# Model-based decomposition of acoustic signals in dispersive environment

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**Résumé –** Les signaux acoustiques transmis dans des environnements dispersifs peuvent être difficile à analyser et à localiser. Le signal propagé dans un environnement dispersif est par sa nature non-stationnaire. Même si à l'émission le signal a une forme simple, ses caractéristiques seront considérablement modifiées et il sera composé, à la réception de plusieurs composantes modales. Dans ce papier, nous présentons une méthode pour la décomposition du signal reçu dans un environnement dispersif sur des fonctions modales issues de la fonction de transfert de la propagation des ondes acoustiques. Les fonctions de base pour la décomposition sont obtenues en variant les paramètres du modèle physique (profondeur et distance) du canal dispersif. Cette approche peut être définie comme la généralisation de la décomposition polynomiale adapté à la classe des signaux étudiés. Le fonctionnement de la méthode est illustré à travers des simulations numériques.

**Abstract –** The acoustic waves transmitted in dispersive environments can be quite complex for decomposition and localization. A signal which is transmitted through a dispersive channel is usually non-stationary. Even if a simple signal is transmitted, it can change its characteristics during the transmission through an underwater acoustic dispersive communication channel. Due to the propagation, the received signal often consists of more components. Only the form of the received signal is known. In this paper, we present a decomposition of received signal based on modal functions obtained from the transfer function of underwater acoustic wave propagation. Basis functions for decomposition are obtained by varying the parameters which characterize the model (depth and range) of the dispersive channel. It can be considered as a generalization of polynomial based decomposition, adjusted to the considered class of signals. The method is numerically tested.

## 1 Introduction

Underwater channels are usually dispersive channels. The dispersive phenomena of underwater propagation is a challenging topic of the research in recent years. There are two main problems in the analysis of dispersive underwater acoustic channels. One problem is that such a systems produces nonlinear transformations of a signal [1–6]. The signal propagation through underwater media is characterized by nonlinear frequency and phase shifts. As a result, different frequency components of a signal are propagated with different speeds, resulting in different time delays at the receiving point. Other problem is that the dispersive channel is usually characterized by a multipath propagation producing multicomponent signals. The multipath propagation often occurs because of the scattering of acoustic signals on the sea bottom.

Signal analysis and processing tools can help in detection and decomposition of received signals. The received signal in a dispersive channel is different from the transmitted signal. The received signal is commonly a complex, multicomponent, non-stationary signal. Typically high frequencies are less disturbed than the lower frequencies in the signal [3, 4]. Because of the non-stationarity of these signals, the time-frequency signal analysis is a suitable tool for their analysis.

Common time-frequency tools for the analysis of this kind of signals are the short-time Fourier transform, its polynomial extensions, the local polynomial Fourier transform, and the dual polynomial Fourier transform [7]. In this paper, we assume that the form of the received signal components is of general form of modes (impulse responses) obtained by solving the transfer function of the underwater acoustic wave propagation. We can decompose the signal using these modes with varying depth and range. The values of the varying parameters are obtained by maximizing the resulting distribution concentration in the transformation domain.

The paper is organized as follows. In Section 2 the signal received from a dispersive channel will be modelled. The dual polynomial Fourier transform is presented in Section 3. The signal decomposition based on the assumed model is presented in Section 4. Numerical results are given in Section 5.

## 2 Modelling of the Received Signals from Dispersive Channels

Let us assume that an underwater acoustic wave is transmitted. We will consider the propagation model as proposed

