On the reconstruction of nonsparse time-frequency signals with sparsity constraint from a reduced set of samples

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Abstract

Nonstationary signals, approximately sparse in the joint time-frequency domain, are considered. Reconstruction of such signals with sparsity constraint is analyzed in this paper. The short-time Fourier transform (STFT) and time-frequency representations that can be calculated using the STFT are considered. The formula for error caused by the nonreconstructed coefficients is derived and presented in the form of a theorem. The results are examined statistically on examples.

Keywords: time-frequency analysis, sparsity, reconstruction, compressive sensing

1. Introduction

Nonstationary signals that cover most of the time and frequency domain may be well localized in the joint time-frequency domain. These signals are dense in both time and frequency, considered separately. However, they could be located within much smaller regions in the joint domain using appropriate

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representations [1, 2, 3, 4, 5, 6]. The basic time-frequency representation is the short-time Fourier transform (STFT). It can be easily related to the Wigner distribution and its cross-terms reduced versions [7]. These representations will be considered in this paper. The signals are sparse in the time-frequency domain if the number of nonzero coefficients in this domain is much smaller than the total number of coefficients. For example, a sum of few nonstationary signal components, being well localized in the STFT at each considered time instant, is a sparse signal in this domain.

A signal that is sparse in a certain domain can be reconstructed with fewer samples than the Shannon-Nyquist sampling theorem requires. Compressive sensing is the field dealing with the problem of signal recovery with reduced number of available samples [8, 9, 10, 11, 12, 13, 14]. Reducing the number of available samples in the analysis manifests as a noise, whose properties in the discrete Fourier transform (DFT) domain are studied in [15]. These results will be used to define reconstruction properties in the STFT case. The influence of noise in the two-dimensional DFT is examined in [16]. If a nonsparse signal is reconstructed with a reduced set of available samples then the noise due to the missing samples of nonreconstructed coefficients will be considered as an additive input noise in the reconstructed signal.

In the compressive sensing literature, only the general bounds for the reconstruction error for nonsparse signals (reconstructed with the sparsity assumption) are derived [10, 17, 18]. In this manuscript, we have presented an exact relation for the expected squared error in approximately sparse or nonsparse signals in the time-frequency domain, reconstructed from a reduced set of signal samples, under the sparsity constraint. The error depends on the number of available samples and the assumed sparsity, that is crucial for any compressive sensing based reconstruction. The results are given in the form of a theorem. Theory is illustrated and checked on statistical examples.

The noise in the reconstructed STFT influences the other time-frequency representations that can be calculated using this STFT. The S-method [6, 7] is considered as an example of such signal representations.

The paper is organized as follows. The theoretical background of compressive sensing and time-frequency signal analysis is presented in Section 2. The theorem and formula of nonsparsity influence on the reconstructed signal is presented in Section 3. The numerical results are given in Section 4. The conclusions are presented in Section 5.

2. Theoretical Background

Let us consider a multicomponent signal

$$x(n)=\sum_{l=1}^{C}x_l(n), \tag{1}$$

where components $x_l(n)$ are nonstationary and the total number of components is C. Assume that the signal is sparse in the STFT domain. The STFT of the discrete-time signal is defined as

$$S_N(n,k)=\sum_{m=-N/2}^{N/2-1}x(n+m)w(m)e^{-j\frac{2\pi}{N}mk}, \tag{2}$$

at an instant n and a frequency k. The window function of length N is $w(m)$. The windowed signal $x(n,m)=x(n+m)w(m)$, which is K-sparse in the STFT domain, can be written in the form

