Decomposition and Analysis of Signals Sparse in the Dual Polynomial Fourier Transform

Isidora Stanković
Univ. Grenoble Alpes/Univ. Montenegro
Grenoble, France/Podgorica, Montenegro
Email: isidoras@ac.me

Miloš Daković
University of Montenegro
Podgorica, Montenegro
Email: milos@ac.me

Cornel Ioana
University of Grenoble Alpes
Grenoble, France
Email: cornel.ioana@gipsa-lab.grenoble-inp.fr

***Abstract*—The acoustic waves transmitted through a dispersive environments can be quite complex for decomposition and localization. A signal which is transmitted through a dispersive channel is usually non-stationary. Even if a simple signal is transmitted, it can change its characteristics (phase and frequency) during the transmission through an underwater acoustic dispersive communication channel. Commonly, several components with different paths are received. In this paper, we present a method for decomposition of multicomponent acoustic signals using the dual polynomial Fourier transform and time-frequency methods. In real-world signals, some disturbances are introduced during the transmission. Common form of disturbances are the sinusoidal signals, making some of the frequency domain signal samples unreliable. Since the signal components can be considered as sparse in the dual polynomial Fourier transform domain, these samples can be omitted and reconstructed using the compressive sensing methods. The acoustic signal decomposition and its reconstruction from a reduced set of frequency domain samples is demonstrated on examples.**

***Keywords*—compressive sensing; dispersive channels; polynomial Fourier transform; sparsity; time-frequency analysis; underwater acoustics**

I. Introduction

The dispersivity in underwater channels has been a challenging topic in the recent years. Many channels with the phenomena of dispersion have been studied. A dispersive channel in underwater acoustics is a system which produces nonlinear signal transformations [1]–[5]. That is, it shifts the propagating signal in the phase which will cause shifts in frequency and time in the received signal. Another characterization of dispersive channels is that it produces multicomponent signals due to multipath propagation which can occur for various reasons. The main one is the scattering of acoustic signals on the sea bottom.

The received signal in a dispersive channel is different from the transmitted signal. It is a complex and non-stationary signal. Because of the non-stationary nature of these signals, the time-frequency signal analysis is a suitable tool for analysis. It can help in detection, extraction and localization of transmitted signals. The most common tool for the analysis of non-stationary signals is the time-frequency signal analysis [6]–[13]. A common problem in practice is strong harmonic disturbances. After these disturbances are removed, the signal components should be reconstructed.

In the theory of sparse signal reconstruction, a signal is sparse if it has only few non-zero components in comparison to the total length of the signal. If the signal is sparse, it can be reconstructed with less measurements [14]–[18]. The considered acoustic signal is sparse in the dual polynomial Fourier transform (DPFT) domain, and the noisy measurements (impulses) occur in frequency domain. The impulses in frequency domain will introduce sinusoids in time domain. These disturbances are removed, and the signal components can be reconstructed by compressive sensing methods, such as the matching pursuit algorithm. In this paper, we present a method for decomposition of a signal which was transmitted through a dispersive environment.

The paper is organized as follows. In Section II, the received signal from a dispersive channel will be modelled and explained. In Section III basic theory of compressive sensing is introduced. The polynomial Fourier transform for analysis and localization of acoustic signals will be presented in Section IV. Numerical results and conclusions are given in Sections VI and VII, respectively.

II. Modelling of the Received Signals from Dispersive Channels

Let us assume that an underwater acoustic wave is transmitted. Assume a linearly frequency modulated (LFM) signal of the form

$$u(n) = e^{j\pi\alpha n^2}. \quad (1)$$

The signal propagates through an isovelocity underwater dispersive channel [2], having the same velocity of sound over all volume [1]–[5]. We will assume that the transmitter is located at the depth of z_t meters. The receiver is located at the depth of z_r meters. The distance between the transmitter and the

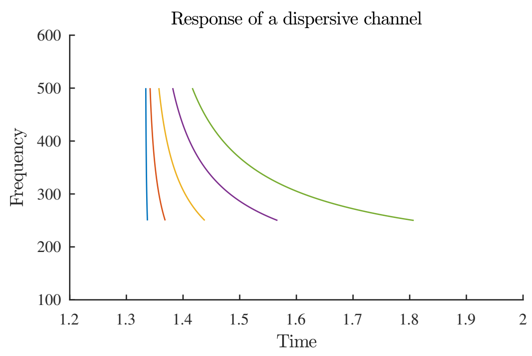

Fig. 1. The time-frequency representation of the impulse response of five modes

receiver is denoted by r. The transfer function of the channel is

$$H(f) = \sum_{m=1}^{+\infty} g_m(z_t) g_m(z_r) \frac{\exp(jk_r(m,f)r)}{\sqrt{k_r(m,f)r}} = \sum_{m=1}^{+\infty} A_t(m,f,r) \exp\big(jk_r(m,f)r\big), \quad (2)$$

where $g_m(z_t), g_m(z_r)$ are the modal functions of the m-th mode for the transmitter and the receiver, respectively. The attenuation rate is $A_t(m,f,r) = A(m,f)/\sqrt{r}$. The transfer function depends on the number of modes, and the modes are dependent on wavenumbers $k_r(m,f)$ [2]

$$k_r(m,f) = \Big(\frac{2\pi f}{c}\Big)^2 - \Big((m-0.5)\frac{\pi}{D}\Big)^2 \quad (3)$$

where D is the channel depth. The sound speed in the case of underwater communications is $c = 1500$ m/s. The modal functions g_m are the solutions [2] of

$$\frac{\partial^2 g}{\partial z^2} + \bigg(\Big(\frac{2\pi f}{c}\Big)^2 - k_r^2(m,f)\bigg) g = 0. \quad (4)$$

