Analysis of Electrical Circuits including Fractional Order Elements

Marko Č. Bošković
Chair of Autom Control
University of Novi Sad
Faculty of Tech Sciences
Novi Sad, Serbia
marko.boskovic@uns.ac.rs

Tomislav B. Šekara
Signals and Systems Dept.
University of Belgrade
School of El Engineering
Belgrade; Serbia
tomi@etf.rs

Budimir Lutovac
Miloš Daković
University of Montenegro
Faculty of El Engineering
Podgorica, Montenegro
budo@ac.me,milos@ac.me

Petar D. Mandić
Mihailo P. Lazarević
University of Belgrade
Faculty of Mech Enginering
Belgrade,Serbia
pmandic@mas.bg.ac.rs,
mlazarevic@mas.bg.ac.rs

Abstract—**This paper deals with the analysis of electrical circuits with classical one-port elements including two novel defined one-port fractional order elements: fractional-order resistive-capacitive RC-α and fractional-order inductive RL-α element. The definitions and analytical relations between current, voltage and power of introduced fractional elements are provided. An example of fractional element realization via ladder electrical circuit composed of classical resistors, capacitors and/or inductors is presented. Several examples are analyzed to illustrate the behavior of electrical circuit with fractional order elements for different values of fractional order α including differentiator/integrator circuits as well as complex circuits without accumulated energy.**

Keywords-electrical circuit; fractional element; resistive-capacitive; resistive-inductive

I. INTRODUCTION

Electrical systems are commonly used for generation, transmission, distribution and processing of electrical signals. They include communication systems, control systems, power systems, signal-processing systems of different complexity. Analysis and synthesis of these systems is in principle based on application of electrical circuit theory. Classical circuit theory is based on application of integer order element models. However, in the last decades, an application of fractional calculus in electrical and other engineering areas is in a large increase since fractional-order models are commonly more accurate than integer-order ones [1,2]. Theoretical aspects of fractional calculus can be found in [3-5]. Addition to the above, integration and differentiation operations of fractional order are able to model non-local and distributed effects. Fractional order systems are successfully applicable in natural and technical phenomena for modeling various processes exhibiting memory and/or stationary effects. Analysis of a circuit with RC and RL models of fractional-order is recently reported in [6-9] and examples of analogue realizations of fractional elements are elaborated in [10-12]. In addition in [13,14] memristive systems of fractional order are elaborated, while papers [15,16] deal with the modeling of transmission lines based on time-fractional telegrapher's equations. An application of fractional models is also found in power electronics for obtaining more accurate converter models [17,18], as well as in fractional order control engineering [19-22] etc.

The aim of the paper is to give a deeper insight into an analysis of electrical circuits with fractional order elements. Two definitions of fractional order elements are provided: resistive-capacitive RC-α and resistive-inductive RL-α element, reported originally in [23]. Definitions of elements are in accordance to a phasor diagrams in terms of correlated time shift between terminal voltage and current. Analogue realization of fractional element is presented for α =0.5, and the same approach is applicable for all the other $\alpha \in (0,1)$. Several examples of electrical circuits with fractional elements are analyzed in Laplace domain and their time responses are determined via numerical inversion of the Laplace transform.

II. FUNDAMENTALS OF FRACTIONAL CALCULUS

Fractional differ-integrator is fractional calculus operator and it arises from generalization of classical differentiation and integration operators. One such operator has transfer function s^α where s is Laplace variable and α is arbitrary real number. For positive α, differ-integrator is a generalization of classical integer order derivative, while for negative α is a generalization of repeated, or n-fold, integral.