$$x(n,m)=\sum_{i=1}^{K}A_i(n)e^{j2\pi mk_i/N} \tag{3}$$

The signal and its STFT in a vector form are

$$\mathbf{S}_N(n)=\mathbf{W}_N\mathbf{H}_N\mathbf{x}(n) \tag{4}$$

$$\mathbf{H}_N\mathbf{x}(n)=\mathbf{W}_N^{-1}\mathbf{S}_N(n), \tag{5}$$

where $\mathbf{S}_N(n)=[S_N(n,0),S_N(n,1),\ldots,S_N(n,N-1)]^T$ is the STFT calculated at time instant n, $\mathbf{x}(n)$ is the original signal (column) vector within the window, $\mathbf{W}_N$ is the DFT matrix of size $N\times N$ with coefficients $W(m,k)=e^{(-j2\pi km/N)}$ and $\mathbf{H}_N$ is a diagonal matrix with the window values at its diagonal. Analysis and reconstruction of the whole signal based on the STFT is straightforward with appropriate overlapping. It is presented in [1, 2, 6].

With the assumption that the signal is sparse in the STFT domain, we can reconstruct it with a reduced number of samples, according to the compressive sensing theory [8, 10, 17, 18, 21].

The number of randomly positioned available samples for the reconstruction is $N_A \ll N$. For a given n the available signal samples are at the positions

$$n + m \in \{n + m_1, n + m_2, ..., n + m_{N_A}\}.$$

The number of unavailable/missing samples is $N_M = N - N_A$. The available samples (measurements) of the windowed signal are then defined as

$$\mathbf{y}_n = [x(n + m_1)w(m_1), \ldots, x(n + m_{N_A})w(m_{N_A})]^T. \tag{6}$$

Note that

$$\mathbf{y}_n = \mathbf{A}\mathbf{S}_N(n),$$

where $\mathbf{A}$ is the measurement matrix. The matrix $\mathbf{A}$ is obtained by keeping the rows of the inverse DFT matrix corresponding to the available samples

$$\mathbf{A} = \begin{bmatrix} \psi_0(m_1) & \psi_1(m_1) & \cdots & \psi_{N-1}(m_1) \\ \psi_0(m_2) & \psi_1(m_2) & \cdots & \psi_{N-1}(m_2) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_0(m_{N_A}) & \psi_1(m_{N_A}) & \cdots & \psi_{N-1}(m_{N_A}) \end{bmatrix} \tag{7}$$

where $\psi_k(m)$ are the inverse DFT matrix coefficients $\psi_k(m) = \frac{1}{N}\exp(j2\pi mk/N)$.

The goal of compressive sensing is to reconstruct the original sparse signal (using its windowed overlapped versions) from the available samples. A general compressive sensing formulation is

$$\min \|\mathbf{S}_N(n)\|_0 \text{ subject to } \mathbf{y}_n = \mathbf{A}\mathbf{S}_N(n).$$

Here we will assume that the initial STFT is calculated using the available samples only

$$S_{N0}(n,k)=\sum_{i=1}^{N_A} x(n+m_i)w(m_i)e^{-j\frac{2\pi}{N}m_i k} \tag{8}$$

$$\mathbf{S}_{N0}(n)=N\mathbf{A}^H\mathbf{y}_n, \tag{9}$$

where superscript H denotes the Hermitian transpose.

The mean and the variance of this STFT, at a given instant n, calculated using the available signal samples only, are [15]

$$E\{S_{N0}(n,k)\}=\sum_{i=1}^{K} N_A A_i(n)\delta(k-k_i) \tag{10}$$

$$\operatorname{var}\{S_{N0}(n,k)\}=N_A\frac{N_M}{N-1}\sum_{i=1}^{K}|A_i(n)|^2\left(1-\delta(k-k_i)\right), \tag{11}$$

where $\delta(k)=1$ only for $k=0$ and $\delta(k)=0$, elsewhere.

In general, time-varying signals are not strictly sparse in the STFT domain. Because of their nature, most of these signals are either approximately sparse or nonsparse. A signal is K-sparse in a transformation domain (in our case, in the STFT domain) if it has only K ($K \ll N$) nonzero coefficients in this domain at positions $k\in\mathbb{K}=\{k_1,k_2,\dots,k_K\}$. Other coefficients, for $k\notin\mathbb{K}$, are zero-valued. A signal is approximately sparse if the coefficients for $k\in\mathbb{K}$ are significantly larger than the coefficients at $k\notin\mathbb{K}$. A signal is not K-sparse if the coefficients for $k\notin\mathbb{K}$ are of the same order as the coefficients at the positions $k\in\mathbb{K}$. If we want to use the compressive sensing based theory for any of these signals the sparsity assumption has to be made. In this paper, we will analyze the error in these signals reconstructed under the K-sparsity assumption in the STFT domain.