It is obvious that the transfer function of a dispersive channel is of a multicomponent structure. The components depend on the wavenumbers $k_r(m,f)$ and their frequencies, on modal functions g_m and the distance r.

The received signal is then

$$x(n) = u(n) * h(n), \quad (5)$$

where $h(n)$ is the impulse response of (2). An ideal time-frequency representation of the impulse response of a dispersive channel environment is shown in Fig. 1. Our goal is to decompose the mode functions, which will make the problem of detecting the transmitted signal straightforward. This decomposition makes compressive sensing methods application possible to use as well. The decomposition method will be formulated within the compressive sensing approach.

In some real-world scenarios, the signal will be received with a kind of disturbance. Here, we will assume that the signal is corrupted with strong sinusoidal disturbances

$$x_d(n) = x(n) + \sum_{l=1}^{N_M} B_i e^{j(\omega_l n + \psi_l)}. \quad (6)$$

The strong periodic disturbances should be detected and removed. Methods for detecting and removing strong disturbances will be presented next.

III. Sparse Signal Reconstruction

Assume a signal $x(n)$, $0 \le n < N$ and its linear transform $X(k)$, which will be defined as

$$X(k) = \sum_n \psi_k(n) x(n) \quad (7)$$

where $\psi_k(n)$ is the basis function of the transform used. In the vector form they are written as

$$\mathbf{x} = [x(0), x(1), \ldots, x(N-1)]^T \quad (8)$$
$$\mathbf{X} = [X(0), X(1), ..., X(N-1)]^T. \quad (9)$$

They are related via $N \times N$ transformation matrix $\mathbf{A}_N$ as

$$\mathbf{X} = \mathbf{A}_N \mathbf{x}. \quad (10)$$

We will assume that signal $x(n)$ is sparse. It means that the signal $\mathbf{x}$ has only $K \ll N$ samples $x(n_1), x(n_2), \ldots, x(n_K)$ that are non-zero. When the signal is sparse in one of its domains, it can be reconstructed with less measurements in one of its transformation domains, i.e. with $N_A < N$. The signal measurements in this case are coefficients of its transform at positions $\mathbb{N}_{\mathbb{A}} = \{k_1, k_2, \ldots, k_{N_A}\}$. The measurement vector is defined by

$$\mathbf{y} = [X(k_1), X(k_2), \ldots, X(k_{N_A})]^T. \quad (11)$$

Vector form of the measurements equation is

$$\mathbf{y} = \mathbf{A}\mathbf{x} \quad (12)$$

where $\mathbf{A}$ is a $N_A \times N$ matrix

$$\mathbf{A} = \begin{bmatrix} \psi_{k_1}(0) & \psi_{k_1}(1) & \cdots & \psi_{k_1}(N-1) \\ \psi_{k_2}(0) & \psi_{k_2}(1) & \cdots & \psi_{k_2}(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{k_{N_A}}(0) & \psi_{k_{N_A}}(1) & \cdots & \psi_{k_{N_A}}(N-1) \end{bmatrix} \quad (13)$$

where $\psi_k(n)$ are the transform coefficients. The matrix is obtained by keeping only the rows of $\mathbf{A}$ corresponding to the available measurements.

The goal of compressive sensing is to reconstruct the signal by minimizing $\mathbf{x}$ using the available measurements $\mathbf{y}$

$$\min \|\mathbf{x}\|_0 \quad \text{subject to} \quad \mathbf{y} = \mathbf{A}\mathbf{x}. \quad (14)$$

It is assumed that the reconstruction conditions are met. The solution of problem (14) can be found in various ways. One of the common algorithms to solve the problem is the orthogonal matching pursuit (OMP) [18]. In the first step of the OMP, the position of the largest component is found

$$n_1 = \arg\max\{\mathbf{x}_0\} \tag{15}$$

using the inital estimate $\mathbf{x}_0 = \mathbf{A}^H\mathbf{y}$, calculated using only the available measurements. A new partial matrix of the matrix $\mathbf{A}$ is formed, omitting all columns except the row which corresponds to the estimated position n_1. New matrix is then $\mathbf{A}_1$. The estimate of the first component in the time domain is

$$\mathbf{x}_1 = (\mathbf{A}_1^H\mathbf{A}_1)^{-1}\mathbf{A}_1^H\mathbf{y}. \tag{16}$$

The signal is reconstructed at the position n_1 and subtracted from the original signal measurements. The estimate of the non-zero position is calculated again with this signal and its maximum position is found at n_2. A new set $\mathbb{K} = \{n_1, n_2\}$ is formed with the corresponding matrix $\mathbf{A}_2$. The new estimate $\mathbf{x}_2$ is calculated and the signal is reconstructed. The procedure is repeated until all K components are reconstructed. For the case when the signal samples are spread, we may use few samples around n_i in each reconstruction step.