In literature can be found many definitions of fractional transformations (fractional derivatives and integrals). The most common definitions are Grunwald-Letnikov and Riemann-Liouville [3-5]: Let mention also Caputo derivative as a variation of Riemann-Liouville differential operator which is also frequently used. The left Riemann-Liouville fractional differ-integral defined as

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (1)$$

where $(n-1<\alpha<n)$ and $\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dt, z\in\mathbb{C}$ is Euler's gamma function. It should be noted that Riemann-Liouville fractional integral can be defined for arbitrary complex order α while here defined only for real order operations since the focus of the paper is modeling of fractional order elements. Important case is $\alpha \in (0,1)$, when definition (1) is reduced to

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau \qquad (2) $$

where a is terminal point on interval. Some important property of Riemann-Liouville derivative is that it reduces to classical, integer order derivative for integer values of order α, and its derivative of constant is non-zero. When fractional derivative is defined common choice for further analysis is Laplace transform. It is usually used to describe the fractional operations in the complex domain and solving fractional integro-differential equations. Besides, it is a starting point for frequency domain analysis. Laplace transform of Riemann-Liouville derivative ${}_0D_t^{\alpha} f(t)$ is

$$ L\left\{ {}_0D_t^{\alpha} f(t) \right\} = s^{\alpha} F(s), \qquad (3) $$

$$ L\left\{ {}_0D_t^{\alpha} f(t) \right\} = s^{\alpha} F(s) - \sum_{k=0}^{n-1} s^k \, {}_0D_t^{\alpha-k-1} f(0). \qquad (4) $$

The expression (4) contains the initial conditions which should be specified when solving fractional differential equations.

III. FRACTIONAL ORDER ONE-PORT ELEMENTS

One-port fractional order elements according to the character of impedance can be classified into two groups: resistive-capacitive and resistive-inductive fractional elements of order α. These elements are noted as RC-α and RL-α elements, and their symbols shown in Fig. 1 are previously introduced in [23].

Figure 1. One-port resistive-capacitive and resistive-inductive fractional element of order α

Let us find mathematical relationships between voltage and current at terminals of RC-α and RL-α fractional order elements. For the purpose of circuit analysis, it is necessary to reference current in the fractional elements to the terminal voltage. Fig. 2 adopts direction of current i in the direction of the voltage drop across the fractional element.

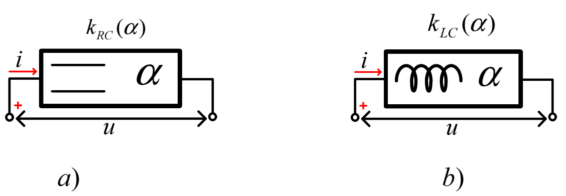


Figure 2. One-port fractional elements with reference directions of current i and voltage u: a) resistive-capacitive RC-α; b)resistive-inductive RL-α

Resistive-capacitive element RC-α in Fig. 2.a is described with equation (1)

$$ i(t) = k_{RC}(\alpha) \left[{}_0D_t^{\alpha} u(t) \right], \qquad (5) $$

where $k_{RC}(\alpha) = (RC)^{\alpha} / R$. From here it is obvious that for α=0 fractional element is pure resistive, while for α=1 is pure capacitive. By applying Laplace transform in (1), impedance of RC-α element in complex domain is obtained as

$$ Z_{RC-\alpha}(s) = \frac{1}{k_{RC}(\alpha) s^{\alpha}} \qquad (6) $$

If a terminal voltage on RC-α element is $u(t) = U\sqrt{2}\cos(\omega t)$ then corresponding current is $i(t) = k_{RC-\alpha} \omega^{\alpha} U \sqrt{2} \cos(\omega t + \alpha\pi/2)$ (Fig. 2.a). From here it is concluded that voltage always lags the current for each $\alpha \in (0,1)$ and justifies the name resistive-capacitive fractional element.

In similar manner, resistive-inductive element RL-α in Fig. 2.b is described with equation (3)

$$ u(t) = k_{RL}(\alpha) \left[{}_0D_t^{\alpha} i(t) \right], \qquad (7) $$

where $k_{RL}(\alpha) = R(L/R)^{\alpha}$ and corresponding impedance in complex domain is

$$ Z_{RL-\alpha}(s) = k_{RL}(\alpha) s^{\alpha}. \qquad (8) $$

As it can be seen from (8), for α=0 fractional element is pure resistive, while for α=1 is pure inductive. If a terminal voltage on RL-α element is $u(t) = U\sqrt{2}\cos(\omega t)$ then corresponding current is $i(t) = \frac{U\sqrt{2}}{k_{RL-\alpha}\omega^{\alpha}} \cos(\omega t - \alpha\pi/2)$ (Fig. 2.b). From here it is concluded that current always lags the voltage for each $\alpha \in (0,1)$ and justifies the name resistive-inductive fractional element.