Signal reconstruction is done using estimation of the nonzero coefficient positions, based on (8) and calculating the unknown coefficients $A_i(n)$ based on the known signal values $x(n+m_i)$. Various reconstruction algorithms can be used. For the numerical verification of the results we will use an iterative form of the OMP algorithm. The reconstruction algorithm used in this paper is an iterative form of the OMP algorithm, introduced in [19, 20]. Since the introduction of compressive sensing, many reconstruction algorithms have been developed. A review of reconstruction algorithms can be found in [21]. The main reason to use the presented algorithm is the fact that it uses the sparsity assumption in an explicit way (producing K nonzero coefficients in the reconstructed signal). Also, its computational complexity is low. Other algorithms that also exploit the sparsity assumption in an explicit way can be used as well.

In the first step, the position of the maximal STFT coefficient is found as

$$k_1 = \arg\max\{\mathbf{S}_{N0}(n)\}.$$

Matrix $\mathbf{A}_1$ is formed from matrix $\mathbf{A}$ by omitting all columns except the column corresponding to k_1. The first STFT estimate is

$$\mathbf{S}_R(n) = (\mathbf{A}_1^H \mathbf{A}_1)^{-1} \mathbf{A}_1^H \mathbf{y}_n.$$

The signal is reconstructed and subtracted from the original signal at the positions of available samples. The STFT estimate is calculated again with this new signal and its maximum position k_2 is found. A new set $\mathbb{K} = \{k_1, k_2\}$ is formed with corresponding matrix $\mathbf{A}_2$. The new estimate $\mathbf{S}_R(n)$ is calculated and the signal is reconstructed. The procedure is repeated K (assumed sparsity) times, with the final reconstruction

$$\mathbf{S}_R(n) = (\mathbf{A}_K^H \mathbf{A}_K)^{-1} \mathbf{A}_K^H \mathbf{y}_n.$$

The reduced measurement matrix $\mathbf{A}_K$ is obtained from $\mathbf{A}$ by selecting the columns corresponding to K detected nonzero coefficient positions

$$\mathbf{A}_K = \begin{bmatrix} \psi_{k_1}(m_1) & \psi_{k_2}(m_1) & \cdots & \psi_{k_K}(m_2) \\ \psi_{k_1}(m_2) & \psi_{k_2}(m_2) & \cdots & \psi_{k_K}(m_2) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{k_1}(m_{N_A}) & \psi_{k_2}(m_{N_A}) & \cdots & \psi_{k_K}(m_{N_A}) \end{bmatrix}. \tag{12}$$

3. Nonsparsity in Time-Frequency Signal Analysis

The reconstruction error of sparse signals is an important topic in compressive sensing. Its general bounds can be found in [10, 17]. An exact formula for the expected squared reconstruction error, with the STFT as a sparsity domain, is presented by the next theorem.

Theorem: *Consider a signal $x(n)$ with time-varying components. Its STFT values are denoted by $\mathbf{S}_N(n) = [S_N(n,0), S_N(n,1), \ldots, S_N(n,N-1)]^T$. The total number of signal samples within a window is N. Assume that the available signal samples are at N_A random positions, defined by $n+m \in \mathbb{N}_A$, and $N_M = N - N_A$ is the number of unavailable/missing samples. The signal is reconstructed under the assumption as it were K-sparse in the STFT domain (with the assumption that the reconstruction conditions are met for this sparsity). The reconstructed signal with K nonzero STFT coefficients at $k \in \mathbb{K}$ is denoted by $\mathbf{S}_{NR}(n)$. The error in the K reconstructed STFT coefficients is:*

$$\|\mathbf{S}_{NK}(n) - \mathbf{S}_{NR}(n)\|_2^2 = K \frac{N_M}{N_A N} \|\mathbf{S}_N(n) - \mathbf{S}_{NK}(n)\|_2^2 . \quad (13)$$