We will assume that the signal is K-sparse in the time domain, and we will consider dual polynomial transform coefficients as measurements.

IV. Dual Polynomial Fourier Transform

Several techniques were developed for the localization non-stationary dispersive channels. Decomposition, localization and reconstruction of sparse signals in the dual polynomial Fourier transform is examined in this paper.

A. Polynomial Fourier transform (PFT)

The idea behind the traditional PFT is to find the parameters where the signal transform achieves the maximum concentration. In this way we can extract all components and localize their positions [6], [7]. Let us assume a signal $x(n)$. Its PFT is calculated as [8]–[10]

$$X_{\alpha_2,\alpha_3,\dots,\alpha_N}(k) = \sum_n x(n)e^{-j\frac{2\pi}{N}(kn+\alpha_2 n^2+\alpha_3 n^3+\dots+\alpha_N n^N)}, \tag{17}$$

where $\alpha_2, \alpha_3, ..., \alpha_N$ are the parameters.

Assume that the analyzed signal is a polynomial phase signal (PPS) of the P-th order

$$x(n) = Ae^{j\frac{2\pi}{N}\sum_{l=1}^{P} a_l n^l}.$$

The signal will be highly concentrated in the PFT space of parameters where the maximum of the transform is achieved (where the transform of this signal is the best concentrated), i.e.

$$(\hat{a}_2, \hat{a}_3, ..., \hat{a}_P) = \arg \max_{(k,\alpha_2,\dots,\alpha_P)} |X_{\alpha_2,\dots,\alpha_P}(k)|. \tag{18}$$

It means that the PFT of a signal $x(n)$ will have the best concentration when $(\alpha_2, ..., \alpha_P) = (a_2, ..., a_P)$. Then the goal to estimate $a_2 \approx \hat{a}_2$, …, $a_P \approx \hat{a}_P$ is achieved.

B. Dual extension of PFT

For the signals whose spectral content is concentrated within short time interval, with changes in frequency the dual PFT (DPFT) is more appropriate tool. Like for PFT, the goal of DPFT is to find the parameters where the transform of the signal produces the highest concentration, meaning maximal sparsity.

The considered signal is a polynomial phase signal

$$X(k) = Ae^{-j\frac{2\pi}{N}\sum_{l=1}^{P} b_l k^l}. \tag{19}$$

in the frequency domain.

The discrete dual PFT is defined as [19]

$$x_{\beta_2,\beta_3\dots,\beta_P}(n) = \sum_k X(k)e^{j\frac{2\pi}{N}(nk+\beta_2 k^2+\dots+\beta_P k^P)}. \tag{20}$$

The maximum of DPFT, i.e., the maximum of the Eq. (20) is achieved when

$$(\hat{b}_1, \hat{b}_2, ..., \hat{b}_P) = \arg \max_{(n,\beta_2,\dots,\beta_P)} |x_{\beta_2,\dots,\beta_P}(n)|. \tag{21}$$

Ideally, the best DPFT concentration is when $(\beta_2, ..., \beta_P) = (b_2, ..., b_P)$. Our goal is to estimate the parameters such that $\hat{b}_2 \approx b_2, \dots, \hat{b}_P \approx b_P$.

A local form of the dual PFT, corresponding to the local PFT (known as LPFT) would be obtained using a frequency domain window function $W(k)$. It reads

$$x_{\beta_2,\beta_3\dots,\beta_P}(n,k) = \sum_l W(l)X(k+l) \times e^{j\frac{2\pi}{N}(nk+\beta_2 k^2+\dots+\beta_P k^P)}.$$

This kind of transform can be used for analysis of quite complex non-stationary acoustic signals in the dispersive media.

V. Sparsity in DPFT

The reconstruction of signals sparse in the PFT representation domain is shown in [20]. In this paper, we will consider the DPFT case when some unavailable coefficients are in the frequency domain (due to denoising procedure on harmonic disturbances).