A. Power of fractional order elements RC-α and RL-α

Instantaneous electrical power on one-port element with respect to the reference directions of voltage and current (Fig. 2.) is defined as

$$ p(t) = u(t) i(t). \qquad (9) $$

On the basis of relation (9), instantaneous power on resistive-capacitive fractional element (Fig. 2.a) is

$$ p_{RC-\alpha}(t) = k_{RC}(\alpha) u(t) \left[{}_0D_t^{\alpha} u(t) \right]. \qquad (10) $$

Corresponding complex power $\underline{S}$ of RC-α element is determined with relation

$$ \underline{S} = \underline{U}\,\underline{I}^* = I^2 \underline{Z}_{RC-\alpha}(j\omega) = \frac{I^2}{\underline{Z}^*_{RC-\alpha}(j\omega)} = P + jQ \qquad (11) $$

where P and Q are active and reactive power, respectively,

$$ P = U^2 k_{RC}(\alpha) \omega^{\alpha} \cos\left(\frac{\alpha\pi}{2}\right), Q = -U^2 k_{RC}(\alpha) \omega^{\alpha} \sin\left(\frac{\alpha\pi}{2}\right), \qquad (12) $$

From (12) it can be calculated apparent power $S=\sqrt{P^2+Q^2}=U^2 k_{RC}(\alpha)\omega^{\alpha}$. It can be concluded from (12) that power factor of resistive-capacitive fractional element is $k=\cos(\alpha\pi/2)$, and complex power is a function of $k_{RC}(\alpha)$ and order α of fractional element.

Analogously to prior, instantaneous power on resistive-inductive fractional element (Fig. 2.b) is

$$p_{RL-\alpha}(t)=k_{RL}(\alpha)i(t)\left[{}_0D_t^{\alpha}i(t)\right]. \tag{13}$$

and corresponding complex power with respect to reference voltage and current directions is

$$\underline{S}=\underline{U}\,\underline{I}^{*}=I^2\,\underline{Z}_{RL-\alpha}(\mathrm{j}\omega)=\frac{I^2}{\underline{Z}^{*}_{RL-\alpha}(\mathrm{j}\omega)}=P+\mathrm{j}Q \tag{14}$$

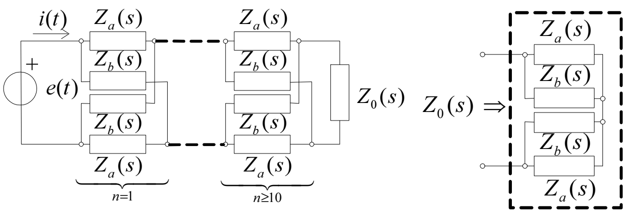
where active and reactive power are given with,

$$P=\frac{U^2}{k_{RL}(\alpha)\omega^{\alpha}}\cos\left(\frac{\alpha\pi}{2}\right), Q=\frac{U^2}{k_{RL}(\alpha)\omega^{\alpha}}\sin\left(\frac{\alpha\pi}{2}\right), \tag{15}$$

From (15) apparent power is $S=\sqrt{P^2+Q^2}=\dfrac{U^2}{k_{RL}(\alpha)\omega^{\alpha}}$. Power factor of resistive-inductive fractional element is $k=\cos(\alpha\pi/2)$, while complex power is is a function of $k_{RL}(\alpha)$ and order α of fractional element.