The K-sparse version of $\mathbf{S}_N(n)$ is denoted by $\mathbf{S}_{NK}(n)$. The elements of vector $\mathbf{S}_{NK}(n)$ are $S_{NK}(n,k) = S_N(n,k)$ for $k \in \mathbb{K}$, and $S_{NK}(n,k) = 0$ for $k \notin \mathbb{K}$. The reconstructed STFT $\mathbf{S}_{NR}(n)$ is formed in the same way, with coefficients for $k \in \mathbb{K}$ being obtained by the reconstruction procedure and the remaining coefficients, for $k \notin \mathbb{K}$ being set to 0.

Notation $\|\mathbf{S}_N(n)\|_2^2$ is used for the expected value of the squared norm-two, i.e. $\|\mathbf{S}_N(n)\|_2^2 = E\{\sum_k |S_N(n,k)|^2\}$.

Proof: Assume that the compressive sensing conditions for the reconstruction are satisfied for the assumed sparsity and the number of available samples [17]. Then we can reconstruct K coefficients $(A_1(n), A_2(n), ..., A_K(n))$ using, for example, the iterative OMP procedure explained at the end of Section 2. The result is $\mathbf{S}_R(n)$ with K reconstructed coefficients. The remaining (nonreconstructed) $N-K$ signal coefficients with amplitudes $(A_{K+1}(n), A_{K+2}(n), ..., A_N(n))$ produce noise in these K reconstructed coefficients. As defined in (11), the noise variance from one nonreconstructed coefficient is

$$|A_i(n)|^2 \, N_A N_M/(N-1). \quad (14)$$

The signal amplitudes in $\mathbf{S}_{N0}(n)$ are proportional to N_A. The amplitudes are recovered to their original values, proportional to N, the same as if all samples were available. The scaling factor is then N/N_A for the reconstructed coefficients. Consequently, the scaling factor for the noise variance in the reconstructed coefficients is $(N/N_A)^2$. That is, the noise variance of a reconstructed coefficient caused by a nonreconstructed coefficient is

$$|A_i(n)|^2 \frac{N^2}{N_A^2} \frac{N_A N_M}{N-1} \cong |A_i(n)|^2 N \frac{N_M}{N_A}. \tag{15}$$

The white noise energy in the reconstructed coefficients of $\mathbf{S}_R(n)$ will be K times larger than the variance in one reconstructed coefficient. The total noise energy caused by the nonreconstructed coefficients ($A_{K+1}(n)$,$A_{K+2}(n)$,..., $A_N(n)$), in K reconstructed coefficients is

$$\|\mathbf{S}_{NR}(n) - \mathbf{S}_{NK}(n)\|_2^2 = KN \frac{N_M}{N_A} \sum_{i=K+1}^{N} |A_i(n)|^2 , \tag{16}$$

where $\mathbf{S}_{NR}(n)$ is obtained from $\mathbf{S}_R(n)$, as defined in the theorem. Energy of the STFT, corresponding to the nonreconstructed coefficients only, can be written as

$$\|\mathbf{S}_N(n) - \mathbf{S}_{NK}(n)\|_2^2 = \sum_{i=K+1}^{N} |NA_i(n)|^2 . \tag{17}$$

From (16) and (17) follows

$$\|\mathbf{S}_{NR}(n) - \mathbf{S}_{NK}(n)\|_2^2 = K \frac{N_M}{N_A N} \|\mathbf{S}_N(n) - \mathbf{S}_{NK}(n)\|_2^2 .$$

