24th International Conference on Information Technology (IT)
Zabljak, 18 – 22 February, 2020

# On Polynomial Approximations of Spectral Windows in Vertex-Frequency Representations

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***Abstract*— Vertex-frequency analysis (VF) can be considered as a generalization of the classical time-frequency analysis. It provides tools and algorithms aiming to characterize the localized signal behavior in the joint vertex-frequency domain. Localized Graph Fourier Transform (LGFT) is an example of such a tool, with a role in the graphs signal processing which is equivalent to the role of the Short-time Fourier transform in traditional signal processing. Bearing in mind the rapidly increasing amounts of data and large dimensions of graphs related to practical applications, the calculation complexity of each tool for the spectral analysis of signals on graphs shall be continuously revisited. As they provide the possibility to calculate VF representations using only local neighborhoods of vertices, without the need for the eigendecomposition, polynomial approximations of spectral windows are commonly used in practice, mostly in the form of the Chebychev approximation. This paper revisits this choice, compares it with two other polynomial approximation approaches, and investigates their influence on the VF-based graph signal analysis and inversion.**

## I. Introduction

Graph signal processing attracted a significant research interest recently [1]–[5]. With graphs acting as signal domains, this particular signal processing framework can be viewed as an extension and generalization of the traditional theory. Large graphs naturally fit into the Big Data context, now concerning the fast-growing number of practical applications. Therefore, the possibility to analyze not the entire graph signal, but rather its local behavior, has become very important for various applications and it is delivered by the vertex-frequency analysis [6]–[13]. VF representations aim to characterize the localized signal behavior in the joint vertex-frequency domain, therefore establishing a natural analogy with the classical time-frequency analysis [14]–[16]. Polynomial LGFT approximations provide the possibility to calculate the LGFT based on the low-order local neighborhood for each considered vertex. As Chebyshev polynomials are dominantly exploited in this context, we investigate how the choice of an alternative polynomial basis influences the results in the context of VF analysis and inversion.

The paper is organized as follows. The basic theory of graphs signal processing is outlined in Section II, while one of the basic VF representations is presented in Section III. Polynomial LGFT approximations are considered in Section IV. Numerical results are given in Section V, while the paper is concluded with Section VI.

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## II. Background Theory

Consider a graph with $N$ vertices, $n \in \mathcal{V} = \{1, 2, \dots, N\}$, connected with edges having associated weights $W_{mn}$, $n = 1, 2, \dots, N$, $m = 1, 2, \dots, N$. Zero-valued weights $W_{mn} = 0$ indicate that the edges $m$ and $n$ are not connected. Weights $W_{mn}$ are used to form the weight matrix $\mathbf{W}$ of size $N \times N$. In the case of unweighted graphs, nonzero elements in $\mathbf{W}$ are equal to unity, while this specific form of the weight matrix is known as the adjacency matrix, $\mathbf{A}$.

In addition to the weight matrix, $\mathbf{W}$, and the adjacency matrix, $\mathbf{A}$, graphs are commonly described by the means of the Laplacian matrix, $\mathbf{L} = \mathbf{D} - \mathbf{W}$, where $\mathbf{D}$ represents the diagonal weight matrix, whose elements on the matrix diagonal are $D_{nn} = \sum_m W_{mn}$.

Spectral analysis of graphs is most commonly based on the eigendecomposition of the graph Laplacian $\mathbf{L}$ or the adjacency matrix, $\mathbf{A}$. The eigenvectors, $\mathbf{u}_k$, and the eigenvalues, $\lambda_k$, of the graph Laplacian are calculated based on the usual definition $\mathbf{L}\mathbf{u}_k = \lambda_k \mathbf{u}_k$, $k = 1, 2, \dots, N$.

Consider a graphs signal, $\mathbf{x} = [x(1), x(2), \dots, x(N)]^T$. The graph Fourier transform (GFT) of such signal is defined as the expansion onto a set of orthonormal basis functions, $\mathbf{u}_k$, $k = 1, 2, \dots, N$, that is

$$X(k) = \mathrm{GFT}\{x(n)\} = \sum_{n=1}^{N} x(n)\, u_k(n). \tag{1}$$

where $\mathbf{u}_k$, with elements $u_k(n)$, $n = 1, 2, \dots, N$, represent eigenvectors of the graph Laplacian, $\mathbf{L}$. The corresponding inverse graph Fourier transform (IGFT) is then defined as