B. Analogue realization of fractional integrator/differentiator

The basic idea to achieve analogue realization of fractional integrator/differentiator is to appropriately connect basic elements: resistors, capacitors and inductors. An example of analogue realization of fractional integrator of order α=1/2 is presented in Fig. 3.a. Input impedance $Z(s)=E(s)/I(s)$ of the circuit in Fig. 3.a for $n\to\infty$ is $Z(s)=\lim_{n\to\infty} Z_{eq}(s)=\sqrt{Z_a(s)Z_b(s)}$ which is practically fulfilled for number of elementary cells $n\geq 10$.



a) *b*)

Figure 3. An example of analogue realization of fractional integrator with ladder structure [23]

For example, if $Z_1(s)=\dfrac{1}{sC}$ and $Z_2(s)=R$ it is obtained fractional integrator of order α=0.5 with RC-based ladder structure ie. $Z(s)=\sqrt{\dfrac{R}{C}}\dfrac{1}{s^{0.5}}$. By numerical simulations it can be concluded that suitable structure of impedance $Z_0(s)$ is basic cell of ladder structure in Fig. 3.*a*, where $Z_a(s)=Z_1(s)/\lambda$ and $Z_b(s)=\lambda Z_2(s)$, $\lambda>0$. In [23] it is concluded that parameter $\lambda\in(0,1]$, affects exclusively the lower cut-off frequency while $\lambda>1$ may be used to regulate upper cut-off frequency.

In similar way it is possible to obtain any realization of fractional integrator $s^{-\alpha}, \alpha\in\mathbb{Q}$ and $\alpha\in(0,1)$ with suitable choice of cells number $m\times n$ in structure in Fig. 3. Parameters *m* and *n* are selected to have large operating frequency range in sense of expected amplitude and phase characteristics. Equivalent procedure can be used to achieve R-C ladder based realization of fractional differentiator $s^{\alpha}, \alpha\in\mathbb{Q}$ and $\alpha\in(0,1)$. Fractional integrators and differentiators may be both realized with R-L structure since there is duality of R-L and R-C structures in terms of fractional transformations.

IV. ANALYSIS OF CIRCUITS WITH FRACTIONAL ELEMENTS

A. Example 1. Electrical circuit with resistive-capacitive element

A simple electric circuit with RC-α element is shown in Fig. 4. Let us find instantaneous value of voltage $u(t)$ at terminals of RC-α fractional element with excitation $e(t)=2h(t)$ [V], where $h(t)$ is a Heaviside function. Values of circuit elements are known: $R=R_1=100\ \Omega$ and $C=12\ \mu$F. Without loss of generality it is assumed that electrical circuit is without accumulated energy for $t<0$.