Without loss of generality, we will consider the analysis to the third order DPFT. Consider that $\mathbf{X}$ has disturbed samples which are found and set as unavailable. The third order DPFT estimated using only the available samples of $\mathbf{X}$ is [21]

$$x_{\beta_2,\beta_3}(n) = \sum_{k \in \mathbb{N}_{\mathbb{A}}} X(k) e^{j\frac{2\pi}{N}(nk+\beta_2 k^2+\beta_3 k^3)} \qquad (22)$$

for

$$X(k) = Ae^{-j\frac{2\pi}{N}(b_1 k+b_2 k^2+b_3 k^3)}. \qquad (23)$$

Assume that parameters β_2, β_3 are found by a direct search over the interval of their possible values. When the parameters are correctly estimated $(\beta_2, \beta_3) = (b_2, b_3)$, the DPFT is

$$x_{b_2,b_3}(n) = \sum_k Ae^{j\frac{2\pi}{N}k(n-b_1)} = A\delta(n-b_1). \qquad (24)$$

Obviously it is sparse. In the case of multicomponent signals, i.e.

$$X(k) = \sum_{m=1}^{M} A_m e^{-j(b_{1m}k+b_{2m}k^2+b_{3m}k^3)}, \qquad (25)$$

the parameters of each component are estimated in iterative way. Without loss of generality, assume that $A_1 > A_2 > \cdots > A_M$. When the first component is matched with

$$(\beta_{21}, \beta_{31}) = (b_{21}, b_{31})$$

we may consider that all other components are spread and negligible. The measurements matrix is obtained from this signal definition (22) assuming that only the values $k \in \mathbb{N}_{\mathbb{A}}$ are available. This relation for various n can be written as

$$\begin{bmatrix} x_{b_{21},b_{31}}(n_1) \\ x_{b_{22},b_{32}}(n_2) \\ \vdots \\ x_{b_{2K},b_{3K}}(n_K) \end{bmatrix} = \mathbf{A}_K^H \begin{bmatrix} X(k_1) \\ X(k_2) \\ \vdots \\ X(k_{N_A}) \end{bmatrix} \qquad (26)$$

where the measurement matix is defined by

$$\mathbf{A}_K = \begin{bmatrix} e^{-j\frac{2\pi}{N}(n_1 k_1+\phi_1)} & \dots & e^{-j\frac{2\pi}{N}(n_K k_1+\phi_1)} \\ \vdots & \ddots & \vdots \\ e^{-j\frac{2\pi}{N}(n_1 k_{N_A}+\phi_{N_A})} & \dots & e^{-j\frac{2\pi}{N}(n_K k_{N_A}+\phi_{N_A})} \end{bmatrix}$$

with $\phi_i = k_i^2 b_{21} + k_i^3 b_{31}$ for $i = 1, \dots, N_A$. Starting from the available values $X(k)$, $k \in \mathbb{N}_{\mathbb{A}}$, we reconstruct the non-zero values in time $[x_{b_{21},b_{31}}(n_1), x_{b_{22},b_{32}}(n_2), \dots, x_{b_{2K},b_{3K}}(n_K)]$ using the iterative OMP procedure, starting with

$$\mathbf{x}_1 = (\mathbf{A}_1^H \mathbf{A}_1)^{-1} \mathbf{A}_1^H \mathbf{y}. \qquad (27)$$

After the DPFT sample at n_1 is reconstructed then the remaining unavailable values $X(k)$ are calculated for the first component. This component is removed from the original measurements. The procedure is repeated for the second component. After the parameters of the second component are found as $(\beta_{22}, \beta_{32}) = (b_{22}, b_{32})$, both the first and second component are reconstructed using both components

$$(\beta_{21}, \beta_{31}) = (b_{21}, b_{31}), \text{ and } (\beta_{22}, \beta_{32}) = (b_{22}, b_{32}).$$

After the first two values are reconstructed, the procedure is continued for all n_i. For the case when the DPFT values are not on the grid, we may use few samples around n_i in the reconstruction. The stopping criterion can be the energy of the remaining signal after the reconstructed components are removed.

VI. Numerical Results

The application of the proposed theory is demonstrated on examples. In the first example a theoretical signal that fully fits the assumed model is considered. A simulated acoustic signal, that only approximately behaves as the assumed theoretical model, is analyzed in other two examples.

Example 1

Let us consider an ideal case of a polynomial phase signal of form (19). Assume that the signal is received with 5 components

$$\begin{aligned} X(k) &= e^{(j2\pi 100k/N)} + e^{(j2\pi 150k/N + j2\pi 0.1k^2/N)} \\ &+ e^{(j2\pi 250k/N + j2\pi 0.1k^2/N + j2\pi 0.00005k^3/N)} \\ &+ e^{(j2\pi 450k/N + j2\pi 0.0001k^3/N)} \\ &+ e^{(j2\pi 600k/N + j2\pi 0.05k^2/N + j2\pi 0.0001k^3/N)} \end{aligned} \qquad (28)$$

with $k = 0, \dots, N-1$ and $N = 1000$. This signal in the time domain, along with the corresponding frequency domain, is presented in Fig. 2 (top). Additionally, we assume that 30% of randomly positioned samples in the frequency domain are strongly disturbed. This will exhibit in the time domain by disturbing sinusoids. The affected signal is shown in Fig. 2 (middle). The disturbance components are first filtered with a simple notch filter and set to zero (hard thresholding). The signal after filtering is shown in Fig. 2 (bottom).