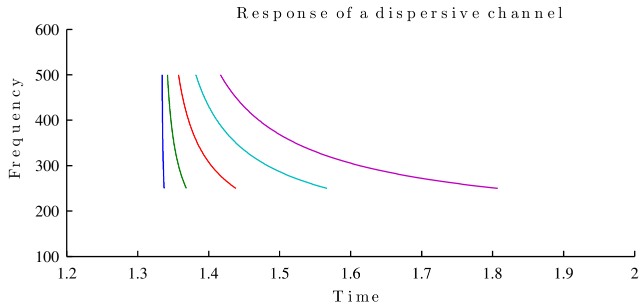


FIGURE 1 – Time-frequency representation of the impulse response of five modes

in [3, 6]. The transfer function for this propagation model is

$$H(f) = \sum_{m=1}^{+\infty} g_m(z_s) g_m(z_r) \frac{\exp(jk_r(m,f)r)}{\sqrt{k_r(m,f)r}} \quad (1)$$

$$\approx \sum_{m=1}^{+\infty} \frac{A(m,f)}{\sqrt{r}} \exp\{jk_r(m,f)r\}, \quad (2)$$

where $g_m(z)$ are the modal functions of the $m$th mode and $k_r(m,f)$ are the horizontal wavenumbers. Values $z_s$ and $z_r$ are the depths in meters of the transmitter and receiver, respectively. Value $r$ represents the distance between the transmitter and receiver. The modal functions are the solutions [6] of

$$\frac{\partial^2 g(z)}{\partial z^2} + \left( \left( \frac{2\pi f}{c} \right)^2 - k_r^2(m,f) \right) g(z) = 0. \quad (3)$$

The sound speed $c$ in the case of underwater communications is $c = 1500$ m/s. In general, the transfer function of a dispersive channel consists of several components. The components depend on the wavenumber and their frequencies. The variable $A_m = A(m,f)/\sqrt{r}$. is the attenuation rate. It depends on $g_m$, $k_r(m,f)$ and $r$. The response to a monochromatic signal, $\exp(j2\pi f_0 n)$, can be written as

$$s_m(n) \approx D_m \exp(j2\pi f_0 n - jk_r(m,f_0)r), \quad (4)$$

where $D_m$ is the depth of the channel. An ideal time-frequency representation of the impulse response of a dispersive channel environment is shown in Fig. 1. It would ideally track the frequency changes in time.

# 3 Dual Polynomial Fourier Transform

The dispersive channels are non-stationary. Therefore, the most suitable tool for their consideration is the time-frequency analysis. Several techniques were developed for the localization in the underwater dispersive channels. Most of them are based on the parametric modeling and short time Fourier analysis, like the technique using the phase continuity of the signals, [3]. In the underwater signals the phase changes in the frequency domain (along with the group delay) are of primary interest. It is the reason why the local polynomial Fourier transform in the frequency domain is more appropriate tool for the analysis that the standard time domain polynomial Fourier transform. We will explain here the procedure of decomposition using the dual polynomial Fourier transform (DPFT) from [8].

The idea of the DPFT is to find the parameters where the signal transformation is maximally concentrated. If the energy is preserved, maximally concentrated distribution produces the maximum of transformation amplitude value as well. Note that the considered underwater signals have significant frequency changes in the spectral content within a short time interval. In that case, the dual PFT is more appropriate tool than its counterpart with parameters in time domain. Using the frequency domain form of the polynomial Fourier transform we can extract the signal components and localize their positions [7].

The discrete DPFT is defined as :

$$x_{\beta_2,\beta_3...,\beta_N}(n) = \int_f X(f) e^{j(2\pi n + \beta_2 f^2 + ... + \beta_N f^N)} df. \quad (5)$$

The maximum of DPFT, defined by Eq.(5), is achieved when

$$(\hat{b}_2, \hat{b}_3, ..., \hat{b}_N) = \arg \max_{(n,\beta_2,...,\beta_N)} |x_{\beta_2,...,\beta_N}(n)|. \quad (6)$$

where $\beta_2, \beta_3, ..., \beta_N$ are the parameters.

Assume that the analyzed signal is a polynomial phase signal (PPS) of the $N$-th order

$$X(f) = A \exp\Big( -j \textstyle\sum_{l=1}^{N} b_l f^l \Big).$$

The signal will be highly concentrated in the PFT space of parameters where the maximum of the transform is achieved (where the transform of this signal is the best concentrated), Eq. (6). That is, the DPFT of a signal $X(f)$ will have the best concentration when $(\beta_2, ..., \beta_N) = (b_2, ..., b_N)$ and the goal to estimate $\hat{b}_2 \approx b_2, \ldots, \hat{b}_N \approx b_N$ is achieved.