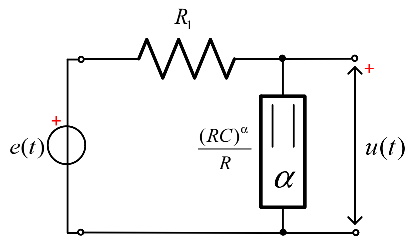


Figure 4. Electrical circuit with resistive-capacitive RC-α element

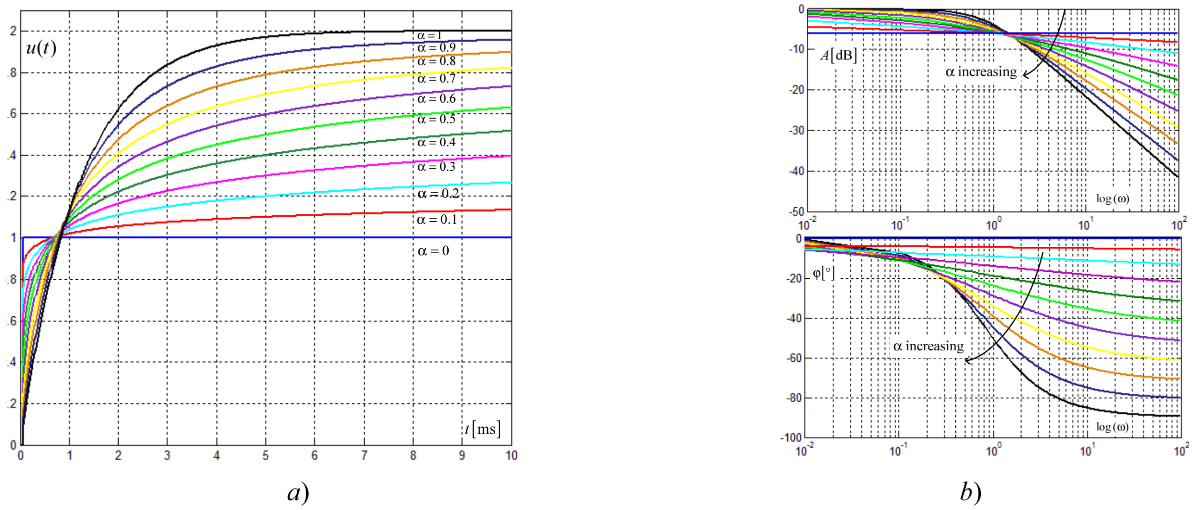


Figure 5. *a*) Instantaneous value of voltage on resistive-capacitive element circuit in Fig. 4.; *b*) Amplitude and phase frequency characteristics of the electrical system in Fig. 4. for different values of parameter $\alpha = 0.1k$, $k = \overline{0,10}$

Instantaneous output voltage (Fig. 5.a) can be found by numerical inversion of Laplace transform of $U(s)=E(s)G(s)$

$$u(t) = L^{-1}\{U(s)\} = L^{-1}\left\{\frac{2}{s}\frac{1}{R_1\dfrac{(RCs)^\alpha}{R}+1}\right\} \tag{16}$$

where $G(s)$ is transfer function of the circuit in Fig. 2a.

Amplitude characteristic of the electrical systems in Fig. 2. which transfer function is $G(s) = \dfrac{1}{(1.2s)^\alpha + 1}$ is

$$A(\omega) = -20\log\sqrt{\left[(1.2\omega)^\alpha \cos\phi(n)+1\right]^2 + (1.2\omega)^{2\alpha}\sin^2\phi(n)}$$

where $\phi(n) = \alpha\pi/2 + \alpha 2n\pi$, while phase characteristic is

$$\varphi(\omega) = -\arctan\left(\frac{(1.2\omega)^\alpha \sin\phi(n)}{(1.2\omega)^\alpha \cos\phi(n)+1}\right).$$

Corresponding amplitude/phase plots are shown in Fig. 5b. Let we note that ambiguity $n\in\mathbb{N}_0$ of amplitude and phase characteristic is solved with selection of a minimum-phase system in Fig. 5.b.

B. *Example 2. Electrical circuit with resistive-capacitive element*

Let us find instantaneous values of the voltage on terminals of resistor R_1 assuming component's values are the same as in Example 1, Heaviside excitation $e(t)=2h(t)$ and circuit without accumulated energy for t<0.

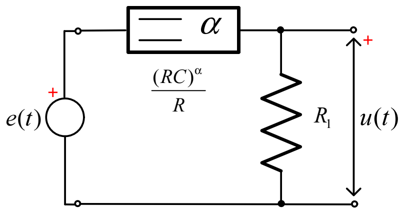


Figure 6. Electrical circuit with resistive-capacitive RC-α element

Analogously to prior example, output voltage (Fig. 7.*a*) is

$$u(t) = L^{-1}\left\{\frac{2}{s}R_1\frac{\dfrac{(RCs)^\alpha}{R}}{R_1\dfrac{(RCs)^\alpha}{R}+1}\right\}, \tag{17}$$

Transfer function of the electrical system in Fig. 6 is $G(s) = \dfrac{(1.2s)^\alpha}{(1.2s)^\alpha + 1}$, and corresponding amplitude/phase plots are shown in Fig. 7.*b*.