For the analysis of this signal we have used the DPFT of the third order as in Eq. (22). Values of β_2, β_3 are varied in the range of -0.2 to 0.2 and -0.3 to 0.3, respectively. The optimal parameter values for various modes are detected iteratively. When we find the first set of parameters β_2, β_3, the peak in the DPFT corresponds to the single component. We can remove the component from the DPFT and continue to estimate other components. This decomposition using the DPFT is shown in Fig. 3.

The decomposition results are non-stationary single component signals. The dual S-method representation as an improved version of short-time Fourier transform [11], [19] is used for displaying the time-frequency content of the individual components and the reconstructed signal. Since all analyzed modes (components) are spread over a wide frequency range,

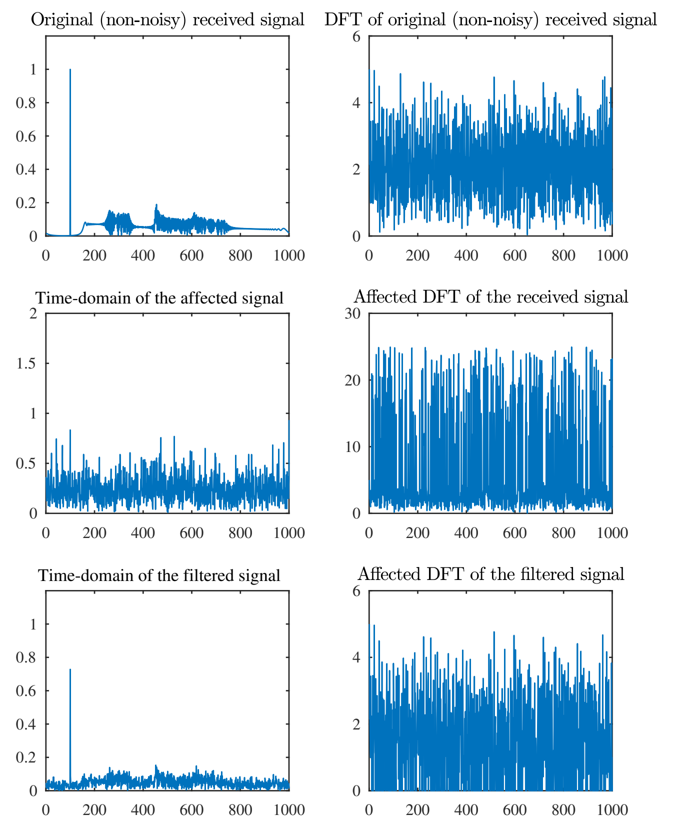

Fig. 2. Received signal in the time domain (left); in the frequency domain (right): without disturbance (top), with disturbance (middle), and the filtered signal (bottom)

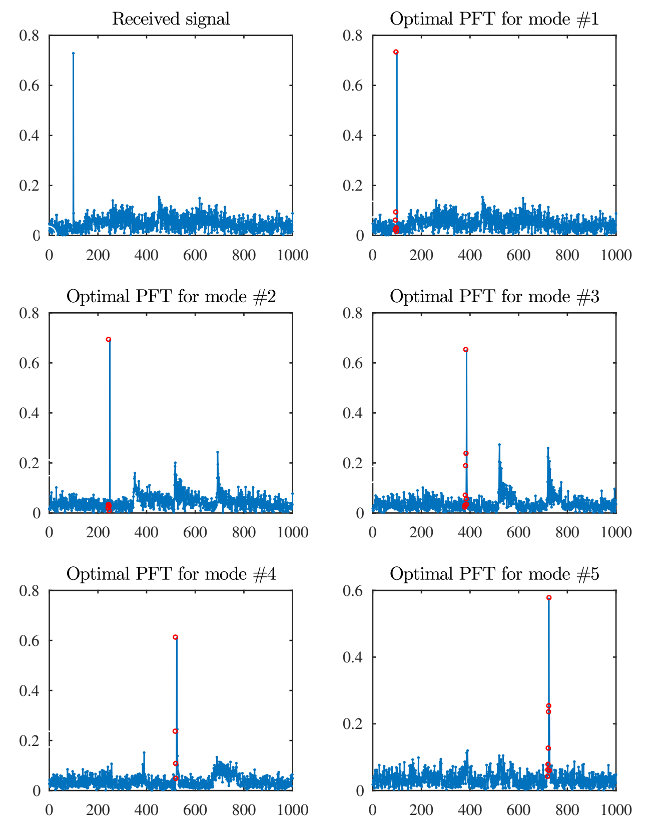

Fig. 3. Decomposition of the components using DPFT

we will analyze the signal in the frequency domain using the dual STFT. It is defined by

$$STFT_D(k,n) = \sum_{p=-N_s/2}^{N_s/2-1} X(p-k)W(p)e^{j\frac{2\pi}{N_s}pn}, \quad (29)$$

where $X(k)$ is Fourier transform of the considered component and $W(k)$ is the analysis window.

Therefore, the dual S-method could be then calculated as

$$SM_D(k,n) = \sum_{i=-L}^{L} STFT_D(k,n+i)STFT_D^*(k,n-i) \quad (30)$$

where $2L+1$ is the time domain window size [6], [7], [11]. In this example, we use $L = 16$ and the Hanning window of size $W_s = 256$.

