Spread-spectrum-modulated signal denoising based on median ambiguity function

Andjela Draganić, Irena Orović, Srdjan Stanković
University of Montenegro, Faculty of Electrical Engineering
Džordža Vašingtona bb, 20000 Podgorica, Montenegro
e-mail: andjelad@ac.me

Abstract- **The paper proposes an application of the median form ambiguity function in direct sequence spread spectrum modulated signals denoising. The observed signals are multicomponent and consisted of short duration sinusoidal components, appearing on different frequencies. The analysis of such multicomponent signals in the time-frequency plane could be disturbed by the unwanted cross-terms. It is shown that the filtering based on the median ambiguity function can completely eliminate the cross-terms. Moreover, beside the cross-terms the observed signal can be disturbed by different types of noise. Impulse noise is considered, due to the fact that this type of noise is common in communications. The signal terms are located around the origin in the ambiguity plane, being symmetric around the y-axis. Since the noise/cross-terms are dislocated from the origin, the filtering does not affect the signal terms providing satisfactory results. After denoising and cross-terms removal, the signal parameters (hop bandwidth and hop time duration) are estimated. The theory is proved by the experimental results.**

***Keywords* – ambiguity function; compressive sensing; FHSS; time-frequency analysis; robust form; wireless communications**

I. INTRODUCTION

Spread spectrum techniques [1] are used to extend the frequency range of the signal, using a code that is unique for each user and uncorrelated with the observed signal. As a result of spreading, a signal with much wider bandwidth compared to the bandwidth of an unmodulated signal, is obtained. The spread spectrum approach increases the number of users of the same transmission medium and, at the same time, decreases the interferences between signals that operate in the same frequency band.

Spread spectrum techniques are in theory known since the beginning of the twentieth century. They have found practical application by the German army during the World War I, when they were used to prevent the interception of signals and the disclosure of confidential information. These techniques are commonly used in communications [2],[4], mobile radio networks, satellite applications, systems for positioning, etc.

There are several types of spread spectrum modulations. The commonly used are direct sequence spread spectrum (DSSS) [4] and frequency hopping spread spectrum (FHSS) [1]-[5].

The second modulation technique is of particular interest in this paper. It is used in the Bluetooth standard and is based on the carrier wave frequency shift in a pseudorandom manner. The pseudorandom sequence defines the signal components frequencies [1]-[5]. Useful signal is being multiplied by the pseudorandom sequence, which results in extending the signal bandwidth. The frequency-hopping pattern represents a sequence of frequencies, and they occupy a certain range called hopping band. The time interval between the two-hops is called a hop interval. The hop bandwidth and hop time duration are among main characteristics of the FHSS signal.

The characteristics of the FHSS signals are observed and extracted from the time-frequency (TF) plane. However, having in mind multicomponent nature of the FHSS signals, cross-terms are common problem in the TF representation. Also, during the transmission through the communication channel, signal may be corrupted by noise. In order to reduce the unwanted terms, we applied the ambiguity-domain filtering [6]-[9]. The robust, median form of the ambiguity function (AF) is used and it is filtered by using the Gaussian kernel adapted to this specific type of signals. It is shown that median AF can provide the TF representation that is cross-terms and noise free.

TF distributions can be combined with the Compressive sensing (CS) approach [10]-[24]. CS provides the possibility to acquire small amount of data from the signal and still be able to recover the whole information about the signal. The aim of TF and CS combination in this paper is to reduce the number of samples from the ambiguity domain, required for providing TF representation from which signal parameters are estimated.

The paper is organized as follows: Section II represent the theoretical background on the spread spectrum modulated signals along with the Cohen class TF distributions. In Section III, the robust form of the AF is described as well as an optimization of the median form AF based on the CS principles. The experimental results are discussed in Section IV and conclusion is in the Section V.

II. THEORETICAL BACKGROUND

The spread spectrum modulation techniques find their usage in the wireless communication standards. FHSS technique is based on changing the carrier frequency from one value to another, according to a priori defined pseudorandom sequence. Extraction of the signal's features is important issue, since it enables identification of the wireless standard. Feature extraction can be made by observing the suitable TF representation, under the condition that it is cross-terms free.