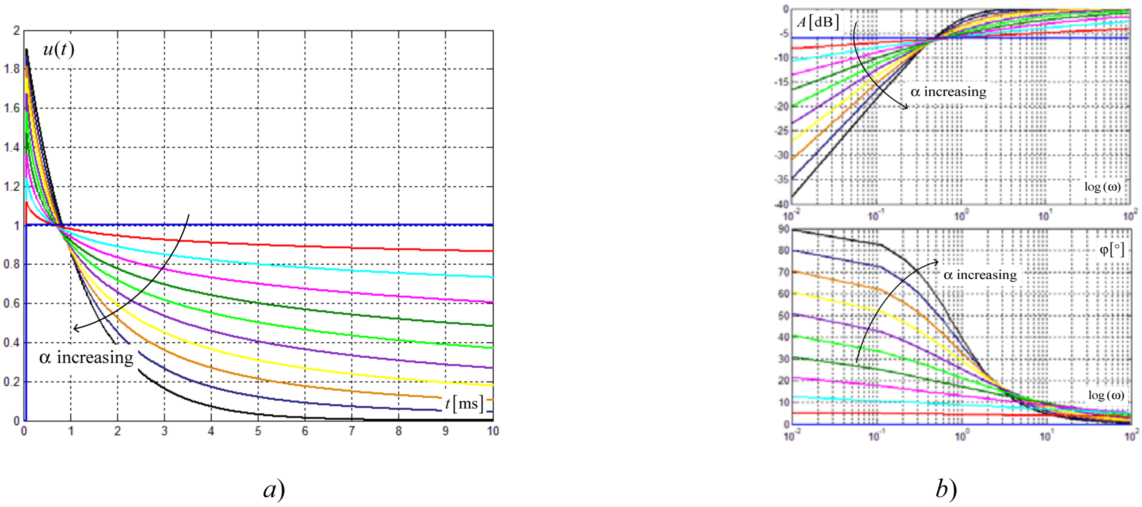


Figure 7. *a*) Instantaneous value of voltage on resistor R_1 in circuit in Fig. 6.; *b*) Amplitude and phase frequency characteristics of the electrical system in Fig. 6. for different values of parameter $\alpha = 0.1k,\ k = \overline{0,10}$

C. Example 3. Complex electric circuit with fractional elements RC-α and RL-α

More complex circuit with resistors and two fractional elements RC-α and RL-α is shown in Fig. 8. Let excitation in is in the form $e(t)=E$ for $t\geq 0_+$ and $e(t)=0$ $t<0_-$, where E=15 V and known values of circuit elements are R=10 Ω, R_1=R_2=100 Ω with R_1C_1=30 ms, L_1/R=1/30 ms. Assuming there is no accumulated energy at t<0, for an excitation the circuit can be solved with some of classical methods: Kirchhoff's voltage and current laws, node-voltage analysis, loop-current method or e.g. by applying Thévenin's, Norton's theorems etc. [24]. Here, analysis of circuit is performed in Laplace domain applying equations (6) and (8) for fractional elements.

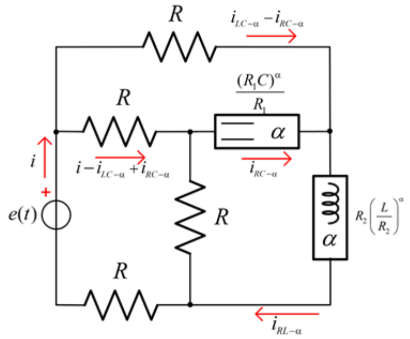


Figure 8. Electrical circuit with both classical and fractional order elements

When Laplace transform of e.g. $I_{RL\text{-}\alpha}(t)=L\{i_{RL\text{-}\alpha}(t)\}$ is calculated, $i_{RL\text{-}\alpha}(t)$ is defined with numerical inversion as it was shown in (16) and illustrated with Fig. 9. Analogously, all the other currents/voltages are unambiguously determined.

$$i_{RL-\alpha}(t) = L^{-1}\left\{\frac{3(30s)^{\alpha}+10}{s(50(s)^{2\alpha}+3(30s)^{\alpha}+300(s/30)^{\alpha}+50)}\right\} \quad (18)$$

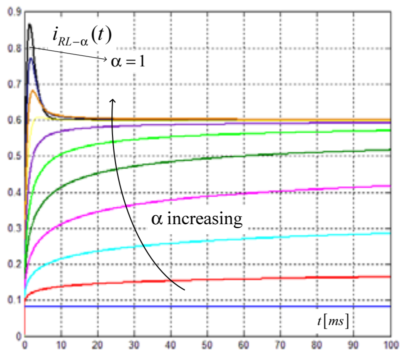


Figure 9. Instantaneous value of the current through resistive-inductive fractional order element in Fig. 8. for different values of of parameter $\alpha = 0.1k,\ k = \overline{0,10}$