# 4 Model-based Decomposition

We have found that the DPFT is more appropriate tool for considered signals that the transform with parameters in the time domain. However, this transformation is quite general and it does not take into account the specific propagation form of the underwater signals. Since we may assume that the components take the form of modal functions, a signal adapted approach to decomposition would be to use the parametrized modal functions in decomposition. Parameters would be the modal index, distance and depth as the unknown parameters.

Therefore, in this paper, we will use the idea as in the DPFT to vary the parameters of the modal functions as the decomposition functions. The goal is to vary the parameters of the transfer function model in the way we would vary the frequency parameters in the DPFT. The received signal $x(n)$ will be then decomposed as :

$$X(m,r,D_m) = \int_f X(f) \exp\left(-jk_r(m,f)r\right) df =$$

$$= \int_f X(f) H_m(f,r,D_m) df \quad (7)$$

$$= \int_f X(f) \exp\left(-j\big((2\pi f/c)^2 - ((m-0.5)\pi/D_m)^2\big)r\right) df,$$

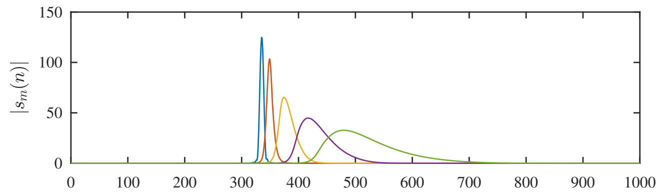


FIGURE 2 – Absolute impulse response of the dispersive channel modes

where $X(f)$ is the Fourier transform of the received signal and $k_r(m,f)$ is defined by

$$k_r(m,f) = \left(2\pi f/c\right)^2 - \left((m-0.5)\pi/D_m\right)^2. \tag{8}$$

The ideal time-frequency representation of modal (here decomposition) functions is presented in Fig. 1.

In [8] the decomposition of the signal by varying the second order DPFT parameters is shown. In this way we maximize DPFT by varying the parameters $\beta_{2,3}$. Here, we assume that the channel depth and the range are parameters and the unknowns. Instead of the polynomial coefficients $\beta_{2,3}$ (as in the DPFT) we will use the parameters $r$ and $D_m$.

The speed and the frequency range in which the underwater acoustic system operates are defined a priori. The values $\hat{r}$ and $\hat{D}_m$ are arbitrarily varied in some expected range. The system will have the highest concentration

$$(\hat{r}, \hat{D}_m) = \arg \max_{(r,D_m)} |X(m,r,D_m)| \tag{9}$$

when these values are close to the true ones, i.e. $\hat{D}_m \approx D_m$ and $\hat{r} \approx r$. An example will be shown in the next section.

# 5 Numerical Results

Let us consider a dispersive underwater channel with five modes. The frequency range $f$ is between $f_{min} = 250$ Hz and $f_{max} = 500$ Hz. The true channel depth and the distance between the transmitter and receiver $r$ are 20 m and 2000 m, respectively. These two parameters are considered as unknown.

The impulse response of each mode independently is shown in Fig. 2. The form of decomposition functions is

$$H_m(f,r,D_m) = A_m \exp\left(-jk_r(m,f)r\right) =$$
$$A_m \exp\left(-j(\left(2\pi f/c\right)^2 - \left((m-0.5)\pi/D_m\right)^2)r\right). \tag{10}$$

where $m$ represents the index of a mode.

The transmitted signal is considered to be a pulse with a short interval, close to a delta function, whose spectrum is then equal to 1, i.e. $U(f) = 1$. The received signal is the convolution between the transmitted signal and the impulse response of the system, with $X(f) = H(f)$.