$$x(n) = \mathrm{IGFT}\{X(k)\} = \sum_{k=1}^{N} X(k)\, u_k(n). \tag{2}$$

## III. Localized graph Fourier transform

As in its classical counterpart, the localized graph Fourier transform (LGFT), admits frequency localization using a spectral domain window. If such a window, centered at the spectral index $k$, is denoted by $H(k-p)$, the LGFT is defined as the inverse GFT of $X(k)$, localized by the spectral window:

$$S(m,k) = \sum_{p=1}^{N} X(p) H_k(\lambda_p)\, u_p(m), \tag{3}$$

that is, the spectral domain LGFT form in (3) can be implemented using band-pass transfer functions $H_k(\lambda_p) = H(k-p)$. If the condition $\sum_{k=0}^{K-1} H_k^2(\lambda_p) = 1$ is satisfied for all $\lambda_p$, $p=1,2,\dots,N$, the inversion is performed using

$$x(n)=\sum_{m=1}^{N}\sum_{k=0}^{K-1}S(m,k)\mathcal{H}_{m,k}(n) \tag{4}$$

where $\mathcal{H}_{m,k}(n)=\sum_{p=1}^{N}H_k(\lambda_p)u_p(m)u_p(n)$. One of such band-pass functions suitable for the inversion is given by

$$H_k(\lambda)=\begin{cases}\sin\left(\frac{\pi}{2}v_x\left(\frac{a_k}{b_k-a_k}(\frac{\lambda}{a_k}-1)\right)\right), & \text{for } a_k<\lambda\le b_k\\ \cos\left(\frac{\pi}{2}v_x\left(\frac{b_k}{c_k-bk}(\frac{\lambda}{b_k}-1)\right)\right), & \text{for } b_k<\lambda\le c_k\\ 0, & \text{elsewhere,}\end{cases} \tag{5}$$

with $a_{k+1}=b_k$, $b_{k+1}=c_k$ and the initial and the last intervals defined as $a_k=a_{k-1}+\frac{\lambda_{\max}}{K-1}$, $b_k=a_k+\frac{\lambda_{\max}}{K-1}$, $c_k=a_k+2\frac{\lambda_{\max}}{K-1}$ with $a_1=0$ and $\lim_{\lambda\to 0}(a_1/\lambda)=1$, for uniform bands within $0\le\lambda\le\lambda_{\max}$. The sine and cosine functions are not differentiable at the interval-end points. Therefore, the argument, $x=\frac{a_k}{b_k-a_k}(\frac{\lambda}{a_k}-1)$, is commonly mapped using a polynomial $v_x(x)$. However, without loss of generality, in his paper we use $v_x(x)=x$.

## IV. Polynomial LGFT Approximation

As $H_k(\lambda_p)$ can be observed as a transfer function of a band-pass graph system, centered at an eigenvalue, $\lambda_k$, and around it, it can be assumed that it has the polynomial form

$$H_k(\lambda_p)=h_{0,k}+h_{1,k}\lambda_p+\cdots+h_{M-1,k}\lambda_p^{M-1}, \tag{6}$$

where $k=0,1,\dots,K-1$, $K$ is the number of bands and $M$ is the polynomial order.

This further implies that, for a given vertex $m$ and spectral index $K$, $S(m,k)$ in (3), assumes the following vector form

$$\mathbf{s}_k=\mathbf{U}H_k(\mathbf{\Lambda})\mathbf{U}^T\mathbf{x}=H_k(\mathbf{L})\mathbf{x}=\sum_{p=0}^{M-1}h_{p,k}\mathbf{L}^p\,\mathbf{x}, \tag{7}$$

where $\mathbf{s}_k$ is the column vector with elements $S(m,k)$, $m=1,2,\dots,N$. The polynomial form in (7) uses only the $(M-1)$-neighborhood in the calculation of the LGFT for each considered vertex, without the need for eigendecomposition analysis. Therefore, it significantly reduces the computational cost. A polynomial approximation of an order $(M-1)$, can be used to implement bandpass LGFT functions, $H_k(\lambda)$, $k=0,1,\dots,K-1$.

The *Chebyshev approximation* is based on Chebyshev polynomials which are defined by

$$T_0(z)=1,\ T_1(z)=z,\dots,T_m(z)=2zT_{m-1}(z)-T_{m-1}(z).$$

for $m\ge 2$ and $-1\le z\le 1$.