In the case when the original signal is K-sparse, i.e. $\mathbf{S}_N(n) = \mathbf{S}_{NK}(n)$, or when all samples are available, i.e. $N_A = N$ and $N_M = 0$, there is no error

$$\|\mathbf{S}_{NR}(n) - \mathbf{S}_{NK}(n)\|_2^2 = 0. \tag{18}$$

4. Numerical Results

Consider a combination of two linear frequency modulated signal components

$$
\begin{aligned}
x(n) = 1.5\exp\left(j192\pi n/N + j48\pi n^2/N^2 + j\varphi_1\right) \\
+\exp\left(j48\pi n/N + j16\pi n^2/N^2 + j\varphi_2\right) \qquad (19)
\end{aligned}
$$

for $0 \leq n \leq 1280$. The STFT is calculated using a Hamming window of the length $N = 256$ with a step in time of 32. Note that the signal is not sparse in the DFT domain since its components sweep almost the whole frequency range. Various numbers of randomly positioned available samples N_A have been considered. The phases φ_1 and φ_2 are random between 0 and 2π. In the reconstruction a K-sparse signal in the STFT domain is assumed, with various $K = 4, 8, 16, 32, 64$. Illustration of the reconstructed signal STFTs for $N_A = 192$ randomly positioned available samples and $K = 8, 16, 32$ is shown in Fig. 1.

The statistical error E_s, and the derived (theoretical) error E_t, in the reconstructed coefficients, are calculated as

$$
E_s = 10\log\left(\|\mathbf{S}_{NK}(n){-}\mathbf{S}_{NR}(n)\|_2^2\right) \qquad (20)
$$

$$
E_t = 10\log\left(K\frac{N_M}{N_A N}\|\mathbf{S}_{N}(n){-}\mathbf{S}_{NK}(n)\|_2^2\right) \qquad (21)
$$

where $\mathbf{S}_N(n)$ is the original STFT of the signal, $\mathbf{S}_{NK}(n)$ is equal to $\mathbf{S}_N(n)$ for its K reconstructed coefficients and $\mathbf{S}_{NR}(n)$ is the reconstructed STFT with K nonzero values. The total reconstruction errors can be calculated as

$$
E_s^{tot} = 10\log\left(\|\mathbf{S}_{N}(n){-}\mathbf{S}_{NR}(n)\|_2^2\right) \qquad (22)
$$

$$
E_t^{tot} = 10\log\left(\left(K\frac{N_M}{N_A N}+1\right)\|\mathbf{S}_{N}(n){-}\mathbf{S}_{NK}(n)\|_2^2\right). \qquad (23)
$$

The reconstruction error values averaged over 100 realizations, calculated using (20), (21), (22), and (23), are shown in Table 1.

The total reconstruction error as a function of the number of available samples is presented in Fig. 2. The number of available samples is varied from 25

Table 1: The error in the reconstructed coefficients and the total error (in dB) for $N_A = 2N/3$ and $N_A = 3N/4$, and various assumed sparsity levels K.

		Error in the reconstructed coefficients		Total error	
N_A	K	Statistics	Theory	Statistics	Theory
$2N/3$	4	-21.4	-21.5	-0.4	-0.4
$2N/3$	8	-19.8	-20.5	-2.3	-2.3
$2N/3$	16	-23.0	-23.5	-8.3	-8.3
$2N/3$	32	-40.9	-41.8	-29.4	-29.5
$2N/3$	64	-53.5	-54.8	-45.1	-45.2
$3N/4$	4	-22.8	-23.3	-0.4	-0.4
$3N/4$	8	-21.9	-22.3	-2.4	-2.4
$3N/4$	16	-25.0	-25.3	-8.4	-8.4
$3N/4$	32	-42.6	-43.6	-29.5	-29.6
$3N/4$	64	-54.4	-56.6	-45.2	-45.5

to 250. We assumed the sparsity values $K = 8, 16, 24,$ and 32. The theoretical results are presented with solid lines and the statistical results are given by dots. Filled marks indicate the region when the reconstruction is possible with a high probability, $N_A \geq 4K$, [17]. Note that any exact recovery can be expected only if $N_A > 2K$.