In the analysis of the multicomponent signals, different TF distributions are used. The goal is to provide good concentration in the TF plane and to avoid cross-terms appearing as a consequence of the TF distribution nature [6]-[8]. Although the Wigner distribution (WD) is the basis for the AF definition, it has problem with the cross-terms occurrence when dealing with the multicomponent signals. Therefore, our focus is on distributions that belong to the Cohen's class, defined starting from the WD as a basis, and introducing a kernel function. By choosing a proper kernel function, the cross-terms and other interferences can be reduced to a great extent or can be completely eliminated.

The distributions from the Cohen class can be defined in a discrete form as follows:

$$C(k,\psi)=\sum_{n=-N/2}^{N/2-1}\sum_{m=-N/2}^{N/2-1} l(m,n)A(m,n)e^{-j\frac{2\pi}{N}mk-j\frac{2\pi}{N}n\psi}, \quad (1)$$

where $l(m,n)$ is a 2D kernel function and $A(m,n)$ is an AF, defined by [8]:

$$A(m,n)=\sum_{t=-N/2}^{N/2-1} x(t+n)x^{*}(t-n)e^{-jmt\frac{2\pi}{N}}. \quad (2)$$

Parameters k, ψ, m and n denote time, frequency, time-lag and frequency-lag coordinates, respectfully. The kernel function acts as a 2D filter of the AF. By properly adjusting the kernel parameters, reduction or complete elimination of the cross terms is possible. Different kernel functions produce various distributions belonging to the Cohen class. For example, Choi-Williams and Zhao-Atlas-Marks distributions to some extent reduce the cross-terms, but their use is limited to a small number of signals. By using the Gaussian kernel type and by adjusting its parameters, satisfactory results can be obtained for different types of signals. Therefore, in this paper we have used a distribution based on the Gaussian kernel:

$$C_G(k,\psi)=\sum_{n=-N/2}^{N/2-1}\sum_{m=-N/2}^{N/2-1} e^{-\left(\frac{m^2}{2\sigma_1^2}+\frac{n^2}{2\sigma_2^2}\right)} A(m,n)e^{-j\frac{2\pi}{N}mk-j\frac{2\pi}{N}n\psi}, \quad (3)$$

where σ_1 and σ_2 are parameters that affect the kernel shape. However, in the case of the noisy signals, Gaussian based distribution fails to provide an accurate representation in the TF plane. Therefore, to overcome this problem, robust Gaussian distribution based on the robust form of the AF is defined.

III. Robust Form of the Ambiguity Function and CS Optimization

A. Robust form of the ambiguity function

Let us describe the robust form of the AF, obtained as a solution of the following optimization problem [8]:

$$A(m,n)=\arg\min_{\varepsilon}\sum_{t=-M/2}^{M/2-1} L(e(m,n,t)) \quad (4)$$

The error function is defined as:

$$e(m,n,t)=x(t+n)x^{*}(t-n)e^{-j\frac{2\pi}{M}mt}-\varepsilon. \quad (5)$$

Then the standard form of the AF is defined using the mean form as follows:

$$A_S(m,n)=mean\left\{x(t+n)x^{*}(t-n)e^{-j\frac{2\pi}{M}mt}\right\}, t\in\left[\frac{-M}{2},\frac{M}{2}\right), \quad (6)$$

while robust, median form is described by using the relation [8]:

$$A_R(m,n)=median\left\{x(t+n)x^{*}(t-n)e^{-j\frac{2\pi}{M}mt}\right\}, \quad t\in[-M/2,M/2) \quad (7)$$

In the case of the mixed noise, the L-form of the AF can be used:

$$A_L(m,n)=\sum_{s=-M/2}^{M/2-1} a_s\left(r_{s\,sort}(m,n)+j\cdot i_{s\,sort}(m,n)\right), \quad (8)$$

where r_{ssort} and i_{ssort} are the sorted elements of real and imaginary part of the autocorrelation function:

$$r_s\in R(m,n)=\left\{\mathrm{Re}\left[x(t+n)x^{*}(t-n)e^{-j\frac{2\pi}{M}mt}\right]\right\}, \quad i_s\in I(m,n)=\left\{\mathrm{Im}\left[x(t+n)x^{*}(t-n)e^{-j\frac{2\pi}{M}mt}\right]\right\}, \quad (9)$$

$m\in$ [-M/2, M/2), and coefficients a_s are described as:

$$a_s=\begin{cases}\dfrac{1}{M-2\rho(M-2)}, & \text{for } s\in[(M-2)\rho, M-(M-2)\rho-1] \\ 0, & \text{outside}\end{cases}. \quad (10)$$

The median form of the AF can be defined by using the relation (8) with $\rho=1/2$.