The finite $(M-1)$-order of the Chebyshev polynomials

$$\bar{P}_{k,M-1}(\lambda)=\frac{c_{k,0}}{2}+\sum_{m=1}^{M-1}c_{k,m}\bar{T}_m(\lambda), \tag{8}$$

where scaling and shifting of the form $\bar{T}_m(\lambda)=T_m(2\lambda/\lambda_{\max}-1)$ is used to map the argument from the interval, $0\le\lambda\le\lambda_{\max}$, to the interval from $-1$ to 1. The polynomial coefficients are calculated using the Chebyshev polynomial inversion property as $c_{k,m}=\frac{2}{\pi}\int_{-1}^{1}H_k((z+1)\lambda_{\max}/2)T_m(z)dz/\sqrt{1-z^2}$.

Therefore, according to (7), the LGFT can be calculated as $\mathbf{s}_k=\bar{P}_{k,M-1}(\mathbf{L})\mathbf{x}$, for $k=0,1,2,\dots,K-1$, with

$$\begin{aligned}\bar{P}_{k,M-1}(\mathbf{L})&=\frac{c_{k,0}}{2}+\sum_{m=1}^{M-1}c_{k,m}\bar{T}_m(\mathbf{L}), \qquad (9)\\ &=h_{0,k}\mathbf{I}+h_{1,k}\mathbf{L}+h_{2,k}\mathbf{L}^2+\cdots+h_{(M-1),k}\mathbf{L}^{M-1}.\end{aligned}$$

The *least squares approximation using monomials* (in this paper it will be abbreviated as LS) assumes that transfer functions $H_k(\lambda)$ are approximated using polynomial $P_{k,M-1}^{LS}(\lambda)=\bar{\alpha}_{0,k}+\bar{\alpha}_{1,k}\lambda+\cdots+\bar{\alpha}_{M-1,k}\lambda^{M-1}$ such that squared error $\int_0^{\lambda_{\max}}|H_k(\lambda)-P_{k,M-1}^{LS}(\lambda)|^2d\lambda$ is minimized. The interval $0\le\lambda\le\lambda_{\max}$ can be, as in the case of Chebyshev approximation, normalized and mapped to $[-1,1]$, to ensure the stability of the calculation, using $z=\frac{2\lambda-\lambda_{\max}}{\lambda_{\max}}$. The introduction of $s_m=\int_{-1}^{1}z^m dz$, $m=0,1,\dots,2M-2$ and $b_m=\int_{-1}^{1}z^mH_k((z+1)\lambda_{\max}/2)dz$, $m=0,1,\dots,M-1$, leads to

$$\begin{bmatrix}s_0 & s_1 & \dots & s_{M-1}\\ s_1 & s_2 & \dots & s_M\\ \vdots & \vdots & \dots & \vdots\\ s_{M-1} & s_M & \dots & s_{2M-2}\end{bmatrix}\begin{bmatrix}\alpha_{0,k}\\ \alpha_{1,k}\\ \vdots\\ \alpha_{M-1,k}\end{bmatrix}=\begin{bmatrix}b_0\\ b_1\\ \vdots\\ b_{M-1}\end{bmatrix},$$

or, in the matrix form, $\mathbf{Sa}=\mathbf{b}$. By solving this system, we get the coefficients $\alpha_{0,k},\alpha_{1,k},\dots,\alpha_{M-1,k}$. As $\lambda=0.5(z+1)\lambda_{\max}$, we have $\sum_{m=0}^{M-1}\alpha_{m,k}z^m=\sum_{m=0}^{M-1}\bar{\alpha}_{i,k}\lambda^m$. The approximation is then

$$P_{k,M-1}^{LS}(\mathbf{L})=\bar{\alpha}_{0,k}\mathbf{I}+\bar{\alpha}_{1,k}\mathbf{L}+\bar{\alpha}_{2,k}\mathbf{L}^2+\cdots+\bar{\alpha}_{(M-1),k}\mathbf{L}^{M-1},$$

based on which $\mathbf{s}_k=\bar{P}_{k,M-1}^{LS}(\mathbf{L})\mathbf{x}$, for $k=0,1,\dots,K-1$.