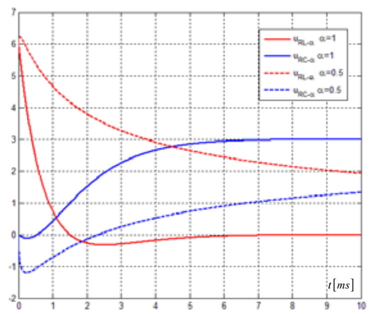


Figure 10. Instantaneous value of voltages $u_{RL-\alpha}(t)$ and $u_{RC-\alpha}(t)$ in Fig. 8. for two values of parameter $\alpha = 1$ (thick lines) and $\alpha = 0.5$ (dashed lines)

Similarly, if excitation in Fig. 8. is sinusoidal, ie. $e(t)=E\sin(50t)$, the analysis can be performed with the same approach in Laplace domain. Instantaneous value of the current $i_{RL\text{-}\alpha}(t)$ in sinusoidal regime is shown in Fig. 11 for $\alpha = 0.2k,\ k = \overline{0,5}$.

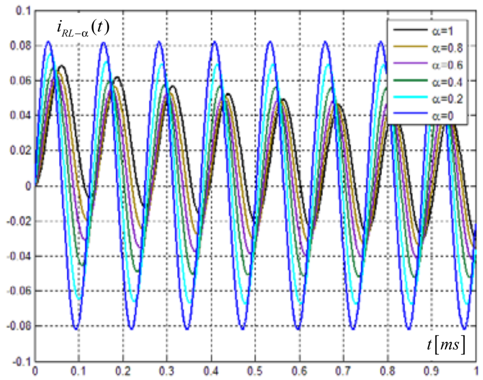


Figure 11. Instantaneous value of the current $i_{RL\text{-}\alpha}(t)$ through RL-α fractional element in Fig. 8. for different values of of parameter $\alpha = 0.2k,\ k = \overline{0,5}$

Let notice from Figs 5 and 7 that fractional-order systems, in general can have the arbitrary slope of amplitude characteristic while for integer order-ones slope is an integer multiple of 20 dB/decade. Presented analysis of the electrical circuits including fractional order elements in Laplace domain is reduced to classical circuit solving techniques. Figs. 9-11 indicate on different behavior of electrical system for various fractional order α. It is obvious in these Figs. that for α=0 both fractional elements are reduced to pure resistive, while for α=1 they are reduced to classical capacitor and inductor. Digital realization of fractional order systems is supported with adequate discretization method as was elaborated in [25].

V. Conclusions

This paper provides an insight in an application of fractional calculus in a circuit theory from the aspect of their analysis in Laplace domain. Two definitions with possible analogue realizations of two fundamental fractional elements are introduced and several examples are done to illustrate behavior of electrical system for different order of fractional elements and justify a reason for more precise modeling of electrical components.

Acknowledgment

Authors gratefully acknowledge the support of Ministry of Edu, Science and Tech Development of the R of Serbia under the Project TR 33020 (T.B.Š) as well as supported by Projects TR 35006, III 41006 (M.P.L.) and TR 33047 (P.D.M).

References

[1] J.Tenreiro Machado, V. Kiryakova, F. Mainardi, Recent history of fractional calculus, Communications in Nonlinear Science and Numerical Simulation, vol. 16, iss. 3, pp.1440-1153, 2011.

[2] In: Lazarević MP (editor), Advanced topics on applications of fractional calculus on control problems, systems tability and modeling,WSEAS; p.202, ISBN:978-960-474-348-3,ID9028., 2014.

[3] K.B. Oldham, J. Spanier, Fractional calculus: theory and applications, differentiation and integration to arbitrary order. Academic Press, NewYork, 1974.

[4] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006

[5] I. Podlubny, Fractional Differential Equations. Academic Press, San Diego, 1999.

[6] A.G. Radwan. K.N. Salama, "Passive and Active Elements Using Fractional $L_\beta C_\alpha$ Circuit", IEEE Trans on Circuits and Systems-I, vol. 58, no. 10, pp. 2388-2397, Oct. 2011.

[7] A.G. Radwan, K.N. Salama, "Fractional-order RC and RL circuits. Circuits Syst. Signal Process, vol. 31, pp. 1901–1915, 2012.