The robust form of the Gaussian TF distribution is obtained by using the robust AF defined as:

$$C_G^R(k,\psi)=\sum_{n=-N/2}^{N/2-1}\sum_{m=-N/2}^{N/2-1} e^{-\left(\frac{m^2}{2\sigma_1^2}+\frac{n^2}{2\sigma_2^2}\right)} A_L(m,n)e^{-j\frac{2\pi}{N}mk-j\frac{2\pi}{N}n\psi}, \quad (11)$$

where $\mathbf{A_L}$ denotes the L-estimate form of the ambiguity function. Due to the nature of the FHSS signals, the median form of $\mathbf{A_L}$ for $\rho=1/2$ is shown to be suitable for dealing with those signals.

B. Undersampling in the ambiguity plane

The number of samples, required for satisfactory representation of the signal's TF distribution, can be reduced.

This can be done by intentionally undersampling the AF, and applying the optimization algorithms to obtain an optimized TF representation. The goal is to reduce the number of ambiguity domain samples, while preserving the TF resolution satisfactory enough for later estimation of the signal parameters: components' hop frequencies, hop intervals and hop time durations.

The CS [9]-[24] is a widely studied approach for recovering missing information in the signal if the signal has domain where can be sparsely represented. Here, as a sparsity signal domain, the TF representation is considered, while the samples are selected from the ambiguity domain [13],[14],[18],[19].

Certain, small number of ambiguity domain samples, is randomly selected from the ambiguity plane. The reconstruction of the AF from a small number of acquired samples, is done by using the optimization algorithms [11],[12],[18]. The 2D CS problem is defined starting from relation between the AF and complex-time distribution:

$$A_R(m,n) = \Upsilon C_G^R(k,\psi)\,, \tag{12}$$

where Υ denotes 2D Fourier transform matrix. The measurements are randomly selected from the ambiguity plane and the measurement matrix $\mathbf{Y}$ is obtained as [18]:

$$Y(m,n) = \mathbf{\Phi} A_R(m,n) = \mathbf{\Phi}\Upsilon C_G^R(k,\psi)\,, \tag{13}$$

where $\mathbf{\Phi}$ is matrix whose elements are 1 and 0, and models random selection of the coefficients. The optimization problem is then formulated as follows:

$$\min_{\mathbf{C}_\mathbf{G}^\mathbf{S}} \left[\lambda \left\| \mathbf{C}_\mathbf{G}^\mathbf{S} \right\|_{\ell_1} + \frac{1}{2}(\mathbf{A}_\mathbf{R} - \mathbf{\Phi}\Upsilon \cdot \mathbf{C}_\mathbf{G}^\mathbf{S}) \right], \tag{14}$$

resulting in an optimized TF distribution $\mathbf{C}_\mathbf{G}^\mathbf{S}$.

IV. EXPERIMENTAL RESULTS

Let us consider three-component FHSS modulated signal, defined as follows:

$$x(t) = \sum_{i=1}^{3} P_i e^{j2\pi f_i t}, \; t = \text{-}1{:}1/50{:}1\text{-}1/50 \,, \tag{15}$$

where it is assumed that the amplitudes P_i are equal to 1 and f_1=10, f_2=7, f_3=2.

The WD of the non-noisy signal is shown in the Figure 1a, while Figure 1b shows the ambiguity function of the non-noisy signal. It can be seen that, even when there is no noise in the signal, the WD produces cross-term between two signal terms.

Let us now observe the WD and the AF when the impulse noise is present in the signal, as shown in Figure 2 (8 noisy peaks occur in the observed signal). The noisy peaks are real valued and therefore only real part of the signal is displayed. The WD and the corresponding AF are shown in Figure 3. It can be seen that differentiation between noise and signal terms is not possible in this case. Therefore, the median form of the ambiguity function is calculated and filtered by the Gaussian kernel function in order to eliminate the unwanted terms. The median ambiguity function is shown in Figure 4a.