For the analysis we have used the model-based decomposition. Variables $D_m$ and $r$ are arbitrarily varied. The value for depth $D_m$ is varied in the range between 0 to 100. The distance value $r$ is varied in the range between 1000 to 3000.

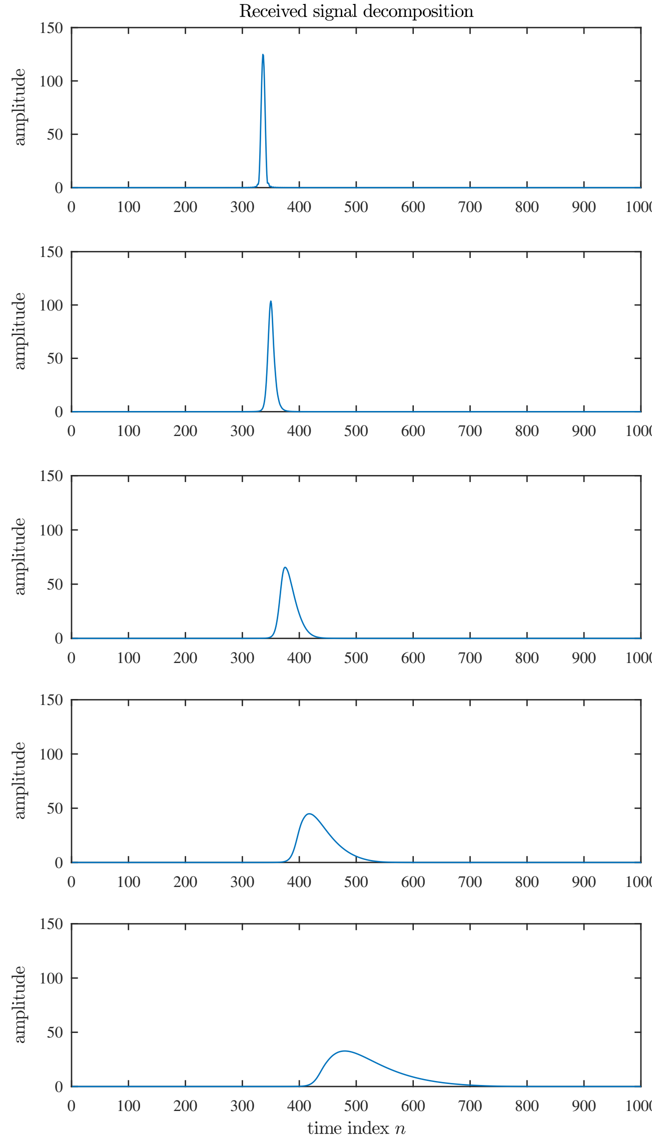


FIGURE 3 – Decomposed modes in the time domain

It has been calculated that the maximal values are found at the position $D_m = 20.0357$ m and $r = 2000$ m. The decomposition of each component is shown in Fig. 3. The sum of the received components and the sum of reconstructed components are shown in Fig. 4.

The decomposition results are non-stationary single component signals. They are shown in the time-frequency domain using the discrete short-time Fourier transform (STFT) representation of the signal. Since all analyzed modes (components) are in a wide frequency range, we will analyze the signal in the frequency domain using the dual STFT. It is defined by

$$STFT(k,n) = \sum_{m=-N_s/2}^{N_s/2} X(m-k)W(m)e^{-j2\pi mn/N_s} \tag{11}$$

where $X(k)$ is the discrete Fourier transform of the considered component and $W(m)$ is the frequency domain analysis window of size $N_s$. In this paper, a Hanning window of size $N_s = 21$ is used.