The received signal is decomposed and reconstructed using the OMP algorithm. The S-method of the reconstructed components is shown in Fig. 4. The sum of normalized representations of the five components is also shown Fig. 4 (bottom right subplot). The original non-noisy signal and the final reconstructed signal are presented in Fig. 5.

Example 2

Here we will analyze the simulated acoustic signal as described in Section II. In this case, the signal is not of an ideal polynomial phase structure.

A signal of form (1) is transmitted through a dispersive channel with $M = 5$ modes. We will assume that the received signal is of form (5), where the signal depends on the transfer function as in Eq. (2) and Eq. (3). The channel depth is $D = 20$ meters and the distance between the transmitter and receiver is $r = 2000$ meters. The frequency range f is between $f_{min} = 250$ Hz and $f_{max} = 500$ Hz. The received signal is shown in Fig. 6 (top left). Amplitude is attenuated $A_m = (6-m)W(f)$, where $W(f)$ is the Hanning window in the frequency domain.

For the analysis of this signal we have used the third order DPFT, with β_2, β_3 being varied in the range of -0.2 to 0.2 and -0.3 to 0.3. The decomposition of the modes is shown in Fig. 6.

For the dual S-method, we use $L = 16$ and the Hanning window of size $W_s = 512$ for the dual STFT calculation. The S-method of the five modes, obtained by decomposition using DPFT before the compressive sensing framework, is shown in Fig. 7. Sum of the normalized representations of the

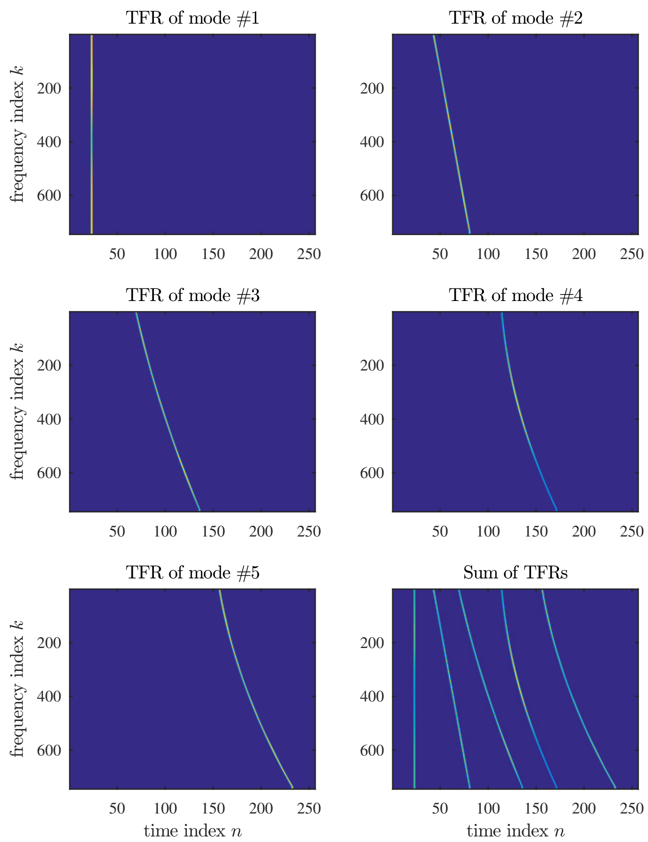

Fig. 4. S-method decomposition of the components

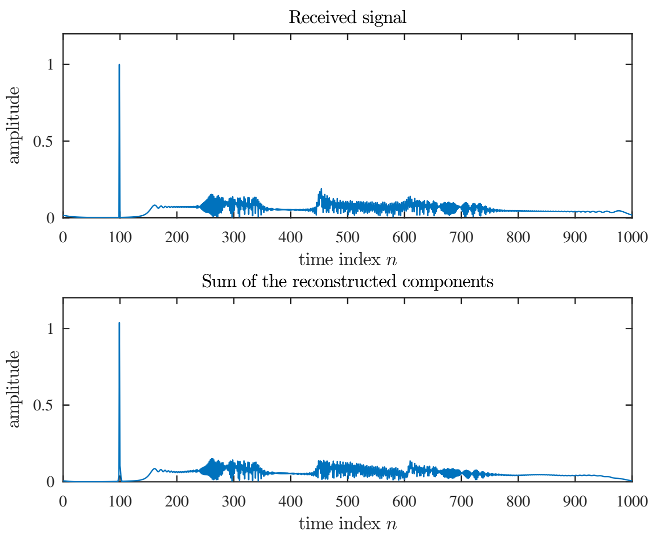

Fig. 5. The original signal (top) and the reconstructed signal (bottom)