[8] T.B. Šekara, M.S. Stanković, "Analogna simulacija frakcionih sistema primjenom α-transformacije, XL konferencija za ETRAN, pp. 465-468, 4-7. jun, Budva, 1996. (in Serbian)

[9] T.B. Šekara, M.S. Stanković, "Analiza jedne klase frakcionih sistema primjenom α-transformacije", XLI konferencija za ETRAN, pp.389-392, 3-6 juna 1997. (in Serbian)

[10] G. W. Bohannan, S. K. Hurst, and L. Springler, "Electrical Component with Fractional-Order Impedance," US20060267595, 11 30, 2006, Utility Patent Application.

[11] G. W. Bohannan, "Analog realization of a fractional control element revisited," in Proc. 41st IEEE Int. Conf. Decision Control, Tutorial Workshop 2: Fractional Calculus Applications in Automatic Control and Robotics, 2002, pp. 203–208.

[12] T.B. Šekara, "Analogna realizacija frakcionih integratora (diferencijatora)", INFOTEH-JAHORINA, Vol. 5, Red. A-7, p. 29-32, March 2006. (in Serbian)

[13] C. Coopmans, I. Petras, and Y. Q. Chen, "Analogue fractional-order generalized memristive devices," in ASME 2009 Int. Design Eng. Tech. Conf. Comput. Inf. Eng. Conf. (IDETC/CIE), San Diego, CA.

[14] M.P. Lazarević, P.D. Mandić, B. Cvetković, T.B. Šekara, B. Lutovac, "Some Electromechanical Systems and Analogies of Mem-systems Integer and Fractional Order", 5th Mediterranean Conference on Embedded Computing, pp. 230-233, MECO 2016, Bar, Montenegro

[15] R.F. Camargo, E. Capelas de Oliveira, E., J. Vaz Jr., On the generalized Mittag-Leffler function and its application in a fractional telegraph equation, Math. Phys. Anal. Geom, vol. 15, pp. 1–16 2012.

[16] S.M. Cvetićanin, D. Zorica, M.R. Rapaić, "Generalized time-fractional telegrapher's equation in transmission line modeling", Nonlinear Dyn, 2017., DOI 10.1007/s11071-016-3322-z

[17] F. Wang, X. Ma, "Fractional order Buck–Boost converter in CCM: modelling, analysis and simulations", vol. 101, pp. 1671-1682, 2014.

[18] C. Wu, G. Si, Y. Zhang, N. Yang, "The fractional-order state-space averaging modelling of the Buck–Boost DC/DC converter in discontinuous conduction mode and the performance analysis", Nonlinear Dyn, vol.79, Issue 1, pp. 689-703, Jan. 2015.

[19] A. Pommier, J. Sabatier, et al., "CRONE Control of a Nonlinear Hydraulic Actuator", Control Engineering Practice 10:391-402, 2002.

[20] J. Sabatier, A. Oustaloup, et al., CRONE Control of Continuous Linear Time Periodic System: Application to a Testing Bench, ISA Transactions 42:421-436, 2003.

[21] I. Podlubny, I. Petráš, B.M. Vinagre, P. O'Leary, L'. Dorčák, Analogue realizations of fractional order controlers, Nonlinear Dynam 29, 281-296, 2002.

[22] B.N. Jakovljević, M. R. Rapaić, Z.D. Jeličić, T.B. Šekara, "Optimization of distributed order fractional PID controller under constraints on robustness and sensitivity to measurement noise", ICFDA 2014.

[23] T.B. Šekara, "Fractional transformations with applications to control systems and electrical circuits", PhD Thesis, Faculty of Electrical Engineering, University of Belgrade (in Serbian), 2006.

[24] Slobodan Milojković, "Teorija električnih kola", Svjetlost Sarajevo 1991., (in Serbian)

[25] M.R.Rapaić, T.B. Šekara, M.P. Lazarević, "On discrete finite-dimensional approximation of linear, infinite dimensional systems",in Fractional Calculus Theory, Mathematics Research Developments, pp. 257-74, 2014. ISBN 978-1-63463-002-3