The *least squares approximation using Legendre polynomials* assumes minimization of $\int_0^{\lambda_{\max}}|H_k(\lambda)-P_{k,M-1}^{Leg}(\lambda)|^2d\lambda$ where $P_{k,M-1}^{Leg}(\lambda)=\bar{\beta}_{0,k}\phi_0(\lambda)+\bar{\beta}_{1,k}\phi_1(\lambda)+\cdots+\bar{\beta}_{M-1,k}\phi_{M-1}(\lambda)$. Polynomials $\phi_0(z)=1$, $\phi_1(z)=z$, $\phi_2(z)=z^2-1/3,\dots$ are called Legendre polynomials and they satisfy Bonnet's recursion formula

$$(m+1)\phi_{m+1}(z)=(2m+1)z\phi_m(z)-m\phi_{m-1}(z). \tag{10}$$

The normalization and shift, using $z=\frac{2\lambda-\lambda_{\max}}{\lambda_{\max}}$, is done, such that $\bar{\phi}_m(\lambda)=\phi_m(2\lambda/\lambda_{\max}-1)$. For each $m=0,1,\dots,M-1$ we calculate $C_m=\int_{-1}^{1}\bar{\phi}_m^2(z)dz$, upon which polynomial coefficients $\beta_m=\frac{1}{C_m}\int_{-1}^{1}H_k((z+1)\lambda_{\max}/2)\bar{\phi}_m(z)dz$ are calculated. As $\lambda=0.5(z+1)\lambda_{\max}$, we have $\sum_{m=0}^{M-1}\beta_{m,k}\phi_m(z)=\sum_{m=0}^{M-1}\bar{\beta}_{i,k}\lambda^m$, which enables us to form, in analogy to the previous approximations,

$$P_{k,M-1}^{Leg}(\mathbf{L})=\bar{\beta}_{0,k}\mathbf{I}+\bar{\beta}_{1,k}\mathbf{L}+\bar{\beta}_{2,k}\mathbf{L}^2+\cdots+\bar{\beta}_{(M-1),k}\mathbf{L}^{M-1}.$$

## V. Numerical results

Observe a graph with $N=100$ vertices, along with the graph signal $x(n)$, as presented in Fig. 1. For the calculation

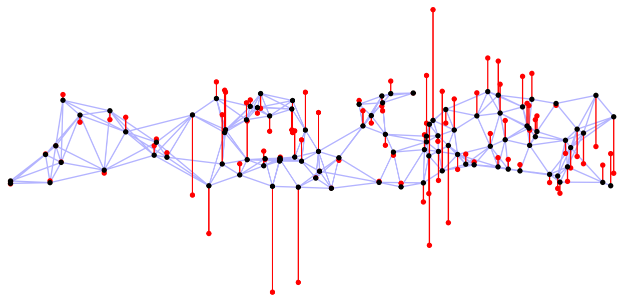

Fig. 1. Observed graph and graph signal.

TABLE I
MSE IN VF INVERSION AND IN POLYNOMIAL APPROXIMATION OF VF FREQUENCY WINDOW. POLYNOMIALS ARE OF ORDER $M = 5$.

| | Chebyshev | | Legendre | | LS | |
|---|---|---|---|---|---|---|
| $K$ | $E_c$ | $E_c^w$ | $E_l$ | $E_l^w$ | $E_{LS}$ | $E_{LS}^w$ |
| 6 | -23.18 | -15.08 | -20.04 | -12.90 | -28.21 | -17.81 |
| 8 | -20.23 | -14.26 | -19.03 | -13.35 | -23.6 | -16.03 |
| 10 | -18.62 | -13.92 | -18.05 | -13.51 | -21.05 | -15.13 |
| 12 | -17.61 | -13.97 | -17.29 | -13.76 | -19.48 | -14.91 |
| 14 | -16.92 | -14.19 | -16.71 | -14.05 | -18.42 | -14.96 |
| 16 | -16.41 | -14.32 | -16.27 | -14.24 | -17.66 | -14.94 |
| 18 | -16.02 | -14.59 | -15.91 | -14.53 | -17.09 | -15.13 |
| 20 | -15.72 | -14.82 | -15.62 | -14.78 | -16.64 | -15.30 |

of the LGFT, $K$ transfer functions (spectral windows) of the form (5) are exploited. As efficient implementations of VF representations require polynomial approximation of relatively small order $M-1$, we investigate how the choice of the polynomial influences the results. To this aim, Chebyschev, Legendre and LS approximations are considered. Original transfer functions, $H_k(\lambda)$, for $K = 16$ frequency bands, and their polynomial approximations are shown in Fig. 2, for the realistic and practically relevant case of $M = 6$. The corresponding LGFTs are shown in Fig. 3. The LS polynomial approximation based VF calculation provides more concentrated representation (last panel) than in the cases of the Chebyshev and Legendre spectral window approximations (second and third panel, respectively). This is in accordance with the results from Fig. 2. The corresponding