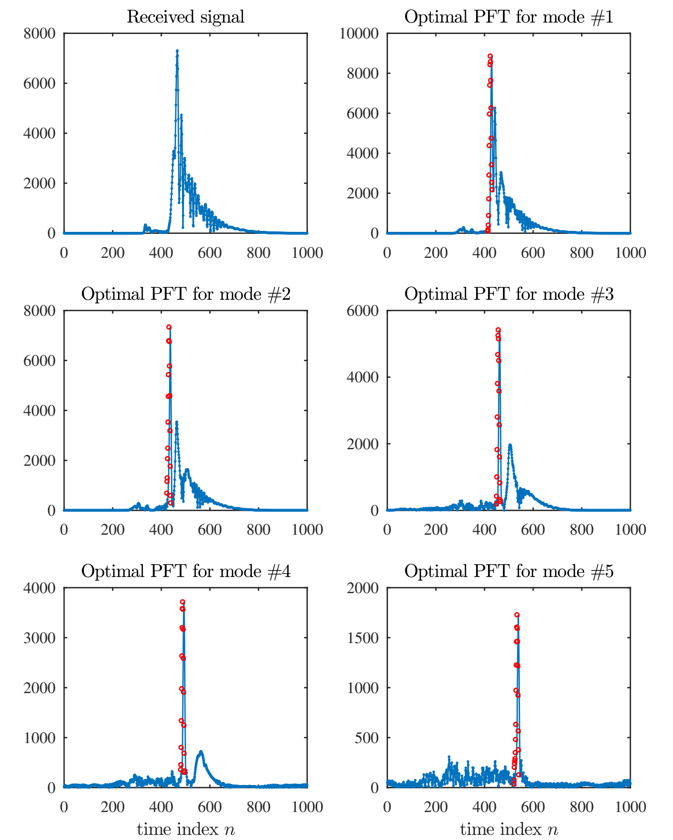

Fig. 6. Decomposition of the modes in time-domain: Received signal (top left); Optimal dual PFT for each mode separately. Samples associated to the current mode are marked with red circles.

five modes is also shown Fig. 7 (bottom right subplot). Sum of the decomposed components and amplitudes of individual components are given in Fig. 8.

Example 3

In this example, we will examine the case when the signal is corrupted with strong sinusoidal disturbances (6).

Assume the case same as in Example 2. The received signal without disturbances is shown in Fig. 9 (top subplots). It is assumed that the received signal has disturbances in frequency domain in the form of high-impulses in 15% of the spectrum. The disturbed received signal is illustrated in Fig. 9 (middle subplots).

Firstly, we remove the components which are affected by the noise (disturbances) using hard thresholding. The noisy spectral samples are considered as unavailable. The frequency and time domains of the received signal after filtering are shown in Fig. 9 (bottom subplots).

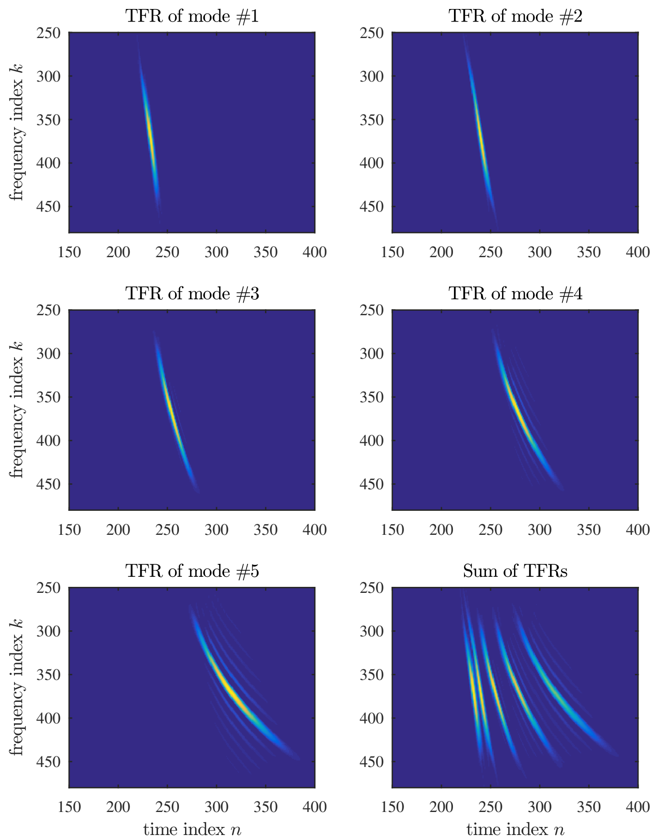

Fig. 7. S-method of the decomposed modes 1, 2, 3, 4 and 5 and sum of the normalized representations of all modes.

