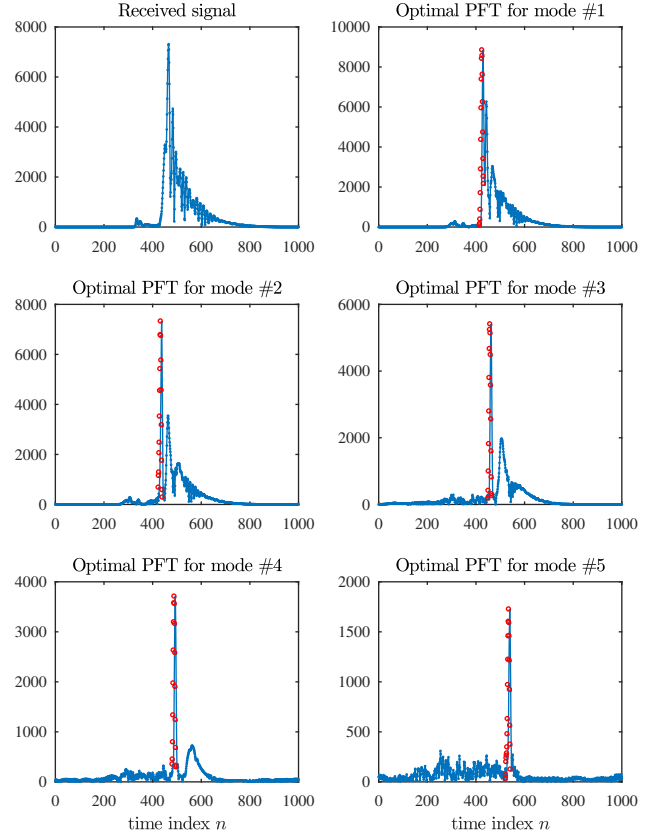


Fig. 3. Decomposition of the modes in time-domain: Received signal (top left); Optimal dual PFT for each mode separately. Samples associated to the current mode are marked with red circles.

using the S-method representation. For the S-method, we need the short-time Fourier transform (STFT) of signal. The STFT is defined as

$$STFT(n, k) = \sum_{m=-N_s/2}^{N_s/2-1} x(m+n)w(m)e^{-j \frac{2\pi}{N_s} mk}, \quad (17)$$

where $w(m)$ is analysis window of length N_s . The S-method is calculated as [6], [8]

$$SM(k, n) = \sum_{p=-L}^L STFT(n, k+p)STFT^*(n, k-p), \quad (18)$$

where $2L + 1$ is window width (in the frequency domain).

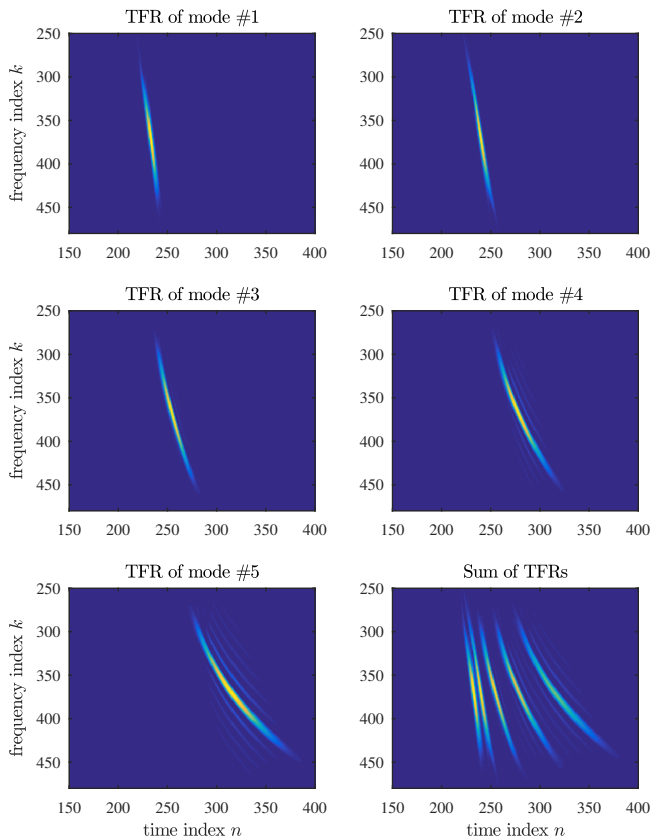


Fig. 4. S-method of the decomposed modes 1, 2, 3, 4 and 5 and sum of the normalized representations of all modes.

Since all analyzed modes (components) are spread over a wide frequency range, we will analyze the signal in the frequency domain using the dual STFT. It is defined by

$$STFT_D(k, n) = \sum_{p=-N_s/2}^{N_s/2-1} X(p-k)W(p)e^{j\frac{2\pi}{N_s}pn}, \quad (19)$$

where $X(k)$ is Fourier transform of the considered component and $W(k)$ is the analysis window.

Therefore, the dual S-method could be then calculated as

$$SM_D(k, n) = \sum_{i=-L}^L STFT_D(k, n+i)STFT_D^*(k, n-i) \quad (20)$$

where $2L+1$ is the time domain window size [6], [8]. In this paper, the S-method with $L=16$ and the Hanning window of size $W_s=512$ for dual STFT calculation is used.

The S-method of the five modes, obtained by decomposition, is shown in first five subplots of Fig. 4. Sum of the normalized representations of the five modes is shown Fig. 4 (bottom right subplot). Sum of the decomposed components and amplitudes of individual components are given in Fig. 5.

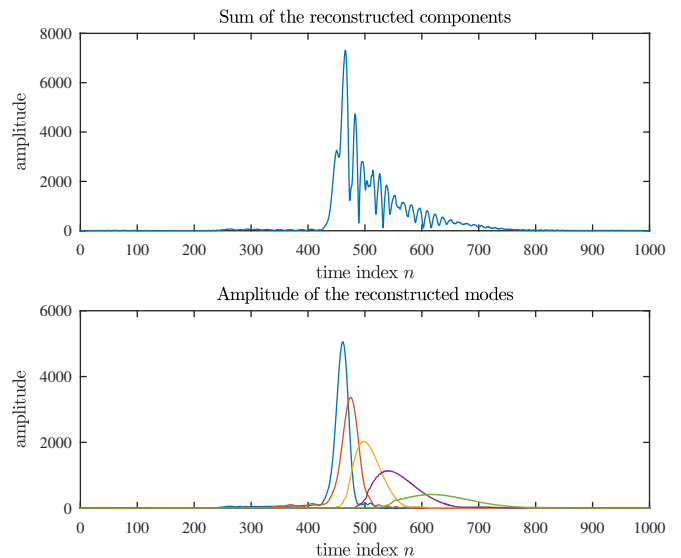


Fig. 5. Sum of the reconstructed modes (top) and amplitudes of individual modes (bottom)

V. CONCLUSIONS

Decomposition of acoustic signals using a dual polynomial Fourier transform is shown in this paper. The signal is considered to be transmitted in a dispersive underwater channel environment. Received signal is decomposed using dual PFT and individual modes are obtained.

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