TABLE II
MSE IN VF INVERSION AND IN POLYNOMIAL APPROXIMATION OF VF FREQUENCY WINDOW. POLYNOMIALS ARE OF ORDER $M = 6$.

| | Chebyshev | | Legendre | | LS | |
|---|---|---|---|---|---|---|
| $K$ | $E_c$ | $E_c^w$ | $E_l$ | $E_l^w$ | $E_{LS}$ | $E_{LS}^w$ |
| 6 | -27.16 | -17.58 | -20.99 | -13.53 | -30.40 | -20.21 |
| 8 | -23.08 | -15.85 | -20.34 | -14.18 | -25.56 | -17.74 |
| 10 | -20.73 | -14.99 | -19.28 | -14.26 | -22.66 | -16.28 |
| 12 | -19.25 | -14.81 | -18.38 | -14.4 | -20.8 | -15.8 |
| 14 | -18.24 | -14.88 | -17.66 | -14.6 | -19.52 | -15.67 |
| 16 | -17.51 | -14.88 | -17.10 | -14.7 | -18.59 | -15.5 |
| 18 | -16.96 | -15.08 | -16.65 | -14.95 | -17.89 | -15.62 |
| 20 | -16.53 | -15.26 | -16.28 | -15.15 | -17.35 | -15.72 |

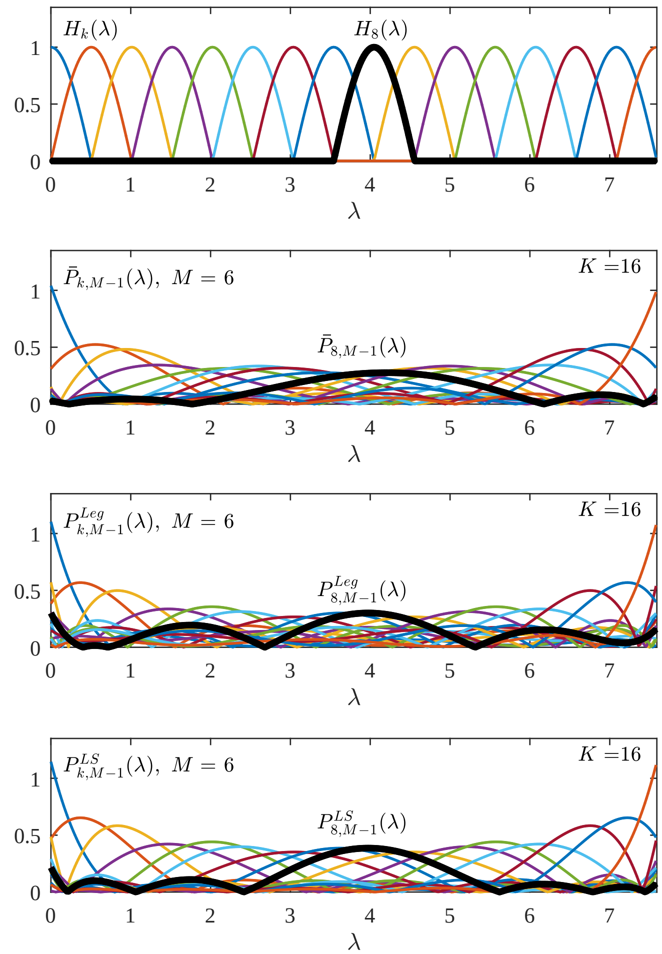


Fig. 2. Exemplar transfer functions in the spectral domain and their polynomial approximations. Polynomials with $M = 6$ are used in the approximation. Top to bottom: original frequency window, Chebyshev, Legendre and LS polynomial approximation.

inversions are shown in Fig. 4.

The described phenomena are investigated with more details in the following experiment. For the number of frequency bands varied from $K = 6$ to $K = 20$, with step 2, we calculate the VF representations using: (i) the original transfer function $H_k(\lambda)$, (ii) Chebyshev, (iii) Legendre, and (iv) the LS based polynomial approximation. The MSEs between the signals obtained trough the inversion of the approximated LGFTs and the inversion with $H_k(\lambda)$, as well as MSEs in the window approximations, are calculated for $M = 5, 6$ and 7, and shown in Tables I, II and III. The MSEs in the inversion calculation are given in columns $E_c$, $E_l$ and $E_{LS}$ for the case of Chebyshev, Legendre and the LS approximations, respectively, while the corresponding MSEs in the window calculations are given in columns $E_c^w$, $E_l^w$ and $E_{LS}^w$. The least squares approximation using monomials (LS) provided the lowest MSEs in all considered cases.