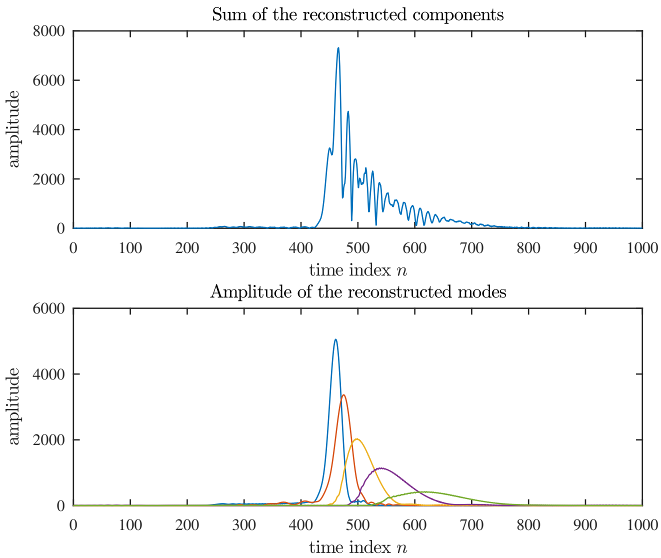

Fig. 8. Sum of the reconstructed modes (top) and amplitudes of individual modes (bottom)

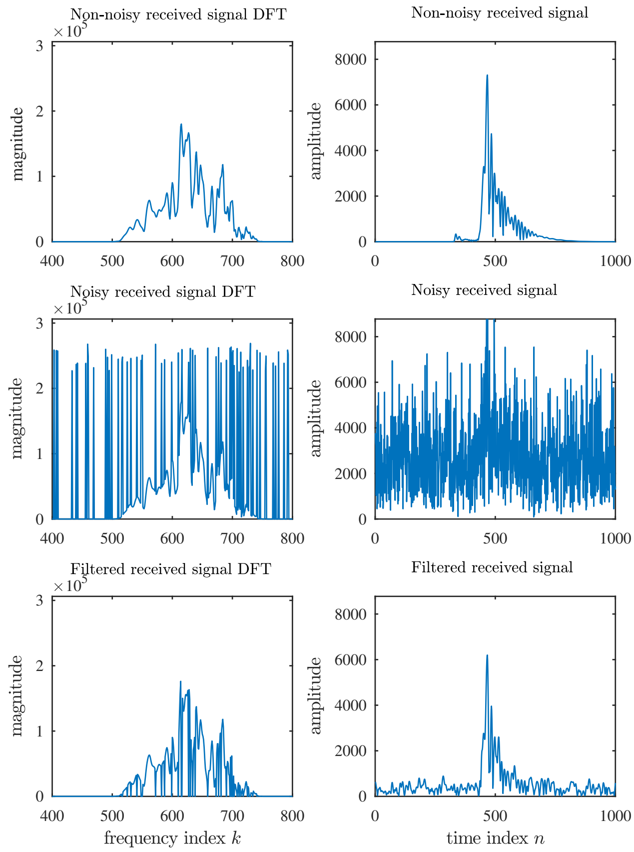

Fig. 9. DFT of received signal (left) and the received signal in time domain (right): without disturbances (top); disturbed (middle) and filtered (bottom)

After that, the parameters are found using a third order DPFT. For the comparison, the received non-noisy signal and the reconstructed signal are shown in Fig. 10.

Since the compressive sensing based reconstruction is able to recover the original values of the disturbed coefficients, the obtained results are almost identical with the results presented in Example 2, when the non-disturbed signal is analyzed.

The presented algorithm belongs to the class of OMP algorithms. Its FPGA realization can be implemented following the one presented in [22]. Other hardware architecture for CS methods presented in [23], [24] can be used also.

VII. Conclusions

Decomposition and reconstruction of acoustic signals are considered. These signals are sparse in the dual polynomial Fourier transform. The analyzed signals are obtained as a result of the transmission through a dispersive underwater channel environment. The received multicomponent signal is decomposed using the dual polynomial Fourier transform. In such a way, individual propagation modes are obtained. The case when the received signal is corrupted with strong

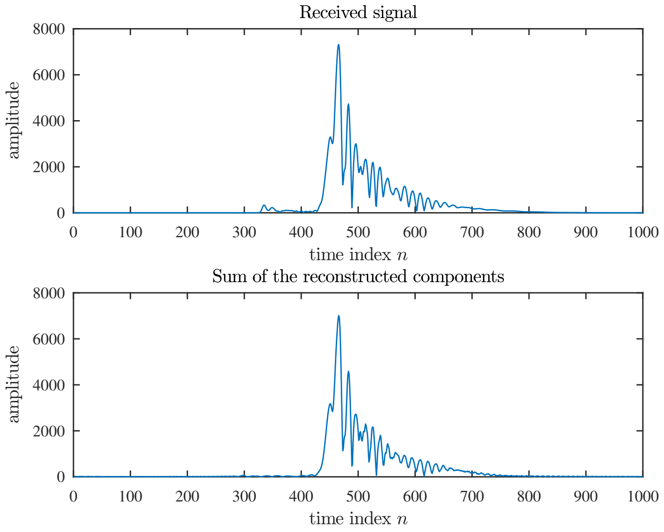

Fig. 10. Comparison: non-noisy received signal (top); signal reconstructed after OMP (bottom)

impulses in the frequency domain, corresponding to harmonic disturbances in the time domain, is analyzed. The original signal is reconstructed using compressive sensing methods, with the dual polynomial Fourier transform sparsity assumption. Further research on this topic could be dedicated to the hardware implementation of the presented method.

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