The results can be easily applied to other time frequency-representations whose realization can be implemented using the STFT. For example, the pseudo Wigner distribution can be calculated as

$$WD(n,k) = \sum_{i=-N/2}^{N/2} S_N(n,k+i)S_N^*(n,k-i). \tag{24}$$

Its cross-terms free (reduced) version, is the S-method

$$SM(n,k) = \sum_{i=-L}^{L} S_N(n,k+i)S_N^*(n,k-i), \tag{25}$$

where L should be sufficiently large to include auto-terms, but not too large to produce cross-terms [7]. The S-method calculated from the reconstructed STFT is shown in Fig. 3.

The noise analysis in these distribution can be easily done based on the derived relations for the noise in the STFT and the results in [22]. Sparse reconstruction of bilinear time-frequency distributions is reviewed in [23].

5. Conclusions

The influence of nonsparsity to the reconstruction of signals that are approximately sparse in the time-frequency domain is analyzed in this paper. The relation for the reconstruction error is derived. The reconstruction results are statistically checked. Statistical results are in high agreement with the derived theoretical results.

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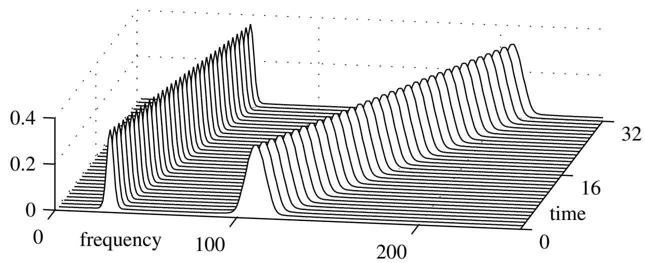
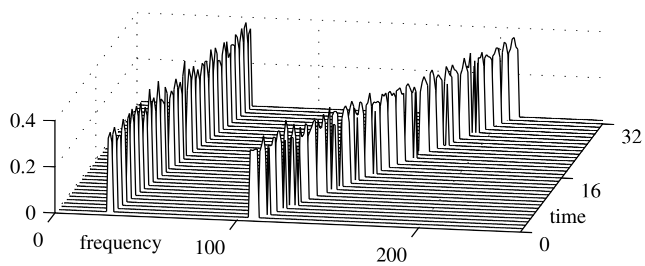
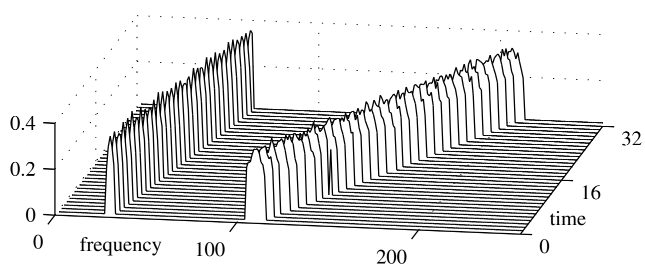


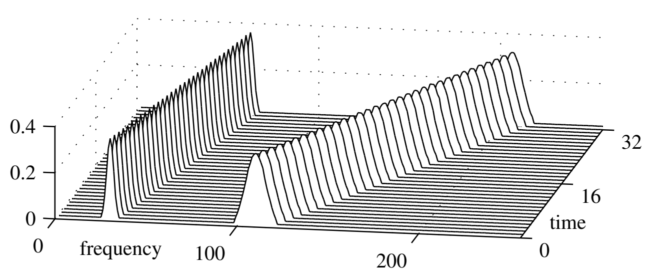
(a) Original STFT



(b) Reconstructed STFT with $K = 8$



(c) Reconstructed STFT with $K = 16$



(d) Reconstructed STFT with $K = 32$

Figure 1: Reconstructed STFT with varying assumed sparsity K with $N_A = 2N/3$ available samples.