## VI. CONCLUSION

We have studied the VF representations obtained using the polynomial approximations of frequency windows and used

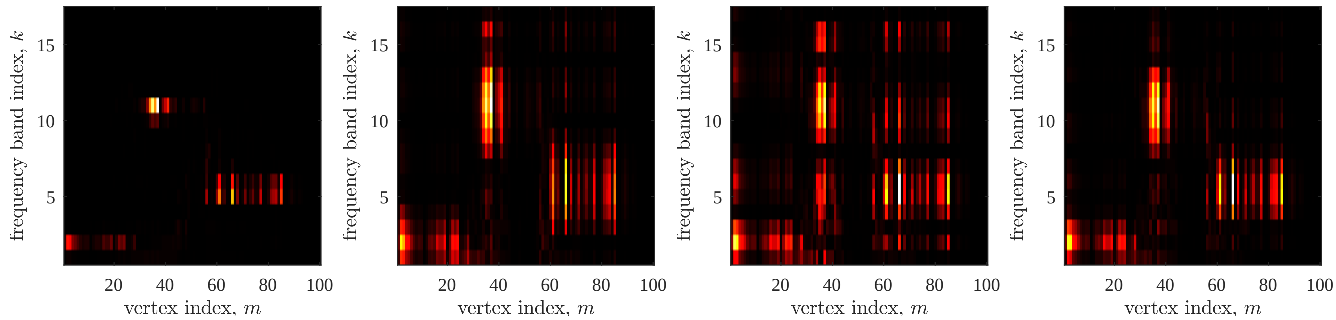


Fig. 3. Vertex-frequency representations calculated based on: exactly calculated frequency localization windows from Fig. 2 (first panel), Chebyshev, Legendre and least squares polynomial approximations from Fig. 2 (second, third and last panel).

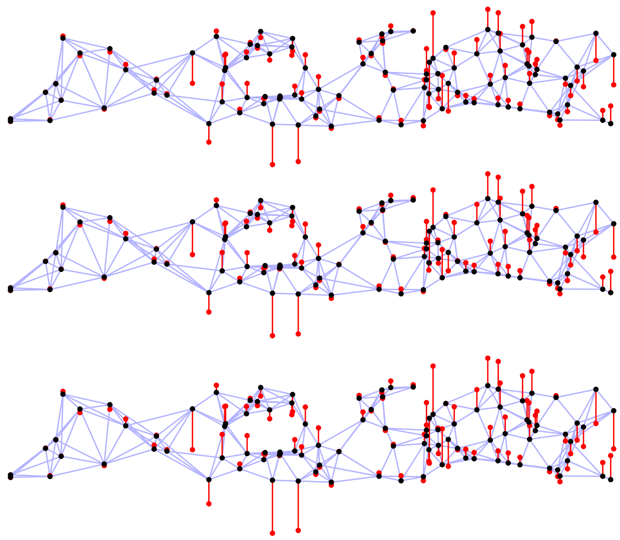

Fig. 4. Signal inversion results based on VF representations from Fig. 3. First panel – inversion of LGFT is calculated based on the Chebyshev polynomial approximation of spectral window, second panel – inversion of LGFT calculated based on the Legendre polynomial approximation), third panel –inversion of LGFT calculated based on the LS approximation.

for the localized characterization and processing of graph signals. Numerical comparisons indicate that the standard least-squares polynomial approximation of frequency window set used in the VF calculations leads to slight improvements, when compared with the results based on the commonly used Chebyshev approximation.