The STFT representation of a sum of five modes is shown Fig. 5 (top). Sum of the decomposed components and the amplitudes of individual components are given in Fig. 5 (bottom). The mean squared error (MSE) in the decomposition is calculated as

$$e = 10 \log \frac{\sum_{k,n}(|STFT_R(k,n)| - |\sum_m STFT_m(k,n)|)^2)}{\sum_{k,n}|STFT_R(k,n)|^2}$$

where $STFT_R(k,n)$ denotes the STFT of the received signal and $STFT_m(k,n)$ are the STFTs of mods (components) of the received signal after the decomposition. The MSE value is $e = -33.897$ dB. The method is not noise sensitive until the threshold for the detections is reached, i.e. when the input signal-to-noise ratio (SNR) is approximately $-5$ dB. When the threshold is reached, the error sharply increases, since some modes are not detected.

The method was compared with the second order dual polynomial decomposition, [8]. Since the mode forms do not fully coincide with the polynomial forms along the frequency, the best means squared error that can be achieved with the second order polynomial approximation was $e = -22.2$ dB.

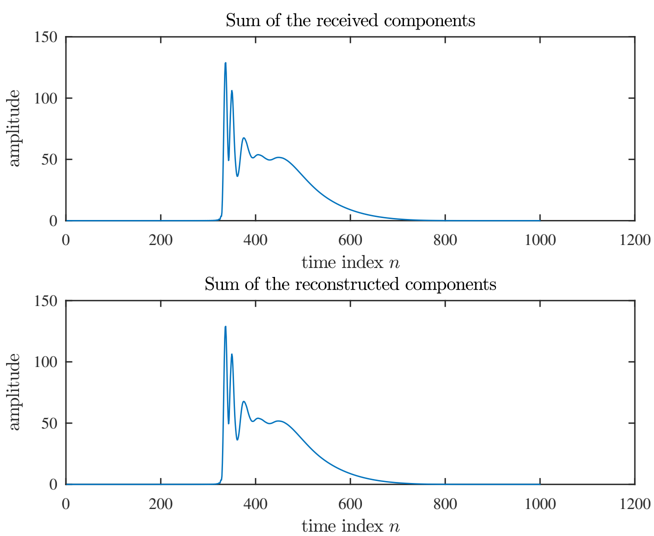


FIGURE 4 – Sum of the components : received (top) ; reconstructed (bottom)

## 6 Conclusions

Decomposition of acoustic signals in a dispersive environment based on the model form is shown. Knowing that the received signal is dependent on the distance between the transmitter and receiver and the channel depth, we can vary these two values to decompose the signals. The decomposition in such way was examined numerically and satisfactory results are obtained.

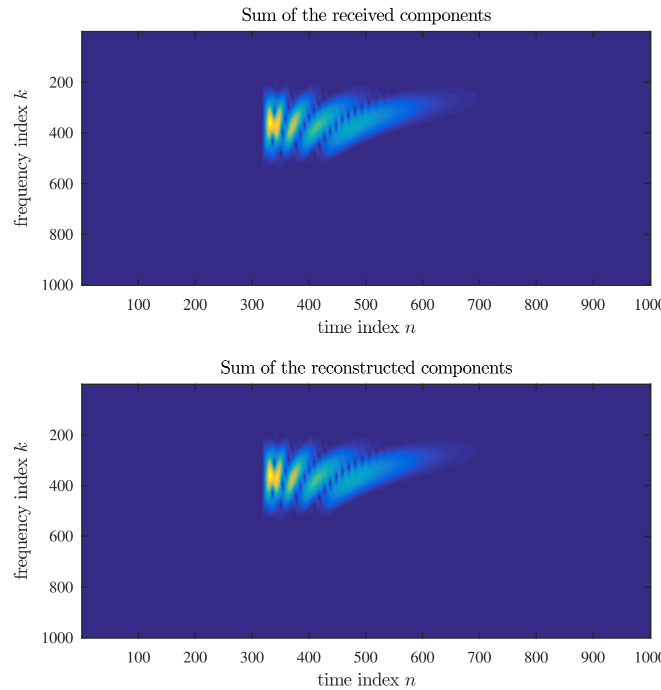


FIGURE 5 – Time-frequency representations of the received (top) and the reconstructed signal (bottom)

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