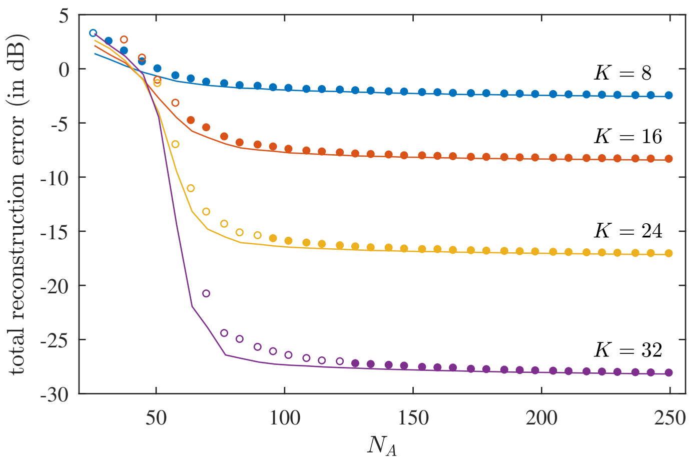
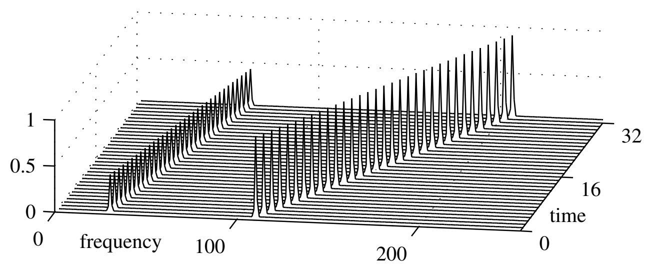
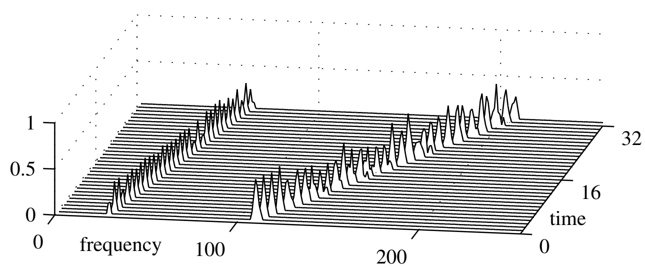


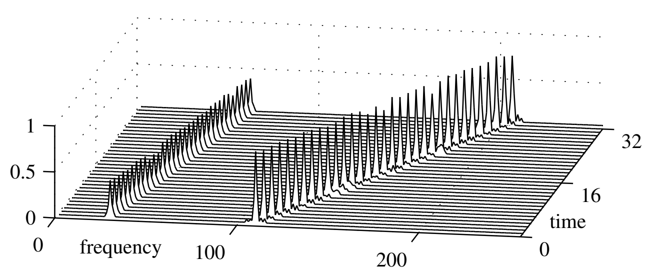
Figure 2: Total reconstruction error as a function of the number of available samples N_A for various assumed sparsity K. Theoretical results are presented by lines and the statistical with dots. Dots for $N_A > 4K$, when the reconstruction is possible with a high probability, are filled.



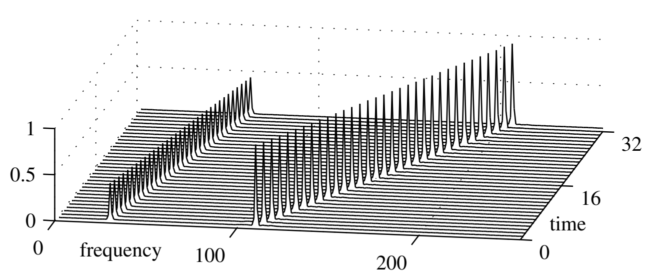
(a) Original SM



(b) The SM calculated from the reconstructed STFT with $K = 8$



(c) The SM calculated from the reconstructed STFT with $K = 16$



(d) The SM calculated from the reconstructed STFT with $K = 32$

Figure 3: The S-method (SM) calculated from the reconstructed STFT with varying assumed sparsity K and $N_A = 2N/3$ available samples.