TABLE III

MSE IN VF INVERSION AND IN POLYNOMIAL APPROXIMATION OF VF FREQUENCY WINDOW. POLYNOMIALS ARE OF ORDER $M = 7$.

| | Chebyshev | | Legendre | | LS | |
|---|---|---|---|---|---|---|
| $K$ | $E_c$ | $E_c^w$ | $E_l$ | $E_l^w$ | $E_{LS}$ | $E_{LS}^w$ |
| 6 | -28.84 | -19.81 | -22.03 | -13.77 | -34.44 | -21.48 |
| 8 | -24.73 | -17.59 | -21.7 | -14.72 | -28.05 | -19.09 |
| 10 | -22.18 | -16.09 | -20.67 | -14.84 | -24.63 | -17.24 |
| 12 | -20.50 | -15.65 | -19.65 | -14.95 | -22.43 | -16.57 |
| 14 | -19.31 | -15.55 | -18.79 | -15.09 | -20.89 | -16.31 |
| 16 | -18.44 | -15.42 | -18.10 | -15.12 | -19.77 | -16.00 |
| 18 | -17.78 | -15.56 | -17.54 | -15.32 | -18.91 | -16.07 |
| 20 | -17.26 | -15.67 | -17.09 | -15.48 | -18.25 | -16.10 |

## REFERENCES


[1] A. Sandryhaila and J. M. Moura, "Discrete signal processing on graphs: Frequency analysis," *IEEE Transactions on Signal Processing*, vol. 62, no. 12, pp. 3042–3054, 2014.

[2] A. Ortega, P. Frossard, J. Kovačević, J. M. Moura, and P. Vandergheynst, "Graph signal processing: Overview, challenges, and applications," *Proc. of the IEEE*, vol. 106, no. 5, pp. 808–828, 2018.

[3] C. J. Quinn, N. Kiyavash, and T. P. Coleman, "Directed information graphs," *IEEE Trans. IT*, vol. 61, no. 12, pp. 6887–6909, 2015.

[4] R. Hamon, P. Borgnat, P. Flandrin, and C. Robardet, "Transformation from graphs to signals and back," in *Vertex-Frequency Analysis of Graph Signals*, pp. 111–139, Springer, 2019.

[5] LJ. Stanković, M. Daković, M. Brajović, and D. Mandic, "A p-Laplacian Inspired Method for Graph Cut,"*27th Telecommunications Forum TELFOR 2019*, Belgrade, Serbia, Nov. 2019.

[6] D. I. Shuman, B. Ricaud, and P. Vandergheynst, "Vertex-frequency analysis on graphs," *Applied and Computational Harmonic Analysis*, vol. 40, no. 2, pp. 260–291, 2016.

[7] D. I. Shuman, B. Ricaud, and P. Vandergheynst, "A windowed graph Fourier transform," in *Proc. IEEE SSP Workshop* , pp. 133–136, 2012.

[8] X.-W. Zheng, Y. Y. Tang, J.-T. Zhou, H.-L. Yuan, Y.-L. Wang, L.-N. Yang, and J.-J. Pan, "Multi-windowed graph Fourier frames," in *Proc. IEEE Int. Conf. Machine Learn. Cyber.*, vol. 2, pp. 1042–1048, 2016.

[9] M. Tepper and G. Sapiro, "A short-graph Fourier transform via personalized pagerank vectors," in *Proc. IEEE Int. Conf. on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 4806–4810, 2016.

[10] L. Stanković, M. Daković, and E. Sejdić, "Vertex-frequency energy distributions," in *Vertex-Frequency Analysis of Graph Signals*, pp. 377–415, Springer, 2019.

[11] L. Stanković, M. Daković, and E. Sejdić, "Vertex-frequency analysis: A way to localize graph spectral components [lecture notes]," *IEEE Signal Processing Magazine*, vol. 34, no. 4, pp. 176–182, 2017.

[12] T. Cioacă, B. Dumitrescu, and M.-S. Stupariu, "Graph-based wavelet multiresolution modeling of multivariate terrain data," in *Vertex-Frequency Analysis of Graph Signals*, pp. 479–507, Springer, 2019.

[13] H. Behjat and D. Van De Ville, "Spectral design of signal-adapted tight frames on graphs," in *Vertex-Frequency Analysis of Graph Signals*, pp. 177–206, Springer, 2019.

[14] L. Stanković, M. Daković, and T. Thayaparan, *Time-frequency signal analysis with applications*. Artech House, 2014.

[15] L. Stanković, M. Brajović, M. Daković, and D. Mandic, "On the Decomposition of Multichannel Nonstationary Multicomponent Signals," *Signal Processing*, Vol. 167, Feb. 2020.

[16] L. Stanković, D. Mandic, M. Daković, and M. Brajović, "Time-frequency decomposition of multivariate multicomponent signals," *Signal Processing*, , Volume 142, January 2018, pp. 468–479,