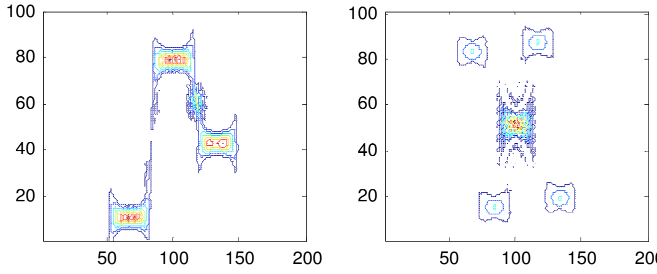


Figure 1. a) the Wigner distribution of non-noisy signal, b) the ambiguity function of non-noisy signal

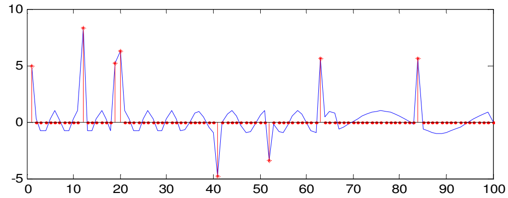


Figure 2: Noisy signal (real part of the signal is displayed). Red marks denote noisy coefficients

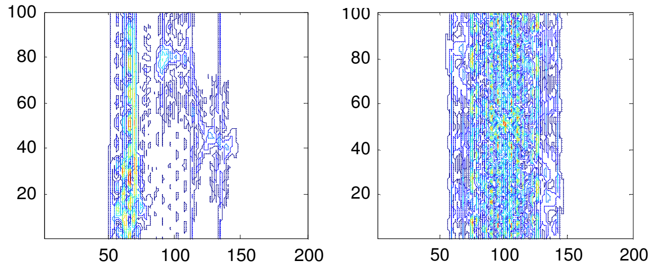


Figure 3: a) Wigner distribution and b) ambiguity function of the signal corrupted by noise

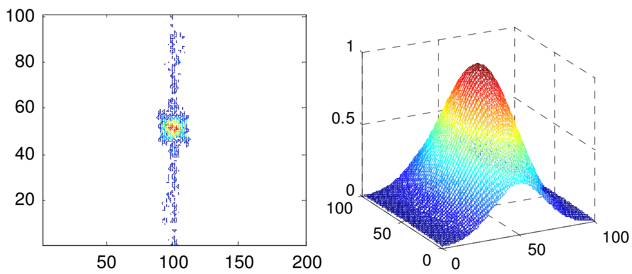
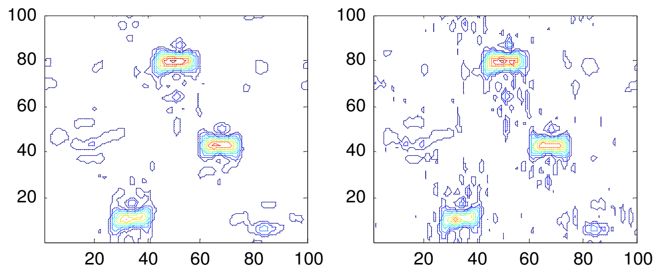


Figure 4: a) Median ambiguity function; b) Gaussian kernel used for the TF distribution calculation

The parameters for the Gaussian kernel are $\sigma_1=\sigma_2=96$. The kernel is shown in Figure 4b, while the TF representation after denoising is shown in Figure 5a.

The filtered ambiguity function can be randomly undersampled in order to obtain the optimized TF representation. In our case, only 27% of the samples from the ambiguity plane is randomly chosen. The optimized TF representation is then calculated by applying the optimization problem according to the relation (14). The obtained TF representation is shown in Figure 5b. The time/frequency durations of signal components, estimated from the original and optimized TF representations, are shown in Table 1.



a) b)

Figure 5: a) Gaussian kernel based TF distribution obtained by using the median ambiguity function; b) The optimized TF distribution obtained from only 27% of the ambiguity domain samples

Table 1: Time (*t*) and frequency (*f*) durations of the signal components, estimated from the original and optimized TF representation

	Component 1		***Component 2***		***Component 3***	
	t	*f*	*t*	*f*	*t*	*f*
TF	15	11	18	80	16	43
Optimized TF	14	11	20	80	18	43

V. Conclusion

The application of the median form ambiguity function for denoising of spread spectrum modulated signals, is proposed in the paper. The signal corrupted by the impulse noise is observed. Beside the noise, the unwanted terms are the cross-terms appearing in the TF and in the ambiguity plane. The fact that the signal components in the ambiguity plane are concentrated around the origin while the noisy components and cross-terms are dislocated, is exploited in this approach. It is shown that the median form of the ambiguity function completely eliminates the unwanted terms from the ambiguity plane. After filtering, the optimization of the ambiguity function size is done by it's undersampling. By using only 27% of the samples from the ambiguity plane, the optimized TF distribution is obtained based on which the signal features are successfully estimated.

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